concerned with so-called stiffness techniques; and those searching for new research problems in finite element methods might find plenty connected with the force method described in this book. Historically, the force method did not receive serious attention in the finite element literature because of difficulties in formulating flexibility matrices for large complicated structural systems. In more recent years, it has been recognized that the force method really involved a discretization of the dual problem in linear elasticity and, as such, can be attacked quite generally if one introduces the notion of stress functions or displacement potentials. This point of view is described in some detail in Chapter 2 of the book, and appropriate stress functions for second-order three-dimensional, and fourth-order two-dimensional problems are presented. Chapter 3 contains a collection of simple examples wherein the ideas outlined in Chapter 2 are used to produce various structural matrices. Chapter 3 is devoted to "strain elements" while Chapter 4 is aimed at the dual problem and describes "stress elements".

Chapter 5 deals with "inconsistent elements" by which the author means elements for which spurious rigid motions or equilibrating stress fields are contained. The author claims that these can be used effectively, but his arguments are not convincing. Chapter 6 is devoted to a readable account of conventional isoparametric elements and contains a number of examples.

Chapter 7 of the book is devoted to "isoparametric stress elements" and is quite an interesting chapter. Here the notions of isoparametrics are applied to the dual problem by representing stress functions as combinations of shape functions used in the coordinate transformations. This is apparently an original idea, and it provides an interesting family of new finite elements whose properties have not been explored mathematically to date.

That the generation of flexibility matrices is not as straightforward as the conventional matrix approach becomes clear in reading Chapters 8, 9, and 10 of the book, in which a great deal of algebra must be used to produce the proper matrices. However, there are some advantages to these methods for problems of singularities as is pointed out in Chapter 11 which is devoted to "cracked finite elements".

The book is unconventional, and it is unlikely that it will be used widely as a textbook for engineers interested in finite element methods, or as a reference for those interested in computational methods. Nevertheless, it does contain a number of new ideas which are worthy of consideration, not only by the practitioner but also by the theoretician.

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46 [12.05.1].—MARVIN SCHAEFER, *A Mathematical Theory of Global Program Optimization,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1973, xvii + 198 pp., 23 cm. Price $12.50.

The title of Schaefer's book would lead the reader to expect a unified mathematical theory of the subject area. The book, however, is something quite different; it is an assortment of optimization techniques mostly couched in mathematical notation. It is composed of two principal parts and three appendices. Part I, "Theoretical Foundations", is concerned with graph-theoretic methods. Part II, "Applications to Global Program Analysis", discusses a number of specific techniques, using some of the results in Part I.

The graph-theoretic methods developed in Part I are, for the most part, quite specific to the requirements of optimization. The main results given in this part are the partitioning of a program graph into intervals (regions with a single entry node); numbering algorithms that display the interval structure of a program graph (the Basic Numbering Algorithm and the Strict Numbering Algorithm); node splitting techniques for transforming irreducible graphs into reducible ones; and applications of the graph connectivity matrix. (A graph is reducible if its sequence of derived graphs leads to a single node. The first element of this sequence is the original graph; successive elements are obtained by treating each interval in the current element as a single node of the next element.)

In the first chapter of Part II, a method is given for determining those variables whose values are available at exit from each node of the program (data flow analysis). The next chapter discusses constant subsumption, common subexpression suppression, and code motion. In constant subsumption, calculations involving constants that might have to be done many times at run time are replaced by single calculations performed at compile time. In common subexpression suppression, multiple occurrences of the calculation of the value of an expression are replaced by a single occurrence of the calculation in the case where the different occurrences are known to produce the same result. In code motion, identical calculations appearing in several different paths are moved to a common point on the paths.

The chapter on loop optimization considers two techniques: invariant expression removal and reduction in strength of operators. An invariant expression of a loop is an expression whose value is known not to change during the execution of the loop; the calculation of the value of such expressions is profitably moved outside the loop. The "reduction in strength" optimization consists of replacing multiplication by addition and, less frequently, exponentiation by multiplication.

A short chapter on safety and execution frequency analysis follows. Some of the problems of insuring that program behavior remains invariant under optimization are discussed here. The remaining chapters contain brief discussions of subroutine linkages, register allocation, and the elimination of dead (unused) variables. The three appendices present, respectively, a collection of selected algorithms in APL; the organization of a specific optimizing compiler; and the problems of partial recompilation in an interactive environment.

Schaefer's book is deeply flawed in two respects. First, it is shot through with minor errors as well as sloppy proofs and explanations. The examples are far far too numerous to list exhaustively, but some samples can be given. On page 15, line 5, the minimal element should be a maximal one. On page 17, the "min" operation does not yield a unique result. On page 70, algorithm 7.12, step 2b, the paths should be restricted to lie within $T(i)$. On page 85, Figure 8.1(b), the "α" in the middle should be an "a".

A second serious flaw lies in the obscurity of the presentation. A particularly noxious example of this is Algorithm 6.19 for node splitting, on page 46, which is heavily encrusted with triply subscripted and superscripted variables. This algorithm is almost impossible to comprehend, and there is no explanation to accompany it; furthermore, it appears to contain errors. Other examples appear in the chapter on vertex ordering algorithms. Three desirable properties of such algorithms are stated at the beginning of the chapter; but nowhere is it either stated or proved that exactly these properties hold for either or both of the two algorithms presented. Furthermore, it is not apparent whether or not the Strict Numbering Algorithm is a special case of the Basic Numbering Algorithm, i.e., whether the order obtained from the SNA could have been obtained from the BNA with an appropriate series of arbitrary choices. Yet another example is the introduction, without explanation, of the variables x_h^M and x_h^m on page 89. It is only after reading several pages on that one becomes aware of the intent of these variables. In a slightly different vein, Appendix II starts with the following paragraph:

> This Appendix describes the architecture of a specific optimizing compiler in which the order of the various analyses is based on pragmatic considerations of the algorithmic source language. A modification of the order of the analyses might be more appropriate for languages of a different type.

However, nowhere is it stated what language is being compiled by this compiler, nor are any references given which might enable the reader to identify either the language or the compiler. Since the author states that the language is a major consideration in the design, the omission seems inexcusable.

It should also be noted that much of the material in Part II is given informally in English, in the form of general heuristics for the writer of an optimizing compiler. While this material is useful, it does not bear the slightest resemblance to a mathematical theory.

In summary, this book can be useful as a compendium of techniques, but it does not develop any unifying principles. For a mathematician, it is unlikely to be of much interest. For a compiler writer, it can serve as a useful source of ideas. On account of its muddy explanations and careless editing, however, it just is not worth the effort of a close reading.

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47 [13.15].— ANDREW B. TEMPLEMAN, Editor, *Engineering Optimization*, Vol. 1, No. 1 (1974); Gordon & Breach, New York, 1974, 69 pp., 26 cm. Price $15.00 for individuals, $52.00 for institutions.

Recognition of the common interest of differing branches of engineering in optimization has prompted the founding of this interesting journal. Although not excluding the publication of "technique" papers which concentrate on algorithms for solving narrowly-stated or algebraically-described optimization problems, the main emphasis will be on the formulation of optimization models and experiences in solving them using existing techniques. Perusal of the first issue shows a breadth of concern from the purely practical ("The cost of obtaining the final design is as significant as the optimality of the final design") to such theoretical/practical considerations as the sensitivity of the optimal design to errors in bounding moments given by plate finite element analysis. The journal should be particularly valuable as a source of case studies and examples of optimization for classroom use. A list of the papers in the first issue is: "The application of optimization techniques in the professional practice," "The optimum design of concrete structures," "Optimality conditions for trusses with nonzero minimum cross-sections," "Heuristic approaches to road network optimization," "The CIRIA optimization study of sewage treatment," "Michell framework for uniform load between fixed supports," and a particularly fine introductory article by the editor, A. B. Templeman, "*Engineering Optimization*—scope and aims."

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