and so are given in base 32. For statistics about the class number distribution see the reviewer's paper in this issue [2].

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1. R. B. LAKEIN & S. KURODA, UMT 38, *Math. Comp.*, v. 24, 1970, pp. 491–493.
2. R. B. LAKEIN, "Computation of the ideal class group of certain complex quartic fields. II," *Math. Comp.*, v. 29, 1975, pp. 137–144 (this issue).

10 [9].—RICHARD B. LAKEIN, *Class Numbers of* 5000 *Quartic Fields* $Q(\sqrt{\pi})$, SUNY at Buffalo, 1973, ms. of 21 computer sheets deposited in the UMT file.

Let P be a rational prime $\equiv 1 \bmod 8$, and $\pi = a + bi$ a Gaussian prime with norm $a^2 + b^2 = P$, normalized so that $a, b > 0, b \equiv 0 \bmod 4$. Then $K = Q(\sqrt{\pi})$ is a totally complex quartic field with no quadratic subfield other than $Q(i)$. The arithmetic of K has many strong analogies to that of a real quadratic field with prime discriminant. In particular, the class number $h(\pi)$ of K is odd.

This table lists the first 5000 primes $P \equiv 1 \bmod 8$ (from $P = 17$ through $P = 226241$), the (normalized) Gaussian prime factor π of P, and the class number $h(\pi)$ of the quartic field $K = Q(\sqrt{\pi})$. The final page of the table lists the cumulative distribution of class numbers for each successive 1000 fields. The distribution of class numbers is very close to that for the first 5000 real quadratic prime discriminants [2]. Details of the method of calculation, as well as the class number distribution, are contained in [1].

AUTHOR'S SUMMARY

1. R. B. LAKEIN, "Computation of the ideal class group of certain complex quartic fields," *Math. Comp.*, v. 28, 1974, pp. 839–846.
2. D. SHANKS, UMT 10, *Math. Comp.*, v. 23, 1969, pp. 213–214.

11 [9].—MORRIS NEWMAN, *A Table of the Coefficients of the Modular Invariant* $j(\tau)$, National Bureau of Standards, Washington, D. C., ms. of 14 pages deposited in the UMT file.

The absolute modular invariant $j(\tau)$, defined by

$$j(\tau) = x^{-1} \prod_{n=1}^{\infty} (1 - x^n)^{-24} \left\{1 + 240 \sum_{n=1}^{\infty} \sigma_3(n) x^n \right\}^3$$

$$= \sum_{n=-1}^{\infty} c(n) x^n = x^{-1} + 744 + 196884x + \cdots,$$

where $x = \exp 2\pi i\tau$ and $\sigma_r(n) = \Sigma_{d/n} d^r$, is the Hauptmodul of the classical modular group Γ. Its coefficients possess many remarkable arithmetic properties, which are set forth in the appended references. For example, the congruence

$$(n + 1)c(n) \equiv 0 \pmod{24},$$

due to D. H. Lehmer [2], implies that $c(n)$ is even except possibly when $n \equiv 7 \pmod 8$. In this case it may be shown that $c(n)$ assumes both even and odd values infinitely often, although necessary and sufficient conditions for $c(n)$ to be odd are still unknown.

The coefficients were first computed for $-1 \leqslant n \leqslant 24$ by H. S. Zuckerman [7] and then for $-1 \leqslant n \leqslant 100$ by A. van Wijngaarden [6]. Here we tabulate the coefficients for $-1 \leqslant n \leqslant 500$. There would seem to be little point in extending the table further, since $c(500)$ is already a number of 120 digits.

The coefficients were computed, using residue arithmetic, by means of the following formula [5]:

$$c(n) = p_{-24}(n + 1) + \frac{65520}{691} \sum_{k=0}^{n} \sigma_{11}(k + 1)p_{-24}(n - k), \qquad n \geqslant 1,$$

where $\sum_{n=0}^{\infty} p_{-24}(n)x^n = \prod_{n=1}^{\infty} (1 - x^n)^{-24}$.

The total computation time on a UNIVAC 1108 system was approximately four minutes.

AUTHOR'S SUMMARY

1. O. KOLBERG, "Congruences for the coefficients of the modular invariant $j(\tau)$ modulo powers of 2," *Univ. Bergen Arbok Naturvit. Rekke*, v. 16, 1961.

2. D. H. LEHMER, "Properties of the coefficients of the modular invariant $J(\tau)$," *Amer. J. Math.*, v. 64, 1942, pp. 488–502.

3. J. LEHNER, "Divisibility properties of the Fourier coefficients of the modular invariant $J(\tau)$," *Amer. J. Math.*, v. 71, 1949, pp. 136–148.

4. J. LEHNER, "Further congruence properties of the Fourier coefficients of the modular invariant $J(\tau)$," *Amer. J. Math.*, v. 71, 1949, pp. 337–386.

5. M. NEWMAN, "Congruences for the coefficients of modular forms and for the coefficients of $j(\tau)$," *Proc. Amer. Math. Soc.*, v. 9, 1958, pp. 609–612.

6. A. VAN WIJNGAARDEN, "On the coefficients of the modular invariant $J(\tau)$," *Nederl. Akad. Wetensch. Proc. Ser. A*, v. 16, 1953, pp. 389–400.

7. H. S. ZUCKERMAN, "The computation of the smaller coefficients of $J(\tau)$," *Bull. Amer. Math. Soc.*, v. 45, 1939, pp. 917–919.

12 [9].—DANIEL SHANKS, *Table of the Greatest Prime Factor of* $N^2 + 1$ *for* $N = 1(1)185000$, 1959, two ms. volumes, each of 185 computer sheets, bound in cardboard covers and deposited in the UMT file.

This table was calculated in 1959 on an IBM 704 system by the p-adic sieve method described completely in [1]. The method is extraordinarily efficient: each division performed is known a priori to have a zero remainder. From the complete factorization of $n^2 + 1$ for $n = 1(1)185000$ I then tabulated only the greatest