

equated to questions of programming technique rather than of methodology. This last is consistent with the very elementary level of the mathematics, and cannot be faulted.

The aforementioned level of the number theoretical material is quite basic, and certainly within the scope of an average undergraduate class. However, because of the tight mathematical presentation, the number theory itself would require considerable teacher supplementation. On the other hand, various attractive examples are provided which motivate at several levels. These include chapters on determining prime factors, calendar analysis, and palindromic numbers; all of these being pointed principally towards related programming problems.

An overall summary view of the content of this book would reveal a modest amount of number theory together with a much larger amount of Fortran programming, both rather compactly presented. Whereas the amount of number theory technique which emerges is quite limited, the programming aspects fare better due to the detailed specimen programs.

Reactions to a noble experiment such as this one are of necessity subjective. This reviewer finds that the proposed marriage of number theory and Fortran IV leaves the former in a somewhat henpecked state. However, connubial matters of this sort are not that cut and dried. The author does succeed in exciting one's interest in the possibility of such a union and a search for an optimal basis of compatibility. Giving a course from this text might very well serve as a good starting point for such a quest.

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44 [4.00].—ROBERT E. O'MALLEY, JR., *Introduction to Singular Perturbations*, Academic Press, New York, 1974, viii + 206 pp., 25 cm. Price \$16.50.

The term "singular perturbations" in this book refers to the study of ordinary differential equations which are modified by adding a small term of higher order of differentiation. The equation $\epsilon y'' + y' + y = 0$, which is a singular perturbation of the first order equation $y' + y = 0$, is a trivial illustration of this concept.

Until the late 1930's this type of problem was almost completely ignored by mathematicians, although the phenomena met in the boundary layer theory of Fluid Dynamics were known to be mathematically described by such differential equations. Since then, the theory of singular perturbations has grown into a substantial field of study, which has attracted numerous mathematicians in many countries. It has been recognized that questions of this sort often have surprising and fascinating mathematical answers and that singular perturbation aspects explain more physical phenomena than anybody would have foreseen forty years ago. The author of this book is one of the leading contributors to the progress in singular perturbation theory in the last ten years.

The book is written by a mathematician in the spirit of mathematics; but it is also, perhaps primarily, intended for users of mathematics in physics, engineering, biology and economics. The mathematical prerequisites are therefore held elementary, essentially at the undergraduate level, and many recondite matters of existence and of asymptotic smallness are omitted, with references to the appropriate literature. In the choice of subjects and of methods, the author has been influenced by his personal preferences and experience. Wherever possible, a unifying principle of uniform approximations that are obtained as the sum of two series is used, the first of which is itself a solution of the differential equation, while the second one involves a "stretched variable"

and describes the asymptotic behavior of the true solution in the narrow intervals of rapid transition, typical in these theories.

After an elementary introductory chapter and a brief review of regular perturbation methods, the author develops a thorough theory of linear boundary problems. This is followed by an equally complete account of nonlinear initial value problems, which includes a section on differential-difference equations with small delay. Such delay problems, while not strictly singular perturbations in the sense of the original definition, have so many analogous features that their inclusion is justified. The next chapter, on nonlinear boundary value problems, described an intrinsically more difficult and less completely explored subject and is, therefore, less general than the preceding one. The remaining three chapters are devoted, respectively, to special types of optimal control problems, a boundary value problem with multiple solutions that arises in chemical reactor theory, and some turning point problems. Here and elsewhere in the book, applications are mentioned for motivation, but are never discussed as such.

The writer's style is lively and natural, and the techniques are well motivated. The author's facility with complicated algebraic manipulations may have led him to describe some of these calculations with less detail than this reviewer would have liked. An attentive reader will, however, be able to fill in these gaps himself. In the chapter on the regulator problem (Chapter 6), a certain amount of previous experience with such problems is expected from the reader. But these are minor objections.

The book is an up-to-date, attractive introduction to a subject whose importance for the applications is bound to grow.

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45 [5.10.3].—J. ROBINSON, *Integrated Theory of Finite Element Methods*, John Wiley & Sons, London, 1973, xxi + 428 pp., 24 cm. Price \$29.50.

First of all, the title of the book is somewhat a misnomer. It does not contain an integrated account of finite elements nor is it a book on the theory of finite element methods. It is a book principally aimed at an engineering audience, which deals primarily with the formulative aspects of finite element methods applied to wide classes of problems in the analysis of elastic structures. While the introductory chapter does outline in broad strokes a review of some of the elementary properties of finite element methods, the book is certainly more readable to those with some experience with the method. The book deals with linear, static, finite-dimensional systems, and matrix notation is used throughout. It contains very little on computational methods, the numerical aspects of finite elements, equation solving, convergence, or on errors inherent in the method.

The first chapter of the book contains a brief account of the basic force method, the displacement method and the closely related direct stiffness method, and three "combined" methods, one of which involves the use of Lagrange multipliers. The principal aim of this chapter is to show various ways in which systems of equations governing large collections of elements can be properly assembled, for various choices of the dependent variables. It does not dwell on how the matrix relations governing individual elements are obtained.

The book contains a considerable amount of material on the force method, a subject in which its author has invested a number of years of work and made some contributions. To date, virtually all mathematical work on finite element methods has been