a pervading sense of realism and practicality that will make it an extremely useful volume for applications of mathematics involving second-order linear ordinary differential equations and the classical special functions. The author has been a well-known contributor to the asymptotic theory of such equations for over twenty years. He has worked on the computational as well as on the theoretical aspects of these problems. In his own research, as in this book, he emphasizes results that can be used to compute, be it with pencil and paper or on electronic machines.

So much is known on the asymptotic approximations to solutions of ordinary linear differential equations that no single book can do justice to this whole body of knowledge. The author has wisely limited himself to differential equations of order two and has omitted all theories that do not imply computational results. A very distinctive feature of this book—and also of the author's own work—is the emphasis on error estimates. Usable, realistic inequalities for the remainder in asymptotic expansions are rarely found in the literature. The author has developed a practical scheme for the derivation of such bounds and he applies it throughout the volume.

The mathematical prerequisites are kept simple: Undergraduate level courses in advanced calculus and complex variable theory, and a first course in ordinary differential equations should suffice. It is true, on the other hand, that the presentation becomes more condensed as the book progresses, and some analytic proofs are described so briefly that the reader has to put in quite a bit of thinking to supply the details. To the serious student of the subject, the many examples and the over 500 exercises will be welcome. The variety and interesting nature of the exercises is impressive.

The first seven chapters contain the essentials of the subject: The classical special functions, the basic properties of second-order linear differential equations, the nature of asymptotic series and the various techniques for obtaining them from integral representations as well as from formal expansions. Chapter 6 is probably the most distinctive section of the book. It describes the author's version of what is frequently called the WKB method, a name he sensibly avoids in favor of the historically more accurate one of Liouville-Green Approximation. As developed by the author, it becomes a very flexible asymptotic tool complete with a general formula for a bound on the remainder. The technique is presented in such a way that it applies to asymptotic problems with a large parameter as well as to large independent variables, to unbounded domains as well as to turning point problems. One price that has to be paid for this generality is some lack of motivation at the beginning. It is not clear to the uninformed why certain terms are treated as small with respect to others. However, as the technique is applied in chapter after chapter, eventually the motivation becomes quite transparent.

Most of the material in the later chapters is of a more specialized nature. It includes, among other things, the Euler-Maclaurin formula, refinements of the saddlepoint method, turning and other transition points, and asymptotic connection problems.

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23 [2.25, 4, 7].—F. W. OLVER, *Introduction to Asymptotics and Special Functions*, Student Edition, Academic Press, Inc., New York, 1974, xii + 297 pp., 24 cm. Price $10.00.

The first seven chapters of the above reviewed volume are well suited to form the basis of a one-semester course. In recognition of this fact, that part of the book has been made available separately as a paperback volume. In view of the high cost of books these days, the author and the publisher are to be commended for this service to the public.

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24 [2.35].—W. MURRAY, Editor, *Numerical Methods for Unconstrained Optimization,* Academic Press, New York, 1972, xi + 144 pp., 24 cm. Price $8.95.

During the last fifteen years the field of unconstrained optimization has experienced a phenomenal rate of growth. As in other fields that have grown at such a fast rate, it is rare to find a book that provides a coherent overview of the subject and also clearly describes the latest important research results. This is such a book, and it is a very welcome addition to numerical analysis, and in particular, optimization literature.

The book provides an excellent survey of unconstrained optimization methods, successfully presenting both theoretical results and practical matters such as computer implementation. The material covered is up-to-date and includes results obtained subsequent to the joint IMA/NPL conference in January 1971 at which the papers were originally presented. Considering the very active roles that all of the contributors to this book have played in extending the frontiers of optimization, this is not surprising. Moreover, this reviewer shares the view of the editor that "most" of the material presented will not become obsolete during the next few years.

As in any book containing the contributions of several authors, the style of the chapters varies considerably. On the whole, however, the book is extremely readable. A description of each chapter follows.

Chapter 1 – *Fundamentals,* authored by W. Murray, outlines some basic theory upon which subsequent chapters rely. This includes: definitions, necessary and sufficient conditions for a minimum, properties of quadratic and convex functions, and methods for minimizing functions of a single variable.

Chapter 2 – *Direct Search Methods,* authored by W. H. Swann, surveys methods which depend only upon values of the objective function; i.e., methods which do not use derivative information. Discussed here are the well-known "pattern search" method of Hooke and Jeeves, Rosenbrock's method, the Davies, Swann and Campey method, the simplex methods of Spendley, Hext, and Himsworth, and of Nelder and Mead, (not to be confused with the simplex method for linear programming), generalized Fibonacci search, and modifications of these methods. Random search, Box's technique of evolutionary operation and the technique of minimizing with respect to each independent variable in turn are also very briefly described. Just after publication of the book, Powell showed that the latter method can fail on differentiable functions.

Chapter 3 – *Problems Related to Unconstrained Optimization,* authored by M. J. D. Powell, is concerned with the solution of two types of problems via unconstrained optimization: nonlinear least-squares and constrained optimization problems. There is a clear and informative discussion of the Gauss-Newton and Marquardt methods and of modified versions of these for dealing with least-squares problems. There is also an excellent discussion of algorithms requiring only function values based upon the generalized secant method and the quasi-Newton approach. For constrained problems, transformation of variables, penalty function methods and Lagrangian methods are discussed. Practitioners who have such problems to solve should take special note of the section