REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

1 [7].—DANIEL SHANKS & JOHN W. WRENCH, JR., *Sums of Reciprocals to* 1,000,000, 1961, ms. of 20 computer sheets deposited in the UMT file.

Herein are tabulated values of the partial sums $\Sigma_1^N n^{-1}$ of the harmonic series for $N = 10^4(10^4)10^6$, truncated to 1060D. These were computed on an IBM 7090 system at the same time that we evaluated π [1] and e [2], and they were intended to be used by the second author in computing Euler's constant, γ, by means of the Euler-Maclaurin formula. However, Knuth [3] computed γ to higher precision before this was completed.

For the sake of comparison we list these sums truncated to 1000D for $N = 10^4$, 10^5, and 10^6, respectively, with $^*(M)^*$ denoting the omission of M digits:

$$9.7876060360\ 4438226417\ ^*(960)^*\ 9216446619\ 7618373424,$$
$$12.0901461298\ 6342794736\ ^*(960)^*\ 7602452004\ 8801442625,$$
$$14.3927267228\ 6572363138\ ^*(960)^*\ 3436083266\ 8760078693.$$

Our value corresponding to $N = 10^4$ agrees in its entirety with the value found to 1275D by Knuth, which has been deposited in the UMT file along with his unpublished table of Bernoulli numbers mentioned on p. 277 of [3].

D. S., J. W. W.

1. DANIEL SHANKS & JOHN W. WRENCH, JR., "Calculation of π to 100,000 decimals," *Math. Comp.*, v. 16, 1962, pp. 76–99.

2. UMT 46, *Math. Comp.*, v. 23, 1969, pp. 679–680.

3. DONALD E. KNUTH, "Euler's constant to 1271 places," *Math. Comp.*, v. 16, 1962, pp. 275–281.

2 [9].—DAVID BALLEW, JANELL CASE & ROBERT N. HIGGINS, *Table of* $\phi(n) = \phi(n + 1)$, South Dakota School of Mines and Technology, 1974, ii + 3 pages, deposited in the UMT file.

There are listed here the 88 solutions of $\phi(n) = \phi(n + 1)$ from $n = 3$ to $n = 2792144$. (Previous tables have listed $n = 1$ also; counting this, there are 89 solutions for $n < 2.8 \cdot 10^6$.) This extends the tables of the 36 solutions to $n = 10^5$ by Lal and Gillard [1] and the 56 solutions to $n = 5 \cdot 10^5$ by Miller [2]. Note that Miller is wrong in stating that the next solution is $n = 525986$. She has omitted $n = 524432$.

A propos my editorial note to [2], there is only one further case in this extension (if I did it correctly). For $n = 2539004$, multiplication (mod n) is isomorphic to multiplication (mod $n + 1$). That is a much more stringent requirement; I do not know if anyone has made a heuristic estimate of whether there are infinitely many such n.

D. S.

1. M. LAL & P. GILLARD, "On the equation $\phi(n) = \phi(n + k)$," *Math. Comp.*, v. 26, 1972, pp. 579–583.
2. KATHRYN MILLER, UMT 25, *Math. Comp.*, v. 27, 1973, pp. 447–448.

3 [9].—B. D. BEACH, H. C. WILLIAMS & C. R. ZARNKE, *Some Computer Results on Units of Quadratic and Cubic Fields*, Scientific Report 31, University of Manitoba, Winnipeg, July 1971.

The table in the appendix lists the class number H and fundamental unit ϵ_0 $(0 < \epsilon_0 < 1)$ of the pure cubic fields $Q(\rho)$ where $\rho = D^{1/3}$. For each cube-free D between 2 and 998 there is listed $H, U, V, W, T,$ and J where

$$\epsilon_0 = (U + V\rho + W\rho^2)/T \quad (1)$$

and J is the length of the period of Voronoi's algorithm. The largest U here is a 330-decimal number for $D = 951$ where $H = 1$. Here, $J = 1352$, and for large U one finds that $J/\log_{10} U \approx 4.1$. Presumably, the mean value of this ratio is analogous to Lévy's constant but its identity is not known to me. The largest H equals 162 here for $D = 813$. Some fields are given twice: e.g., $Q((12)^{1/3}) = Q((18)^{1/3})$ and so its ϵ_0 is given in two forms. Happily, the H then agree—in all cases that I checked.

A direct comparison with Wada's units to $D = 249$, see [1], is not possible since Wada gives the reciprocal $\epsilon = 1/\epsilon_0 = (A + B\rho + C\rho^2)/E$ instead. It is of some interest to argue which unit is preferable. Usually, U, V, W have only one-half the decimals of A, B, C; for example, for $D = 239$, U has 94 decimals while A has 188. But for applications, ϵ is usually preferable. Thus, in evaluating the regulator $R = |\log \epsilon_0|$, the formula (1) can suffer catastrophic loss of significance since ϵ_0 may be exceedingly small. Of course, one can obtain ϵ from ϵ_0 by

$$\epsilon = (U^2 - DVW) + (W^2 D - UV)\rho + (V^2 - UW)\rho^2$$

if $T = 1$. So, for such large $U, V, W, R = \log(3U^2 - 3DVW)$ will be very accurate.

The text describes Voronoi's algorithm and refers to earlier, less extensive tables by Markov, Cassels, Selmer, etc.

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1. H. WADA, RMT 15, *Math. Comp.*, v. 26, 1972, pp. 302–303.