SCHOENBERG, I., *Splines and Histograms* (52 pages). Histosplines are splines which "interpolate" histograms in the sense of preserving area. A variety of theorems and constructions are given for univariate and bivariate histosplines.

DE BOOR, C., *Appendix to "Splines and Histosplines" by I. J. Schoenberg* (30 pages). This appendix presents and analyzes the tensor product nature of bivariate histosplines and develops algebraic machinery for their efficient calculation.

STRAUS, E., *Real Analytic Functions as Ratios of Absolutely Monotonic Functions* (12 pages). A discussion is given of various results on the problem of determining under what conditions analytic functions, positive on a real segment, can be expressed as the ratio of two absolutely monotonic functions.

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51 [2.05, 2.35, 3.25].—F. A. LOOTSMA, Editor, *Numerical Methods for Non-Linear Optimization*, Conference sponsored by Science Research Council, University of Dundee, Scotland, 1971, Academic Press, New York, 1972, xiv + 439 pp., 24 cm. Price $25.—.

This book is a collection of 29 papers presented at the Conference on Non-Linear Optimization held in Dundee, Scotland, in the summer of 1971. Within this general area the range of topics is rather broad. Ten papers treat general unconstrained optimization problems, with the total divided about equally between papers on theoretical results and those reporting experimental investigations; especially noteworthy in the former class is Dixon's paper on the equivalence of a large set of variable metric methods when exact line searches are performed. Three additional papers treat the special problem of non-linear least squares.

Three papers attack the difficult and important problem of finding or identifying global minima rather than merely local ones; McCormick's survey paper, in particular, can serve as a good introduction to this relatively unexplored area.

The remaining thirteen papers treat a variety of topics in constrained optimization, mainly from theoretical viewpoints; papers are included on quadratic programming, linearly-constrained programming, complementarity problems and especially on penalty-function methods. The general reader may find particularly informative the survey by Fletcher on linearly-constrained programming and that by Lootsma on solving constrained problems by means of solutions to unconstrained problems.

The majority of papers in this collection are specialized in the sense that they are probably of primary interest to the expert or practitioner in the field rather than to the general reader or novice who would like to get a general introduction to the area;

most specialists but few nonspecialists will want to own this book. A few of the papers, however, could be quite informative for the general reader; the above-mentioned papers of Lootsma, Fletcher, and McCormick are in this class, as are, to a somewhat lesser extent, the reports of computational experiments by Himmelblau and by Sargent and Sebastian.

J. W. D.

52 [4, 5].—GUNTER H. MEYER, *Initial Value Methods for Boundary Value Problems*, Academic Press, New York, 1973, x + 220 pp., 24 cm. Price $14.50.

MELVIN R. SCOTT, *Invariant Imbedding and its Applications to Ordinary Differential Equations. An Introduction*, Addison-Wesley, Reading, Mass., 1973, 215 pp. Price $19.50, cloth, $11.50, paperbound.

Although these books deal with the same topic, there is remarkably little overlap. When they are to be distinguished in this review, the author's initials will be used. The books are monographs devoted to the technique of invariant imbedding as applied mainly to two point boundary value problems for ordinary differential equations. There are several ways of viewing invariant imbedding. The original view is to consider a particular problem as imbedded in a family of problems with the length of the interval as a parameter. The invariant imbedding equations are differential equations for the unknown boundary values as a function of this parameter. These equations turn out to have specified initial values, in this respect, they are simpler than the original problem. Moreover, the approach is a natural one to some problems, e.g. when only the missing boundary values are of interest, or for parameter studies varying the interval length, or when a free boundary is to be located. Another view of invariant imbedding is as a kind of shooting method. A third view appropriate to linear problems is based on Riccati equations and is a natural approach to Sturm-Liouville problems.

When the ordinary differential equation is nonlinear, the imbedding equations are partial differential equations. They are the principal object of the theory in GHM and are thoroughly investigated using characteristic theory. Numerical aspects of the solution of the partial differential equations are treated as well. MRS is almost wholly concerned with linear problems, which lead to initial value problems for ordinary differential equations. His analysis is based on the Riccati approach and particular attention is given to the computation of the solution (as opposed to the missing boundary values). GHM is the more demanding of mathematical background, but both books place modest demands on the reader. Both books are clearly written and well structured, though both would have profited by more careful proof reading.

It is frequently the case that the invariant imbedding equations are stable and represent an effective way to solve boundary value problems. The authors do not attempt to evaluate the approach in comparison to alternatives, though MRS explicitly