over GF(p) are known. More generally, in this paper the authors give a primitive polynomial of the third kind of degree n over GF(p^d) for each p, d, n satisfying $p < 10^2$, $p^d < 10^3, p^{dn} < 10^6$. Each GF($p^d$) is the exponential representation of [1, Section 3] as defined by the polynomial given here of degree d over GF(p). Under the natural lexicographic order on GF$[p^d, x]$, each of these polynomials is the first primitive polynomial of the third kind of its degree over GF(p^d). They were obtained through a search option in a software package developed by the authors and based on techniques described in [1]. Exhaustive tables of prime polynomials and the three kinds of primitive polynomials have been compiled for the smaller cases and degrees, portions of which will appear in due time. Those given in this paper are to be found on a microfiche card at the back of this journal.

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1. J. T. B. BEARD, JR., "Computing in GF(q)," *Math. Comp.*, v. 28, 1974, pp. 1159–1166.

Factorization Tables for $x^n - 1$ Over GF(q)

By Jacob T. B. Beard, Jr.* and Karen I. West

Abstract. These tables give the complete factorization of $x^n - 1$ over GF(q), $q = p^a, 2 \leqslant n \leqslant d$ as below, together with the Euler Φ-function of $x^n - 1$ whenever $\Phi(x^n - 1) < 10^8$.

$q = 2;$ $d = 32$	$q = 3;$ $d = 27$	$q = 11; d = 15$
$q = 2^2; d = 16$	$q = 3^2; d = 15$	$q = 13; d = 15$
$q = 2^3; d = 16$	$q = 5;$ $d = 25, n \neq 23^\dagger$	$q = 17; d = 15$
$q = 2^4; d = 16$	$q = 5^2; d = 10$	$q = 19; d = 12$
$q = 2^5; d = 12$	$q = 7;$ $d = 15$	$q = 23; d = 10$

This paper gives the complete factorization of $x^n - 1$ over GF(q), $q = p^a$, as indi-

cated in the abstract. These tables are to be found at the back of this journal on a micro-fiche card. In addition, the generalized Euler Φ-function is given whenever $\Phi(x^n - 1) < 10^8$. The representation used for $GF(p^a)$, $a > 1$, is discussed in [1, Section 3], while $GF(p)$ is represented as usual by the integers *modulo* p. Briefly, for $a > 1$ the additive identity of $GF(p^a)$ is denoted by Z, while $\alpha \in GF(p^a)^* = \{0, 1, \cdots, p^a - 2\}$ is an exponent for a cyclic generator for $GF(p)^*$. For appropriate table headings, the defining polynomial $F(x)$ of $GF(p^a)$ is given and remains the same as listed in [2]. Each table gives n, the prime factorization of $x^n - 1$, and $\Phi(x^n - 1) < 10^8$. All non-linear polynomials are given in ascending order by degree, with the variable factor suppressed in each term. Linear factors are given in the form $x - \alpha$, displaying the root α. Hence the factorization

$$f(x) = (1 - a_1)(1 - a_2)(b_0 + b_1 + 1)(c_0 + c_1 + c_2 + 1)$$

represents the product

$$f(x) = (x - a_1)(x - a_2)(b_0 + b_1 x + x^2)(c_0 + c_1 x + c_2 x^2 + x^3).$$

Also, each factorization is naturally ordered according to the degrees of the factors.

The tables were obtained using a software package developed by the authors and run on a Xerox $\Sigma 7$. The importance of the factorization of $x^n - 1$ over $GF(q)$ is well known. In particular, this output enables the computation of all admissible q-polynomials (Ore, [3]) for elements of $GF(q^n)$, since each such monic q-polynomial

$$\sum_{i=0}^{m} a_i x^{q^i} \in GF[q, x^q]$$

corresponds to a divisor

$$\sum_{i=0}^{m} a_i x^i$$

of $x^d - 1$ over $GF(q)$ for some divisor d of n. This allows a significant improvement of the corresponding algorithm in [1, Section 4].

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1. J. T. B. BEARD, JR., "Computing in $GF(q)$," *Math. Comp.*, this issue.

2. J. T. B. BEARD, JR. & K. I. WEST, "Some primitive polynomials of the third kind," *Math. Comp.*, v. 28, 1974, pp. 1166–1167.

3. O. ORE, "Contributions to the theory of finite fields," *Trans. Amer. Math. Soc.*, v. 36, 1934, pp. 243–274.

ADDENDUM TO

FACTORIZATION TABLES FOR $x^n - 1$ OVER GF(q)   (this issue, pp.   )

by

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TABLE I - TABLE XIV

1. <u>Description</u>.   For complete details and references, see the text of this paper. The representation used for $GF(p^a)$, a > 1, is discussed in [1], while GF(p) is represented as usual by the integers <u>modulo</u> p. The defining polynomial F(x) of $GF(p^a)$ appears in the appropriate headings. All non-linear polynomials are given in ascending order by degree with the variable factor suppressed in each term, including the defining polynomials F(x). Each table gives n, the prime factorization of $x^n - 1$, and the generalized Euler function $\phi(x^n - 1) < 10^8$. Linear factors are given in the form x - α, displaying the root α. Finally, each factorization is naturally ordered according to the degrees of the factors.

TABLE I

FACTORS OF X^N-1 OVER GF(2), $2 \le N \le 32$

N	FACTORS	PHI
2	(1+1) (1+1)	2
3	(1+1) (1+1+1)	3
4	(1+1) (1+1) (1+1) (1+1)	8
5	(1+1) (1+1+1+1+1)	15
6	(1+1) (1+1) (1+1+1) (1+1+1)	24
7	(1+1) (1+1+0+1) (1+0+1+1)	49
8	(1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1)	128
9	(1+1) (1+1+1) (1+0+0+1+0+0+1)	189
10	(1+1) (1+1) (1+1+1+1+1) (1+1+1+1+1)	480
11	(1+1) (1+1+1+1+1+1+1+1+1+1+1)	1023
12	(1+1) (1+1) (1+1) (1+1) (1+1+1) (1+1+1) (1+1+1) (1+1+1)	1536
13	(1+1) (1+1+1+1+1+1+1+1+1+1+1+1+1)	4095
14	(1+1) (1+1) (1+1+0+1) (1+1+0+1) (1+0+1+1) (1+0+1+1)	6272
15	(1+1) (1+1+1) (1+1+0+0+1) (1+0+0+1+1) (1+1+1+1+1)	10125
16	(1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1)	32768
17	(1+1) (1+0+0+1+1+1+0+0+1) (1+1+1+0+1+0+1+1+1)	65025
18	(1+1) (1+1) (1+1+1) (1+1+1) (1+0+0+1+0+0+1) (1+0+0+1+0+0+1)	96768
19	(1+1) (1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1)	262143
20	(1+1) (1+1) (1+1) (1+1) (1+1+1+1+1) (1+1+1+1+1) (1+1+1+1+1) (1+1+1+1+1)	491520
21	(1+1) (1+1+1) (1+1+0+0+1) (1+0+0+1+1) (1+1+1+0+0+1+0+0+1) (1+0+0+1+0+0+1+1+1)	583443
22	(1+1) (1+1) (1+1+1+1+1+1+1+1+1+1+1) (1+1+1+1+1+1+1+1+1+1+1)	2095104
23	(1+1) (1+1+0+0+0+1+1+1+0+0+1) (1+0+1+0+1+1+1+0+0+0+1+1)	4190209
24	(1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1) (1+1+1) (1+1+1) (1+1+1) (1+1+1) (1+1+1) (1+1+1) (1+1+1) (1+1+1)	6291456
25	(1+1) (1+1+1+1+1) (1+0+0+0+0+1+0+0+0+0+1+0+0+0+0+1+0+0+0+0+1)	15728625
26	(1+1) (1+1) (1+1+1+1+1+1+1+1+1+1+1+1+1) (1+1+1+1+1+1+1+1+1+1+1+1+1)	33546240
27	(1+1) (1+1+1) (1+0+0+1+0+0+1) (1+0+0+0+0+0+0+0+0+1+0+0+0+0+0+0+0+0+1)	49545027
28	(1+1) (1+1) (1+1) (1+1) (1+1+0+1) (1+1+0+1) (1+1+0+1) (1+1+0+1) (1+0+1+1) (1+0+1+1) (1+0+1+1) (1+0+1+1)	
29	(1+1) (1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1)	
30	(1+1) (1+1) (1+1+1) (1+1+1) (1+1+0+0+1) (1+1+0+0+1) (1+0+0+1+1) (1+0+0+1+1) (1+1+1+1+1) (1+1+1+1+1)	
31	(1+1) (1+0+0+1+0+1) (1+0+1+0+0+1) (1+0+1+1+1+1) (1+1+0+0+1+1) (1+1+1+0+1+1) (1+1+1+1+0+1)	
32	(1+1) (1+1)	

TABLE II

FACTORS OF X^N-1 OVER $GF(2^2)$: $F(X) = 1+1+1$, $2 \leq N \leq 16$

N	FACTORS	PHI
2	(0=0) (0=0)	12
3	(0=0) (0=1) (0=2)	27
4	(0=0) (0=0) (0=0) (0=0)	192
5	(0=0) (0+1+0) (0+2+0)	675
6	(0=0) (0=0) (0=1) (0=1) (0=2) (0=2)	1728
7	(0=0) (0+0+Z+0) (0+Z+0+0)	11907
8	(0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0)	49152
9	(0=0) (0=1) (0=2) (1+Z+Z+0) (2+Z+Z+0)	107163
10	(0=0) (0=0) (0+1+0) (0+1+0) (0+2+0) (3+2+0)	491200
11	(0=0) (0+2+0+0+1+0) (0+1+0+0+2+0)	3139587
12	(0=0) (0=0) (0=0) (0=0) (0=1) (0=1) (0=1) (0=?) (0=2) (0=2) (0=2)	7077888
13	(0=0) (0+1+Z+2+Z+1+0) (0+2+Z+1+Z+2+0)	50307075
14	(0=0) (0=0) (0+0+Z+0) (0+0+Z+0) (0+Z+0+0) (0+Z+0+0)	
15	(0=0) (0=1) (0=2) (1+0+0) (2+0+0) (0+1+0) (1+1+0) (0+2+0) (2+2+0)	
16	(0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0)	

TABLE III

FACTORS OF X^N-1 OVER $GF(2^3)$: $F(X) = 1+0+1+1$, $2 \leq N \leq 16$

N	FACTORS	PHI
2	(0=0) (0=0)	56
3	(0=0) (0+0+0)	4+1
4	(0=0) (0=0) (0=0) (0=0)	358+
5	(0=0) (0+0+0+0+0)	28665
6	(0=0) (0=0) (0+0+0) (0+0+0)	225792
7	(0=0) (0=1) (0=2) (0=3) (0=4) (0=5) (0=6)	823543
8	(0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0)	1+680064
9	(0=0) (0+0+0) (0+3+0) (0+5+0) (0+6+0)	
10	(0=0) (0=0) (0+0+0+0+0) (0+0+0+0+0)	
11	(0=0) (0+0+0+0+0+0+0+0+0+0+0)	
12	(0=0) (0=0) (0=0) (0=0) (0+0+0) (0+0+0) (0+0+0) (0+0+0)	
13	(0=0) (0+1+4+1+0) (0+2+1+2+0) (0+4+2+4+0)	
14	(0=0) (0=0) (0=1) (0=1) (0=2) (0=2) (0=3) (0=3) (0=4) (0=4) (0=5) (0=5) ,(0=6) (0=6)	
15	(0=0) (0+0+0) (0+0+2+2+0) (0+2+2+0+0) (0+0+0+0+0)	
16	(0=0) (0=0) (0=0) (0=n) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0)	

TABLE IV

FACTORS OF X^N-1 OVER $GF(2^4)$: $F(X) = 1+0+0+1+1$, $2 \le N \le 16$

N	FACTORS	PHI
2	(0=0) (0=0)	
3	(0=0) (C=5) (0=10)	240
4	(0=0) (0=0) (0=0) (0=0)	3375
5	(0=0) (0=3) (0=6) (0=9) (0=12)	61440
6	(0=0) (0=0) (0=5) (0=5) (0=10) (0=10)	759375
7	(0=0) (0+0+Z+0) (0+Z+0+0)	13824000
8	(0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0)	
9	(0=0) (0=5) (0=10) (5+Z+Z+0) (10+Z+Z+0)	
10	(0=0) (0=0) (0=3) (0=3) (0=6) (0=6) (0=9) (0=9) (0=12) (0=12)	
11	(0=0) (0+10+0+0+5+0) (0+5+0+0+10+0)	
12	(0=0) (0=0) (0=0) (0=0) (0=5) (0=5) (0=5) (0=5) (0=10) (0=10) (0=10) (0=10)	
13	(0=0) (0+4+1+0) (0+8+2+0) (0+1+4+0) (0+2+8+0)	
14	(0=0) (0=0) (0+0+Z+0) (0+0+Z+0) (0+Z+0+0) (0+Z+0+0)	
15	(0=0) (0=1) (0=2) (0=3) (0=4) (0=5) (0=6) (0=7) (0=8) (0=9) (0=10) (0=11) (0=12) (0=13) (0=14)	
16	(0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0)	

N	FACTORS	PHI
2	(0=0) (0=0)	
3	(0=0) (0+0+0)	992
4	(0=0) (0=0) (0=0) (0=0)	31713
5	(0=0) (0+0+0+0+0)	1015808
6	(0=0) (0=0) (0+0+0) (0+0+0)	32505825
7	(0=0) (0+0+Z+0) (0+Z+0+0)	
8	(0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0)	
9	(0=0) (0+0+0) (0+Z+Z+0+Z+Z+0)	
10	(0=0) (0=0) (0+0+0+0+0) (0+0+0+0+0)	
11	(0=0) (0+1+0) (0+2+0) (0+4+0) (0+8+0) (0+16+0)	
12	(0=0) (0=0) (0=0) (0=0) (0+0+0) (0+0+0) (0+0+0) (0+0+0)	

TABLE VI

FACTORS OF X^N-1 OVER GF(3) , $2 \leq N \leq 27$

10

N	FACTORS	PHI
2	(1=1) (1=2)	4
3	(1=1) (1=1) (1=1)	18
4	(1=1) (1=2) (1+0+1)	32
5	(1=1) (1+1+1+1)	160
6	(1=1) (1=1) (1=1) (1=2) (1=2) (1=2)	324
7	(1=1) (1+1+1+1+1+1)	1456
8	(1=1) (1=2) (1+0+1) (2+1+1) (2+2+1)	2048
9	(1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1)	13122
10	(1=1) (1=2) (1+1+1+1+1) (1+2+1+2+1)	26600
11	(1=1) (2+2+1+2+0+1) (2+0+1+2+1+1)	117128
12	(1=1) (1=1) (1=1) (1=2) (1=2) (1=2) (1+0+1) (1+0+1) (1+0+1)	209982
13	(1=1) (2+2+0+1) (2+0+1+1) (2+1+1+1) (2+2+2+1)	913982
14	(1=1) (1=2) (1+1+1+1+1+1+1) (1+2+1+2+1+2+1)	2119936
15	(1=1) (1=1) (1=1) (1+1+1+1) (1+1+1+1) (1+1+1+1)	9447840
16	(1=1) (1=2) (1+0+1) (2+1+1) (2+2+1) (2+0+1+0+1) (2+0+2+0+1)	13107200
17	(1=1) (1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1)	86093440
18	(1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=2) (1=2) (1=2) (1=2) (1=2) (1=2) (1=2) (1=2) (1=2)	
19	(1=1) (1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1)	
20	(1=1) (1=2) (1+0+1) (1+2+0+1+1) (1+1+1+1+1) (1+1+0+2+1) (1+2+1+2+1)	
21	(1=1) (1=1) (1=1) (1+1+1+1+1+1+1) (1+1+1+1+1+1+1) (1+1+1+1+1+14)	
22	(1=1) (1=2) (2+2+1+2+0+1) (1+2+2+2+0+1) (2+0+1+2+1+1) (1+0+2+2+2+1)	
23	(1=1) (2+2+2+1+1+0+2+0+2+0+0+1) (2+0+0+1+0+1+0+2+2+1+1+1)	
24	(1=1) (1=1) (1=1) (1=2) (1=2) (1=2) (1+0+1) (1+0+1) (1+0+1) (2+1+1) (2+1+1) (2+1+1) (2+2+1) (2+2+1) (2+2+1)	
25	(1=1) (1+1+1+1+1) (1+0+0+0+0+1+0+0+0+0+1+0+0+0+0+1+0+0+0+0+1)	
26	(1=1) (1=2) (1+2+0+1) (2+2+0+1) (2+0+1+1) (2+1+1+1) (1+2+1+1) (1+0+2+1) (1+1+2+1) (2+2+2+1)	
27	(1=1) (1=1)	

TABLE VIII

FACTORS OF X^N-1 OVER $GF(3^2)$: $F(X) = Z+1+1$, $2 \leq N \leq 15$

N	FACTORS	PHI
2	(0=0) (0=4)	64
3	(0=0) (0=0) (0=0)	448
4	(0=0) (0=2) (0=4) (0=6)	4096
5	(0=0) (0+5+0) (0+7+0)	81280
6	(0=0) (0=0) (0=0) (0=4) (0=4) (0=4)	419904
7	(0=0) (4+3+5+0) (4+1+7+0)	4239872
8	(0=0) (0=1) (0=2) (0=3) (0=4) (0=5) (0=6) (0=7)	16777216
9	(0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0) (0=0)	
10	(0=0) (0=4) (0+1+0) (0+3+0) (0+5+0) (0+760)	
11	(0=0) (4+4+0+4+4+Z+0) (4+Z+0+4+0+0)	
12	(0=0) (0=0) (0=0) (0=2) (0=2) (0=2) (0=4) (0=4) (0=6) (0=6) (0=6)	
13	(0=0) (4+4+Z+0) (4+Z+0+0) (4+0+0+0) (4+4+4+0)	
14	(0=0) (0=4) (0+3+1+0) (0+1+3+0) (4+3+5+0) (4+1+7+0)	
15	(0=0) (0=0) (0=0) (0+5+0) (0+5+0) (0+5+0) (0+7+0) (0+7+0) (0+7+0)	

TABLE VIII   12.

FACTORS OF X^N-1 OVER GF(5) , $2 \le N \le 25$

N	FACTORS	PHI
2	(1=1) (1=4)	16
3	(1=1) (1+1+1)	96
4	(1=1) (1=2) (1=3) (1=4)	256
5	(1=1) (1=1) (1=1) (1=1) (1=1)	2500
6	(1=1) (1=4) (1+1+1) (1+4+1)	9216
7	(1=1) (1+1+1+1+1+1+1)	62496
8	(1=1) (1=2) (1=3) (1=4) (2+0+1) (3+0+1)	147456
9	(1=1) (1+1+1) (1+0+0+1+0+0+1)	1499904
10	(1=1) (1=1) (1=1) (1=1) (1=4) (1=4) (1=4) (1=4)	6250000
11	(1=1) (4+1+1+4+2+1) (4+3+1+4+4+1)	39037504
12	(1=1) (1=2) (1=3) (1=4) (1+1+1) (4+2+1) (4+3+1) (1+4+1)	84934656
13	(1=1) (1+1+4+1+1) (1+2+1+2+1) (1+3+0+3+1)	
14	(1=1) (1=4) (1+1+1+1+1+1+1) (1+4+1+4+1+4+1)	
15	(1=1) (1=1) (1=1) (1=1) (1=1) (1+1+1) (1+1+1) (1+1+1) (1+1+1) (1+1+1)	
16	(1=1) (1=2) (1=3) (1=4) (2+0+1) (3+0+1) (2+0+0+0+1) (3+0+0+0+1)	
17	(1=1) (1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1)	
18	(1=1) (1=4) (1+1+1) (1+4+1) (1+0+0+1+0+0+1) (1+0+0+4+0+0+1)	
19	(1=1) (4+4+2+4+2+2+3+0+1) (4+0+2+3+3+1+3+1+1)	
20	(1=1) (1=1) (1=1) (1=1) (1=1) (1=2) (1=2) (1=2) (1=2) (1=3) (1=3) (1=3) (1=3) (1=3) (1=4) (1=4) (1=4) (1=4) (1=4)	
21	(1=1) (1+1+1) (1+0+2+2+2+0+1) (1+1+1+1+1+1+1) (1+4+3+1+3+4+1)	
22	(1=1) (1=4) (1+3+4+4+1+1) (4+1+1+4+2+1) (1+1+4+4+3+1) (4+3+1+4+4+1)	
*23	----	
24	(1=1) (1=2) (1=3) (1=4) (2+0+1) (3+0+1) (1+1+1) (2+1+1) (3+2+1) (4+2+1) (3+3+1) (4+3+1) (1+4+1) (2+4+1)	
25	(1=1) (1=1)	

*Based on the CPU time of an inconclusive run, we are reasonably certain that $(x^{23} - 1)/(x - 1)$ has two prime factors of degree 11 or else is prime. We conjecture it is prime.

TABLE IX 13

FACTORS OF X^N-1 OVER $GF(5^2)$: $F(X) = 2+1+1$, $2 \leq N \leq 10$

N	FACTORS	PHI
2	(0=0) (0=12)	576
3	(0=0) (0=8) (0=16)	13824
4	(0=0) (0=6) (0=12) (0=18)	331776
5	(0=0) (0=0) (0=0) (0=0) (0=0)	9375000
6	(0=0) (0=4) (0=8) (0=12) (0=16) (0=20)	
7	(0=0) (17+5+13+0) (12+1+17+0)	
8	(0=0) (0=3) (0=6) (0=9) (0=12) (0=15) (0=18) (0=21)	
9	(0=0) (0=8) (0=16) (4+Z+Z+0) (20+Z+Z+0)	
10	(0=0) (0=0) (0=0) (0=0) (0=0) (0=12) (0=12) (0=12) (0=12) (0=12)	

TABLE X   14

FACTORS OF X^N-1 OVER GF(7) , $2 \leq N \leq 15$

N	FACTORS	PHI
2	(1=1) (1=6)	36
3	(1=1) (1=2) (1=4)	216
4	(1=1) (1=6) (1+0+1)	1728
5	(1=1) (1+1+1+1+1)	14400
6	(1=1) (1=2) (1=3) (1=4) (1=5) (1=6)	46656
7	(1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1)	705894
8	(1=1) (1=6) (1+0+1) (1+3+1) (1+4+1)	3981312
9	(1=1) (1=2) (1=4) (3+0+0+1) (5+0+0+1)	25264224
10	(1=1) (1=6) (1+1+1+1+1) (1+6+1+6+1)	
11	(1=1) (1+1+1+1+1+1+1+1+1+1+1)	
12	(1=1) (1=2) (1=3) (1=4) (1=5) (1=6) (1+0+1) (2+0+1) (4+0+1)	
13	(1=1) (1+1+1+1+1+1+1+1+1+1+1+1+1)	
14	(1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=6) (1=6) (1=6) (1=6) (1=6) (1=6) (1=6)	
15	(1=1) (1=2) (1=4) (1+1+1+1+1) (2+1+4+2+1) (4+1+2+4+1)	

TABLE XI

FACTORS OF $X^N - 1$ OVER GF(11) , $2 \leq N \leq 15$

N	FACTORS	PHI
2	(1=1) (1=10)	100
3	(1=1) (1+1+)	1200
4	(1=1) (1=10) (1+0+1)	12000
5	(1=1) (1=3) (1=4) (1=5) (1=9)	100000
6	(1=1) (1=10) (1+1+1) (1+10+1)	1440000
7	(1=1) (10+4+5+1) (10+6+7+1)	17689000
8	(1=1) (1=10) (1+0+1) (10+3+1) (10+8+1)	
9	(1=1) (1+1+1) (1+0+0+1+0+0+1)	
10	(1=1) (1=2) (1=3) (1=4) (1=5) (1=6) (1=7) (1=8) (1=9) (1=10)	64625060
11	(1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1) (1=1)	
12	(1=1) (1=10) (1+0+1) (1+1+1) (1+5+1) (1+6+1) (1+10+1)	
13	(1=1) (1+1+1+1+1+1+1+1+1+1+1+1+1)	
14	(1=1) (1=10) (1+6+4+1) (10+4+5+1) (1+4+6+1) (10+6+7+1)	
15	(1=1) (1=3) (1=4) (1=5) (1=9) (1+1+1) (9+3+1) (5+4+1) (3+5+1) (4+9+1)	

TABLE XII

FACTORS OF X^N-1 OVER GF(13), $2 \leq N \leq 15$

N	FACTORS	PHI
2	(1-1) (1-12)	144
3	(1-1) (1-3) (1-9)	1728
4	(1-1) (1-5) (1-8) (1-12)	20736
5	(1-1) (1+1+1+1+1)	342720
6	(1-1) (1-3) (1-4) (1-9) (1-10) (1-12)	2985984
7	(1-1) (1+3+1) (1+5+1) (1+6+1)	56899584
8	(1-1) (1-5) (1-8) (1-12) (5+0+1) (8+0+1)	
9	(1-1) (1-3) (1-9) (4+0+0+1) (10+3+0+1)	
10	(1-1) (1-12) (1+1+1+1) (1+12+1+12+1)	
11	(1-1) (1+1+1+1+1+1+1+1+1+1+1)	
12	(1-1) (1-2) (1-3) (1-4) (1-5) (1-6) (1-7) (1-8) (1-9) (1-10) (1-11) (1-12)	
13	(1-1) (1-1) (1-1) (1-1) (1-1) (1-1) (1-1) (1-1) (1-1) (1-1) (1-1) (1-1) (1-1)	
14	(1-1) (1-12) (1+3+1) (1+5+1) (1+6+1) (1+7+1) (1+8+1) (1+10+1)	
15	(1-1) (1-3) (1-9) (1+1+1+1+1) (3+1+9+3+1) (9+1+3+9+1)	

TABLE XIII

FACTORS OF $X^N - 1$ OVER GF(17) , $2 \leq N \leq 15$

N	FACTORS	PHI
2	(1-1) (1-16)	256
3	(1-1) (1+1+1)	4608
4	(1-1) (1-4) (1-13) (1-16)	65536
5	(1-1) (1+1+1+1+1)	1336320
6	(1-1) (1-16) (1+1+1) (1+16+1)	24233664
7	(1-1) (1+1+1+1+1+1+1)	
8	(1-1) (1-2) (1-4) (1-8) (1-9) (1-13) (1-15) (1-16)	
9	(1-1) (1+1+1) (1+3+1) (1+4+1) (1+10+1)	
10	(1-1) (1-16) (1+1+1+1+1) (1+16+1+16+1)	
11	(1-1) (1+1+1+1+1+1+1+1+1+1+1)	
12	(1-1) (1-4) (1-13) (1-16) (1+1+1) (16+4+1) (16+13+1) (1+16+1)	
13	(1-1) (1+5+2+4+2+5+1) (1+13+2+12+2+13+1)	
14	(1-1) (1-16) (1+1+1+1+1+1+1) (1+16+1+16+1+16+1)	
15	(1-1) (1+1+1) (1+1+1+1+1) (1+11+15+5+1) (1+5+15+11+1)	

TABLE XIV

FACTORS OF X^N-1 OVER GF(19) , $2 \le N \le 12$

N	FACTORS	PHI
2	(1-1) (1-18)	324
3	(1-1) (1-7) (1-11)	5832
4	(1-1) (1-18) (1+0+1)	116640
5	(1-1) (1+5+1) (1+15+1)	2332800
6	(1-1) (1-7) (1-8) (1-11) (1-12) (1-18)	34012224
7	(1-1) (1+1+1+1+1+1+1)	
8	(1-1) (1-18) (1+0+1) (18+6+1) (18+13+1)	
9	(1-1) (1-4) (1-5) (1-6) (1-7) (1-9) (1-11) (1-16) (1-17)	
10	(1-1) (1-18) (1+4+1) (1+5+1) (1+1+9+1) (1+15+1)	
11	(1-1) (1+1+1+1+1+1+1+1+1+1+1)	
12	(1-1) (1-7) (1-8) (1-11) (1-12) (1-18) (1+0+1) (7+0+1) (11+0+1)	

TABLE XV

FACTORS OF X^N-1 OVER GF(23) , $2 \le N \le 10$

N	FACTORS	PHI
2	(1+1) (1+22)	484
3	(1+1) (1+1+1)	11616
4	(1+1) (1+22) (1+0+1)	255552
5	(1+1) (1+1+1+1+1)	6156480
6	(1+1) (1+22) (1+1+1) (1+22+1)	
7	(1+1) (22+9+10+1) (22+13+14+1)	
8	(1+1) (1+22) (1+0+1) (1+5+1) (1+18+1)	
9	(1+1) (1+1+1) (1+0+0+1+0+0+1)	
10	(1+1) (1+22) (1+1+1+1+1) (1+22+1+22+1)	