and describes the asymptotic behavior of the true solution in the narrow intervals of rapid transition, typical in these theories.

After an elementary introductory chapter and a brief review of regular perturbation methods, the author develops a thorough theory of linear boundary problems. This is followed by an equally complete account of nonlinear initial value problems, which includes a section on differential-difference equations with small delay. Such delay problems, while not strictly singular perturbations in the sense of the original definition, have so many analogous features that their inclusion is justified. The next chapter, on nonlinear boundary value problems, described an intrinsically more difficult and less completely explored subject and is, therefore, less general than the preceding one. The remaining three chapters are devoted, respectively, to special types of optimal control problems, a boundary value problem with multiple solutions that arises in chemical reactor theory, and some turning point problems. Here and elsewhere in the book, applications are mentioned for motivation, but are never discussed as such.

The writer's style is lively and natural, and the techniques are well motivated. The author's facility with complicated algebraic manipulations may have led him to describe some of these calculations with less detail than this reviewer would have liked. An attentive reader will, however, be able to fill in these gaps himself. In the chapter on the regulator problem (Chapter 6), a certain amount of previous experience with such problems is expected from the reader. But these are minor objections.

The book is an up-to-date, attractive introduction to a subject whose importance for the applications is bound to grow.

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45 [5.10.3].—J. ROBINSON, *Integrated Theory of Finite Element Methods*, John Wiley & Sons, London, 1973, xxi + 428 pp., 24 cm. Price $29.50.

First of all, the title of the book is somewhat a misnomer. It does not contain an integrated account of finite elements nor is it a book on the theory of finite element methods. It is a book principally aimed at an engineering audience, which deals primarily with the formulative aspects of finite element methods applied to wide classes of problems in the analysis of elastic structures. While the introductory chapter does outline in broad strokes a review of some of the elementary properties of finite element methods, the book is certainly more readable to those with some experience with the method. The book deals with linear, static, finite-dimensional systems, and matrix notation is used throughout. It contains very little on computational methods, the numerical aspects of finite elements, equation solving, convergence, or on errors inherent in the method.

The first chapter of the book contains a brief account of the basic force method, the displacement method and the closely related direct stiffness method, and three "combined" methods, one of which involves the use of Lagrange multipliers. The principal aim of this chapter is to show various ways in which systems of equations governing large collections of elements can be properly assembled, for various choices of the dependent variables. It does not dwell on how the matrix relations governing individual elements are obtained.

The book contains a considerable amount of material on the force method, a subject in which its author has invested a number of years of work and made some contributions. To date, virtually all mathematical work on finite element methods has been concerned with so-called stiffness techniques; and those searching for new research problems in finite element methods might find plenty connected with the force method described in this book. Historically, the force method did not receive serious attention in the finite element literature because of difficulties in formulating flexibility matrices for large complicated structural systems. In more recent years, it has been recognized that the force method really involved a discretization of the dual problem in linear elasticity and, as such, can be attacked quite generally if one introduces the notion of stress functions or displacement potentials. This point of view is described in some detail in Chapter 2 of the book, and appropriate stress functions for second-order three-dimensional, and fourth-order two-dimensional problems are presented. Chapter 3 contains a collection of simple examples wherein the ideas outlined in Chapter 2 are used to produce various structural matrices. Chapter 3 is devoted to "strain elements" while Chapter 4 is aimed at the dual problem and describes "stress elements".

Chapter 5 deals with "inconsistent elements" by which the author means elements for which spurious rigid motions or equilibrating stress fields are contained. The author claims that these can be used effectively, but his arguments are not convincing. Chapter 6 is devoted to a readable account of conventional isoparametric elements and contains a number of examples.

Chapter 7 of the book is devoted to "isoparametric stress elements" and is quite an interesting chapter. Here the notions of isoparametrics are applied to the dual problem by representing stress functions as combinations of shape functions used in the coordinate transformations. This is apparently an original idea, and it provides an interesting family of new finite elements whose properties have not been explored mathematically to date.

That the generation of flexibility matrices is not as straightforward as the conventional matrix approach becomes clear in reading Chapters 8, 9, and 10 of the book, in which a great deal of algebra must be used to produce the proper matrices. However, there are some advantages to these methods for problems of singularities as is pointed out in Chapter 11 which is devoted to "cracked finite elements".

The book is unconventional, and it is unlikely that it will be used widely as a textbook for engineers interested in finite element methods, or as a reference for those interested in computational methods. Nevertheless, it does contain a number of new ideas which are worthy of consideration, not only by the practitioner but also by the theoretician.

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46 [12.05.1].—MARVIN SCHAEFER, *A Mathematical Theory of Global Program Optimization,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1973, xvii + 198 pp., 23 cm. Price $12.50.

The title of Schaefer's book would lead the reader to expect a unified mathematical theory of the subject area. The book, however, is something quite different; it is an assortment of optimization techniques mostly couched in mathematical notation. It is composed of two principal parts and three appendices. Part I, "Theoretical Foundations", is concerned with graph-theoretic methods. Part II, "Applications to Global Program Analysis", discusses a number of specific techniques, using some of the results in Part I.

The graph-theoretic methods developed in Part I are, for the most part, quite