REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

39 [2.05, 2.10, 6.15].—L. M. DELVES & J. WALSH, Editors, *Numerical Solution of Integral Equations,* Clarendon Press, Oxford, 1974, 335 pp., 24 cm. Price $14.50.

The book consists of a collection of papers, by various authors, presented at the University of Manchester Summer School in July 1973, dealing with methods and principles in the numerical solution of integral and integro-differential equations.

The material is divided into three parts. Part 1, consisting of the first five chapters, gives a brief overview of the mathematical tools necessary in the numerical analysis of integral equations. The topics discussed are: the theory of linear integral equations, numerical integration, linear algebra, functional analysis, and approximation theory.

Part 2, Chapters 6–18, deals with the actual numerical methods for solving various integral equations, including Fredholm equations of first and second kinds, Volterra equations of first and second kinds, various ordinary and partial integro-differential equations, and nonlinear equations and systems. This is a reasonably complete and up-to-date survey of known numerical techniques. Both theoretical and practical aspects are considered.

Part 3, Chapters 19–25, is a selection of various applications, such as potential and flow problems, water waves, and diffraction and scattering. These are examples of the usefulness of integral equation techniques in the solution of a variety of problems from engineering and physics.

The papers in this volume are expository and written in a rather informal style. The material is generally presented in outline form, without proofs or undue rigor, emphasizing principles rather than technical or algorithmic detail. For more specific information the reader will have to consult the cited references; most papers fortunately include a good bibliography. The apparent aim of the book is to provide the reader with a quick and painless introduction to the use of integral equations in practical applications. In this aim it has succeeded quite well.

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40 [7.00].—R. B. DINGLE, *Asymptotic Expansions: Their Derivation and Interpretation,* Academic Press, New York and London, 1973, xv + 521 pp., 24 cm. Price $38.00.

This is a valuable book on asymptotics. The author acknowledges he is a theoretical physicist; but when it comes to the subject of asymptotics, his use of theory as it relates to mathematical rigor in establishing that certain series are asymptotic, as the concept is usually employed, plays a minor role. The tempo is set in the opening sentence of what is called the prologue. "Throughout this book, the designation 'asymptotic series' will be reserved for those series in which for large values of the variable at all phases the terms first progressively decrease in magnitude, then reach a minimum and thereafter increase." More on this point, he later states that the "exposition will be heuristic and descriptive rather than rigorously doctrinaire." The author cites examples of such asymptotic series dating from the times of Stirling, Euler and Maclaurin. He notes the Poincaré definition for an asymptotic power series and criticizes it because of certain deficiencies which can arise in the pragmatic use of such series.

For convenience, let x be real and positive,

$$F(x) = f(x) + g(x), \qquad f(x) = \sum_{k=0}^{n-1} a_k x^{-k} + R_n(x).$$

According to Poincaré, if M_n is free of x,

$$|R_n(x)| \leqslant M_n x^{-n}, \quad \text{i.e. } R_n(x) = O(x^{-n}) \quad \text{as } x \longrightarrow \infty,$$

then the infinite series $\Sigma_{k=0}^{\infty} a_k x^k$ is asymptotic to $f(x)$ and one writes

$$f(x) \sim \sum_{k=0}^{\infty} a_k x^{-k}.$$

If $g(x)$, for example, is of the form

$$g(x) = e^{-\lambda x} h(x), \qquad \lambda > 0, \qquad h(x) = \sum_{k=0}^{n-1} h_k x^{-k} + O(x^{-n}), \qquad x \longrightarrow \infty,$$

then the series $\Sigma_{k=0}^{\infty} a_k x^{-k}$ is also asymptotic to $F(x)$. Hence a series can be asymptotic to more than one function. In practice, we desire to use the series for finite values of x. Here $F(x)$ cannot always be efficiently approximated by $f(x)$ as the contribution of $g(x)$ might be significant.

The Poincaré definition is a theoretical guide and serves to characterize the dominant part of the contribution to $F(x)$ as $x \longrightarrow \infty$. The definition can also apply to the function $e^{\lambda x}[F(x) - f(x)]$. I am sure Poincaré recognized the disparity between the definition of an asymptotic series and its use in numerical evaluation. The above discussion shows that to safely use asymptotic series, one should know the complete representation for $F(x)$; and if $F(x)$ is approximated by $f(x)$ which in turn is approximated by $\Sigma_{k=0}^{n-1} a_n x^{-k}$, then one should have bounds for the errors committed by truncation. The theory states only that $x^n |R_n(x)|$ be bounded as $x \longrightarrow \infty$. It does not say how sharp the bound must be.

The author's criticism of the Poincaré definition and his interpretation of the state of affairs as being vague and severely limited as to accuracy and range of applicability is not fair. Thus in his work the formal results obtained are not proved asymptotic in the Poincaré sense (or in a more generalized sense not noted here), and there is little on error bounds.

In his research which began around 1955, the author concentrated on deriving the 'complete asymptotic representation' of functions including the exponentially small terms, if any. A considerable part of the present volume is based on this research. Most examples treated are of hypergeometric type which are a special case of Meijer's G-function. In a series of papers dating from 1946, Meijer* derives the "complete asymptotic representation" of the G-function. I find it strange that this work is totally ignored except for mention of a single Meijer paper in the references on p. 55. Numerous other important references are omitted.

A brief outline of the work follows. Chapter 1 is a "behavioral survey" of asymptotics. Chapter 2 is concerned with the derivation of asymptotic series from converging series. Chapter 3 deals with the conversion of power series into integral representations.

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*C. S. MEIJER, "On the G-function. I–VIII," *Nederl. Akad. Wetensch. Proc. Ser. A*, v. 49, 1946, pp. 227–237, 344–356, 457–469, 632–641, 765–772, 936–943, 1063–1072, 1165–1175. A rather complete summary of this work is given in Y. L. LUKE, *The Special Functions and Their Approximations*, Vol. 1, Academic Press, New York, 1969. See also *Math. Comp.*, v. 26, 1972, pp. 297–299.

Chapters 4–11 treat the derivation of asymptotic expansions, both uniform and nonuniform, from various types of integral representations. Derivation of asymptotic series, both uniform and nonuniform, from homogeneous and inhomogeneous differential equations is taken up in Chapters 12–20. In Chapters 21–26, there is presented the theory of terminants, a topic related to the idea of converging factors. In this connection, it is known that in many cases approximations based on finite sections of asymptotic series can be weighted to produce converging series which are far more efficient than using a finite section of the asymptotic series up to and including the smallest term augmented by the portion furnished by the converging factor method. This aspect is not considered.

In summary, the volume contains a wealth of information. Though much of it is formal and error estimates are wanting, the tome is valuable for its many approximations and ideas.

Y. L. L.

41 [13.40].—MARTIN GREENBERGER, JULIUS ARONOFSKY, JAMES L. MCKENNEY & WILLIAM F. MASSY, *Networks for Research and Education: Sharing Computer and Information Resources Nationwide,* The M.I.T. Press, Cambridge, 1974, xv + 418 pp., 24 cm. Price $12.50.

This book presents the papers, discussions and analyses of three working seminars and twelve workshop reports held in late 1972 and early 1973 sponsored by NSF and conducted by EDUCOM. The seminars were designed to help identify the central issues in building and operating networks on a national basis. The term networks was used to designate "the more general set of activities and arrangements whereby computers and communications are used for extensive resource sharing by a large number of separate, independent organizations". The main area chosen was networking for research and education on the national level.

The seminar themes included:

1. User characteristics and needs–discussions on the way in which computing and information are used.

2. Organizational matters–discussions on topics of network management, institutional relations, user organizations and regional computing systems.

3. Operations and funding–discussions on computers and communications, software systems and operating procedures, applications development and user services and network economics and funding.

In an overview, the editors discuss highlights of the issues covered and present conclusions and recommendations of the seminars they feel were identified during the discussions. The conclusions presented are worth restating here:

1. Computer networking must be acknowledged as an important new mode for obtaining information and computation. It is a real alternative that needs to be given serious attention in current planning and decision making.

2. The major problems to be overcome in applying networks to research and education are political, organizational, and economic in nature rather than technological.

3. Networking does not in and of itself offer a solution to current deficiencies. What it does offer is a promising vehicle with which to bring about important changes in user practices, institutional procedures, and government policy that can lead to effective solutions.

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