

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

49 [2, 3, 4].—JOSEF STOER, *Einführung in die Numerische Mathematik*. I; J. STOER & R. BULIRSCH, *Einführung in die Numerische Mathematik*. II, Springer-Verlag, New York, 1972; I: ix + 250 pp., 21 cm. Price \$4.70, II: ix + 286 pp., 21 cm. Price \$5.50.

Here is a modern introductory textbook for German-speaking people by leading German numerical analysts. Volume I covers interpolation, quadrature, linear equations, least squares, and the zeros of functions. An example of the scope of the work is the coverage of both the Cooley-Tukey algorithm and splines in the chapter on interpolation.

Volume II (written with R. Bulirsch) covers eigenvalues, ordinary differential equations, and the iterative solution of large systems of linear equations. An example treated at some length in Volume II is a realistic re-entry problem for a space vehicle.

Of all the English textbooks, one is reminded most of Isaacson and Keller's "Analysis of Numerical Methods".

There are problems at the end of each chapter and the book is in paperback.

B. P.

50 [2.05, 2.35].—A. MEIR & A. SHARMA, Editors, *Spline Functions and Approximation Theory*, Birkhäuser Verlag, Basel, Switzerland, 1973, 386 pp., 25 cm. Price Fr. 64.—.

This book is the proceedings of a symposium held at the University of Alberta in May, 1972. There are 15 papers and 4 abstracts and, as the title suggests, there is considerable emphasis on spline functions (seven papers). Some of the papers present original results and others survey various areas. Almost all of the papers are well done and some are excellent. The topics are so diverse that generalities do not apply; we present the author, title, length (in typewritten pages), and a very short summary for each of the 15 papers. The reviewer hopes the authors will forgive him if he has been unable to exhibit the entire content of their papers in two sentences or less.

BERENS, H., *Pointwise Saturation* (20 pages). This is a survey and reformulation of a wide variety of saturation results. The unifying theme in the presentation is the use of relationships with differential properties.

DAVIS, C., *A Combinatorial Problem in Best Uniform Approximation* (26 pages). For a function  $f(x)$  which oscillates  $N$  times, we wish to determine the best approximation among those functions which oscillate  $m < N$  times. Various results are given including some leading to computational algorithms.

- DE BOOR, C., *Good Approximation by Splines with Variable Knots* (16 pages). This is a survey of various results and approaches to obtaining good (as opposed to best) knots for spline approximation. Emphasis is placed on obtaining knots efficiently which are valid for broad classes of functions.
- DE VORE, R. & RICHARDS, F., *Saturation and Inverse Theorems for Spline Approximation* (10 pages). A simpler proof is given of the saturation theorem for spline approximation where the knots satisfy a "mixing" condition. That this condition forces "bad" choices of knots in many situations can be seen by comparing the results with the convergence results of the preceding paper.
- DITZIAN, F. & MAY, C., *Saturation Classes for Exponential Formulae of Semi-Groups of Operators* (18 pages). The optimal rate of convergence (and corresponding saturation class) is obtained for approximation by the "exponential formulae". The context is  $C_0$  semi-groups of operators  $T(t)$ , and the analysis involves the infinitesimal generator.
- FIELDS, J. & ISMAIL, M., *On Some Conjectures of Askey Concerning Completely Monotonic Functions* (12 pages). Askey analyzed the  $L^p$  convergence of Lagrange interpolation at the zeros of  $P_n^{(\alpha, \beta)}(z)$  and made some conjectures concerning the positivity of certain Cesàro means. The authors have established five of these conjectures and give proofs for three of them here.
- GAUTHIER, P., *Une Application de la Théorie de l'Approximation a l'Etude des Fonctions Holomorphes* (6 pages). A very short proof by approximation theoretic methods is given for two recent results on meromorphic and holomorphic functions.
- JEROME, J., *Linearization in Certain Nonconvex Minimization Problems and Generalized Spline Projections* (50 pages). This long paper presents a variety of extensions of Jerome's earlier work on nonconvex minimization. Particular consideration is given to the minimization of a general bilinear form (not necessarily nonnegative) on a Sobolev space.
- LYCHE, T. & SCHUMACHER, L., *On the Convergence of Cubic Interpolating Splines* (22 pages). Conditions are established on a partition of an interval, so that natural cubic spline interpolation converges for all continuous functions. These are analogous to conditions of Marsden, Meir, Sharma and others for periodic splines.
- MOTZKIN, T., SHARMA, A. & STRAUS, E., *Averaging Interpolation* (44 pages). Polynomial interpolation with respect to certain linear functionals (averaging interpolation) is studied along with the related concept of relative unsolvence. Many results are established involving existence, uniqueness and representation.
- MUNTEANU, M., *On the Construction of Multidimensional Splines* (32 pages). A survey is given of methods to construct classes of splines which are combined interpolation and smoothing.
- OSTROWSKI, A., *On Error Estimates A Posteriori in Iterative Procedures* (10 pages). A survey is given of various approaches to obtain a posteriori error estimates for iterative computations.

- SCHOENBERG, I., *Splines and Histograms* (52 pages). Histosplines are splines which "interpolate" histograms in the sense of preserving area. A variety of theorems and constructions are given for univariate and bivariate histosplines.
- DE BOOR, C., *Appendix to "Splines and Histosplines" by I. J. Schoenberg* (30 pages). This appendix presents and analyzes the tensor product nature of bivariate histosplines and develops algebraic machinery for their efficient calculation.
- STRAUS, E., *Real Analytic Functions as Ratios of Absolutely Monotonic Functions* (12 pages). A discussion is given of various results on the problem of determining under what conditions analytic functions, positive on a real segment, can be expressed as the ratio of two absolutely monotonic functions.

JOHN R. RICE

Purdue University  
 Division of Mathematical Sciences  
 Lafayette, Indiana 47907

- 51 [2.05, 2.35, 3.25].—F. A. LOOTSMA, Editor, *Numerical Methods for Non-Linear Optimization*, Conference sponsored by Science Research Council, University of Dundee, Scotland, 1971, Academic Press, New York, 1972, xiv + 439 pp., 24 cm. Price \$25.—.

This book is a collection of 29 papers presented at the Conference on Non-Linear Optimization held in Dundee, Scotland, in the summer of 1971. Within this general area the range of topics is rather broad. Ten papers treat general unconstrained optimization problems, with the total divided about equally between papers on theoretical results and those reporting experimental investigations; especially noteworthy in the former class is Dixon's paper on the equivalence of a large set of variable metric methods when exact line searches are performed. Three additional papers treat the special problem of non-linear least squares.

Three papers attack the difficult and important problem of finding or identifying global minima rather than merely local ones; McCormick's survey paper, in particular, can serve as a good introduction to this relatively unexplored area.

The remaining thirteen papers treat a variety of topics in constrained optimization, mainly from theoretical viewpoints; papers are included on quadratic programming, linearly-constrained programming, complementarity problems and especially on penalty-function methods. The general reader may find particularly informative the survey by Fletcher on linearly-constrained programming and that by Lootsma on solving constrained problems by means of solutions to unconstrained problems.

The majority of papers in this collection are specialized in the sense that they are probably of primary interest to the expert or practitioner in the field rather than to the general reader or novice who would like to get a general introduction to the area;