

R. W. Gatterdam: The Higman theorem for primitive-recursive groups—A preliminary report (6 pp.).

S. Lipschutz: On the word problem and T -fourth-groups (10 pp.).

J. McCool and A. Pietrowski: On a conjecture of W. Magnus (4 pp.).

R. McKenzie and R. J. Thompson: An elementary construction of unsolvable word problems in group theory (18 pp.).

T. G. McLaughlin: A non-enumerability theorem for infinite classes of finite structures (4 pp.).

A. W. Mostowski: Uniform algorithms for deciding group theoretic problems (28 pp.).

B. H. Neumann: The isomorphism problem for algebraically closed groups (10 pp.).

H. Schiek: Equations over groups (6 pp.).

L. Wos and G. Robinson: Maximal modules and refutation completeness: semidecision procedures in automatic theorem proving.

It is not possible to comment on these papers in a general review, but it should at least be mentioned that the paper by B. H. Neumann is preparing the ground for the recent Boone-Higman theorem which characterises the intersection of the algebraically closed groups.

An appendix contains fifty research problems. With one exception, the name of the person raising the question is not given. In several cases, it would be very hard to guess since the problem is “in the air”.

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34 [4].—HANS J. STETTER, *Analysis of Discretization Methods for Ordinary Differential Equations*, Springer Verlag, New York, 1973, xvi + 338 pp., 24 cm. Price \$44.40.

In the eleven years since Henrici's now classical book “Discrete Variable Methods in Ordinary Differential Equations” (J. Wiley & Sons, 1962) appeared, there have been a plethora of new and variants of existing methods in the literature, particularly for the initial value problem which is the major concern of the exhaustive analysis in this excellent book. Professor Stetter places these methods in a solid mathematical framework, and in doing so, extends the existing theory and presents many new results. To quote from the preface: “This text is *not* an introduction to the use of finite-difference methods; rather, it assumes that the reader has a knowledge of the field, preferably including practical experience in the computational solution of differential equations,” and from the end of Chapter 1: “In the remainder of this treatise the many practical aspects of the numerical solution of ordinary

differential equations by discretization methods will *not* be considered, . . .” My sole objection to the first statement is the word “text”. This treatise provides a fundamental and complete background for those concerned with the theoretical development of the field. However, in addition to assuming a good intuitive grasp of the practical aspects necessary to put the results in perspective, it assumes a moderate mathematical sophistication. This is a book neither for an engineer wanting to know “How to solve it”, nor for a beginning student. (Further, its price makes me unwilling to require it as a text, even for an advanced seminar following a course based on, for example, Henrici’s text (op. cit.), although it puts the current state of the field into shape such that a course of this nature would be well-structured and exciting.)

The book starts with an abstract discussion of discretization methods and their stability, consistency and convergence. Particular emphasis is placed on the asymptotic expansion of the local and global discretization error, a topic in which Stetter has made many new contributions, both in previous papers and in the pages of this book. The second chapter specializes the ideas to discretization methods for initial value problems and introduces some important ideas, in particular “strong exponential stability”. This refers to the exponential reducing property of a method for a class of exponentially stable differential equations, and is as close as the book comes to discussing stiff problems. However, it is possible that the approach will provide the tools for an analysis of this currently open area. The four remaining chapters cover virtually all classes of methods. In addition to explicit R-K and linear multistep methods, the analysis is shown to be applicable to implicit R-K, cyclic multistep, predictor-corrector, hybrid (off step points), power series, Groebner-Knapp-Wanner, R-K-Felhberg, Nordsieck and extrapolation methods. Stetter’s asymptotic error theory sheds new light on Butcher’s concept of “effective order”. (Further important results on global asymptotic error estimation can be found in Stetter’s recent paper “Economical Global Error Estimation”, in Proceedings of the International Symposium on Stiff Differential Equations, edited by R. A. Willoughby, expected in 1974, Plenum Press.)

Generally, the analysis is performed assuming a “coherent grid”. This is one in which the grid is specified by $t_n = t_{n-1} + h\theta(t_n)$ where θ is a piecewise constant strictly positive function. The lack of an analysis for more general grid points leaves out an important open area of research, as the vast majority of practical algorithms not only vary the step size frequently, but also vary the method from step to step.

Although criticism of this first class work is presumptuous, permit me to make two observations. Firstly, this book starts with abstractions and then gives examples (but there are an ample number). I feel that it would have been easier to read if the examples had preceded and motivated each definition. Since the reader is assumed to be familiar with discretization methods, they could have provided good introductory material for each concept. Secondly,

it is necessary to read this book from the beginning in order to read any later section. Although there is a lot of cross referencing, there is no table of notations, and by the midpoint there are so many notational conventions (e.g. which type of letter represents a member of which space, which letter represents the local error, etc.) that a quick run through the book is not possible. However, the time spent in carefully reading the first two chapters is amply rewarded by the density of material presented in relatively few pages of the last four chapters.

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35 [5]. — GILBERT STRANG & GEORGE J. FIX, *An Analysis of the Finite Element Method*, Prentice-Hall, Englewood Cliffs, N. J., 1973, xiv + 306 pp., 24 cm. Price \$16.—.

Few are able to present sophisticated analytical developments in a simple, practical, and easily digestible form. Strang and Fix have succeeded! Their book shows the mark of men who have contributed to the development of the field. They get right to the heart of the subject matter, without introducing extraneous mathematical embellishments. Their prose is a pleasure to read, their mathematical taste is elegant, and their numerical techniques are simply described and evaluated for efficiency. This book will be a delightful graduate text and reference work for a broad audience. Its spirit can best be described by quoting from the Preface:

“Its purpose is to explain the effect of each of the approximations that are essential for the finite element technique to be computationally efficient. We list here some of these approximations:

- (1) interpolation of the original physical data
- (2) choice of a finite number of polynomial trial functions
- (3) simplification of the geometry of the domain
- (4) modification of the boundary conditions
- (5) numerical integration of the underlying functional in the variational principle
- (6) roundoff error in the solution of the discrete system.

“These questions are fundamentally mathematical, and so are the authors. Nevertheless, this book is absolutely not intended for the exclusive use of specialists in numerical analysis. On the contrary, we hope it may help establish closer communication between the mathematical engineer and the mathematical analyst. It seems to us that the finite element method provides a special opportunity for this communication: the theory is attractive, the applications are growing, and best of all, the method is so new that the gap