where

$$F_{(n+1)!} = S_{n+1}(F_{n!})[C(f_{n+1}(x))]$$

and $C(f_{n+1}(x))$ in $(F_{n!})_{n+1}$ is the companion matrix of $f_{n+1}(x)$. Remembering the natural isomorphism between $(K_{n!})_{n+1}$ and $(K)_{(n+1)!}$ for arbitrary fields K, we see that $\mathrm{GF}(p^{(n+1)!})$ is a subfield of $(\mathrm{GF}(p))_\infty$ and has order $p^{(n+1)!}$. Furthermore, $\mathrm{GF}(p^{1!}) \subset \cdots \subset \mathrm{GF}(p^{n!}) \subset \mathrm{GF}(p^{(n+1)!})$. We define $\mathrm{GF}(p^{\infty!}) = \bigcup_{n=1}^{\infty} \mathrm{GF}(p^{n!})$ and are done.

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Some Primitive Polynomials of the Third Kind

By Jacob T. B. Beard, Jr.* and Karen I. West

Abstract. This paper gives the first primitive polynomial of the third kind of degree n over $\mathrm{GF}(p^d)$ for each p, d, n satisfying $p < 10^2, p^d < 10^3, p^{dn} < 10^6$.

In the preceding paper [1, Section 3] Beard introduced an exponential representation for $\mathrm{GF}(p^d)$ which allows full use of its multiplicative structure and permits direct rational calculations in $\mathrm{GF}(p^d)$. As indicated in [1, Section 4], such representations are easily and quickly obtained once primitive polynomials of the third kind of degree d

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over GF(p) are known. More generally, in this paper the authors give a primitive polynomial of the third kind of degree n over GF(p^d) for each p, d, n satisfying $p < 10^2$, $p^d < 10^3, p^{dn} < 10^6$. Each GF($p^d$) is the exponential representation of [1, Section 3] as defined by the polynomial given here of degree d over GF(p). Under the natural lexicographic order on GF$[p^d, x]$, each of these polynomials is the first primitive polynomial of the third kind of its degree over GF(p^d). They were obtained through a search option in a software package developed by the authors and based on techniques described in [1]. Exhaustive tables of prime polynomials and the three kinds of primitive polynomials have been compiled for the smaller cases and degrees, portions of which will appear in due time. Those given in this paper are to be found on a microfiche card at the back of this journal.

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Factorization Tables for $x^n - 1$ Over GF(q)

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Abstract. These tables give the complete factorization of $x^n - 1$ over GF(q), $q = p^a$, $2 \leqslant n \leqslant d$ as below, together with the Euler Φ-function of $x^n - 1$ whenever $\Phi(x^n - 1) < 10^8$.

$q = 2$; $d = 32$	$q = 3$; $d = 27$	$q = 11$; $d = 15$
$q = 2^2$; $d = 16$	$q = 3^2$; $d = 15$	$q = 13$; $d = 15$
$q = 2^3$; $d = 16$	$q = 5$; $d = 25, n \neq 23^{\dagger}$	$q = 17$; $d = 15$
$q = 2^4$; $d = 16$	$q = 5^2$; $d = 10$	$q = 19$; $d = 12$
$q = 2^5$; $d = 12$	$q = 7$; $d = 15$	$q = 23$; $d = 10$

This paper gives the complete factorization of $x^n - 1$ over GF(q), $q = p^a$, as indi-

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†*Added at galley by the authors.* $(x^{23} - 1)/(x - 1)$ is prime in GF[5, x] by 33. Theorem in Dickson's *Linear Groups*.

ADDENDUM TO

SOME PRIMITIVE POLYNOMIALS OF THE THIRD KIND (this issue, pp.)

by

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Table A gives the first primitive polynomial of the third kind of degree n over $GF(p^d)$ for each p,d,n satisfying $p < 10^2$, $p^d < 10^3$, $p^{dn} < 10^6$. For complete details on the representation for $GF(p^d)$ see the texts of this paper and "Computing in $GF(q)$", both in this issue.

TABLE A

GF(2): $1 + x + x^2$

$1 + x^2 + x^3$

$1 + x^3 + x^4$

$1 + x + x^2 + x^4 + x^5$

$1 + x^5 + x^6$

$1 + x^6 + x^7$

$1 + x + x^2 + x^7 + x^8$

$1 + x + x^4 + x^8 + x^9$

$1 + x + x^4 + x^9 + x^{10}$

$1 + x + x^3 + x^{10} + x^{11}$

$1 + x + x^2 + x^4 + x^6 + x^{11} + x^{12}$

$1 + x + x^2 + x^{12} + x^{13}$

$1 + x^2 + x^3 + x^{13} + x^{14}$

$1 + x^{14} + x^{15}$

$1 + x + x^4 + x^{15} + x^{16}$

$1 + x + x^3 + x^{16} + x^{17}$

$1 + x + x^5 + x^{17} + x^{18}$

$1 + x + x^5 + x^{18} + x^{19}$

GF(2^2): $1 + 0x + 0x^2$

$2 + 1x + 0x^2 + 0x^3$

$1 + 0x + 0x^3 + 0x^4$

$1 + 0x^4 + 0x^5$

$1 + 0x + 0x^5 + 0x^6$

$1 + 0x^2 + 0x^6 + 0x^7$

$1 + 0x + 0x^2 + 0x^7 + 0x^8$

$1 + 0x^2 + 0x^8 + 0x^9$

GF(2^3): $1 + 0x + 0x^2$

$3 + 0x^2 + 0x^3$

$3 + 0x^3 + 0x^4$

$3 + 0x^2 + 0x^4 + 0x^5$

$1 + 0x^5 + 0x^6$

GF(2^4): $1 + 0x + 0x^2$

$1 + 0x^2 + 0x^3$

$7 + 0x + 0x^3 + 0x^4$

GF(2^5): $1 + 0x + 0x^2$

$5 + 0x^2 + 0x^3$

GF(2^6): $1 + 0x + 0x^2$

$13 + 0x^2 + 0x^3$

GF(2^7): $3 + 0x + 0x^2$

GF(2^8): $1 + 0x + 0x^2$

GF(2^9): $1 + 0x + 0x^2$

GF(3):	$2 + x + x^2$
	$1 + 2x + x^2 + x^3$
	$2 + x^3 + x^4$
	$1 + 2x + x^4 + x^5$
	$2 + x^3 + x^5 + x^6$
	$1 + x^2 + x^6 + x^7$
	$2 + 2x^2 + 2x^3 + x^7 + x^8$
	$1 + 2x + x^8 + x^9$
	$2 + 2x + x^3 + x^9 + x^{10}$
	$1 + 2x + x^{10} + x^{11}$
	$2 + x + x^2 + x^3 + x^{11} + x^{12}$
GF(3^2):	$5 + 0x + 0x^2$
	$7 + 1x + 0x^2 + 0x^3$
	$1 + 0x^3 + 0x^4$
	$1 + 4x + 0x^4 + 0x^5$
	$5 + 0x + 0x^5 + 0x^6$
GF(3^3):	$1 + 0x + 0x^2$
	$2 + 1x + 0x^2 + 0x^3$
	$1 + 0x^3 + 0x^4$
GF(3^4):	$11 + 0x + 0x^2$
	$31 + 1x + 0x^2 + 0x^3$
GF(3^5):	$1 + 0x + 0x^2$
GF(3^6):	$1 + 0x + 0x^2$
GF(5):	$2 + x + x^2$
	$2 + x^2 + x^3$
	$2 + 4x + x^3 + x^4$
	$3 + 2x + x^2 + x^4 + x^5$
	$2 + x^5 + x^6$
	$2 + x^6 + x^7$
	$3 + x + x^2 + x^7 + x^8$
GF(5^2):	$1 + 0x + 0x^2$
	$13 + 0x^2 + 0x^3$
	$13 + 0x^2 + 0x^3 + 0x^4$
GF(5^3):	$9 + 0x + 0x^2$
GF(5^4):	$1 + 0x + 0x^2$
GF(7):	$3 + x + x^2$
	$2 + x + x^2 + x^3$
	$3 + x + x^3 + x^4$
	$4 + x^4 + x^5$
	$5 + 2x + x^2 + x^5 + x^6$
	$4 + 6x + x^6 + x^7$
GF(7^2):	$11 + 0x + 0x^2$
	$13 + 0x^2 + 0x^3$
GF(7^3):	$5 + 0x + 0x^2$

GF(11):	$7 + x + x^2$	GF(29):	$3 + x + x^2$
	$3 + x^2 + x^3$		$3 + x^2 + x^3$
	$8 + x^3 + x^4$		$2 + x^3 + x^4$
	$4 + x + x^4 + x^5$	GF(29^2):	$13 + 0x + 0x^2$
GF(11^2):	$1 + 0x + 0x^2$	GF(31):	$12 + x + x^2$
GF(13):	$2 + x + x^2$		$9 + x^2 + x^3$
	$2 + x^2 + x^3$		$13 + x^3 + x^4$
	$2 + x + x^3 + x^4$	GF(31^2):	$1 + 0x + 0x^2$
	$6 + x + x^4 + x^5$	GF(37):	$5 + x + x^2$
GF(13^2):	$1 + 0x + 0x^2$		$17 + x^2 + x^3$
GF(17):	$3 + x + x^2$		$22 + x^3 + x^4$
	$7 + x^2 + x^3$	GF(41):	$12 + x + x^2$
	$7 + x + x^3 + x^4$		$11 + x^2 + x^3$
	$5 + x^4 + x^5$		$26 + x^3 + x^4$
GF(17^2):	$19 + 0x + 0x^2$	GF(43):	$3 + x + x^2$
GF(19):	$2 + x + x^2$		$9 + x^2 + x^3$
	$6 + x^2 + x^3$		$3 + x + x^3 + x^4$
	$2 + x^3 + x^4$	GF(47):	$13 + x + x^2$
GF(19^2):	$1 + 0x + 0x^2$		$2 + x^2 + x^3$
GF(23):	$7 + x + x^2$		$5 + x^3 + x^4$
	$6 + x^2 + x^3$	GF(53)	$5 + x + x^2$
	$20 + x^3 + x^4$		$2 + x^2 + x^3$
GF(23^2):	$7 + 0x + 0x^2$	GF(59)	$2 + x + x^2$
			$9 + x^2 + x^3$

GF(61):	$2 + x + x^2$	GF(79):	$3 + x + x^2$
	$6 + x^2 + x^3$		$2 + x^2 + x^3$
GF(67):	$12 + x + x^2$	GF(83):	$2 + x + x^2$
	$6 + x^2 + x^3$		$11 + x^2 + x^3$
GF(71):	$11 + x + x^2$	GF(89):	$6 + x + x^2$
	$8 + x^2 + x^3$		$6 + x^2 + x^3$
GF(73):	$11 + x + x^2$	GF(97):	$5 + x + x^2$
	$5 + x^2 + x^3$		$5 + x^2 + x^3$