$|S_p|$ is bounded below by $1/3$. The smallest S_p here is one of the aforementioned $A = 1$, namely, $p = 170647, A = 1, B = 159, S_p = -0.3333334056$. (The table lists $S_p = -0.335414$ for this p, showing that four decimals are corrupted in adding up the 85 thousand cosines.) The existence of such small S_p illustrates the marked distinction between these cubic sums and the quadratic Gauss Sums with n^2 instead of n^3 in (1). Then, $|S_p| = \sqrt{p}$, as is well known. For other recent work, see Cassels [6] and the references cited there.

D. S.

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6 [9].—MARIE NICOLE GRAS, "Méthodes et algorithmes pour le calcul numérique du nombre de classes et des unités des extensions cubiques cycliques de Q," Institut de mathématiques pures, Grenoble, 1972–1973. Tables 1–4.

For any product $m = p_1 \cdot p_2 \cdot \cdots \cdot p_n$ of distinct primes $p \equiv 1 \pmod 3$ there are 2^{n-1} distinct cyclic cubic fields of discriminant m^2 and for $m = 9 \cdot p_1 \cdot \cdots \cdot p_n$ there are 2^n such fields. Altogether there are 630 fields with $m < 4000$. Table 1 lists each such m with (A) its prime decomposition; (B) its appropriate representation $4m = a^2 + 27b^2$; (C) its class number h; and, in most cases, (D) $\mathrm{tr}(\epsilon)$ and $\mathrm{tr}(\epsilon^{-1})$. These latter integers give the equation

$$x^3 = \mathrm{tr}(\epsilon)x^2 - \mathrm{tr}(\epsilon^{-1})x + 1$$

satisfied by the fundamental units and having a discriminant m^2k^2 for some index $k \geqslant 1$. When $\mathrm{tr}(\epsilon)$ and $\mathrm{tr}(\epsilon^{-1})$ are too large, they are omitted here since they were not obtained with the precision used. (These large units are only missing from Table 1 for some cases of $h = 1$ or 3 when $\zeta_k/\zeta(1)$ is relatively large because one or more small primes split in the field. The first units missing are those for $m = 919$ which has $h = 1$ and both 2 and 3 as splitting primes.)

This table, and those that follow, were computed by a new, interesting method described in Marie Gras's paper [1]. The tables are more easily extended to larger m by this method if h is large. There are known criteria for $9|h$ and $4|h$, [2], [3]. Table 2 continues with 154 more $m < 10^4$ having $9|h$ while Table 3 contains 119 $m < 10^4$ having $4|h$. These two tables overlap some. Sometimes, units are missing, as before.

Table 4 contains all m between $4 \cdot 10^3$ and $2 \cdot 10^4$ having a representation $4m = a^2 + 27$ or $1 + 27b^2$ or $9 + 27b^2$. In these 89 fields, $\mathrm{tr}(\epsilon)$ and $\mathrm{tr}(\epsilon^{-1})$ are never missing since they are known a priori. They equal $\pm 1/2(a \mp 3)$, $\pm 3/2(9b \mp 1)$ and $\pm 3/2(3b \mp 1)$, respectively. These units are relatively small and the class numbers, correspondingly, are relatively large. The largest is $h = 129$ for $m = 97 \cdot 181 = (1 + 27 \cdot 51^2)/4$.

These tables of cyclic cubic fields go far beyond earlier tables of Hasse, Cohn and Gorn, and Godwin. For the "simplest cubics", having $4m = a^2 + 27$, the reviewer has gone further [4] using an entirely different method.

D. S.

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7 [9].—WELLS JOHNSON, *The Irregular Primes to* 30000 *and Related Tables*, ms. of 28 computer pages (+ 1 introductory page), deposited in the UMT file, June 1974.

This unpublished table constitutes an appendix to a paper published elsewhere in this issue. The 13-column table presents the complete list of 1619 irregular pairs $(p, 2k)$ with $p < 30000$ together with some computations which depend upon this list. The table shows that Fermat's Last Theorem is true for all prime exponents $p < 30000$. In addition, the tables of [1], [2], [3] are completed to 30000, so that the cyclotomic invariants of Iwasawa are completely determined for primes within this range. The computations were performed on the PDP-10 computer at Bowdoin College.

AUTHOR'S SUMMARY

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