Since $E(m)$ approximately doubles each line, *empirically* the number of solutions with $n \leqslant N$ can be expected to increase something like $N^{0.3}$. There is little doubt that there are infinitely many solutions, but this has not been proven.

However, a $N^{0.3}$ growth does suggest that there are only finitely many triples:

$$\phi(n) = \phi(n + 1) = \phi(n + 2);$$

and, in fact, none is known besides the single triple $n = 5186$ discovered long ago.

Of these three hundred and six solutions, I find that there are only twenty-three where the $\phi(n)$ residue classes prime to n have the same abelian group under multiplication (mod n) as the $\phi(n + 1)$ classes have (mod $n + 1$). These twenty-three are determined from the listed factorizations as in [2, Theorem 43, p. 93]. The twenty-three are $n =$

1	3	15	104
495	975	22935	32864
57584	131144	491535	2539004
3988424	6235215	7378371	13258575
17949434	25637744	26879684	29357475
32235735	41246864	48615735	

For example, for the largest n here, the $\phi(n) = \phi(n + 1)$ residue classes both have the abelian group

$$C(2) \times C(2) \times C(2) \times C(12) \times C(230208).$$

It is not unlikely that there are infinitely many solutions even with this much more stringent requirement, but note that there are none at all in the second half of the range of n here.

D. S.

1. DAVID BALLEW, JANELL CASE & ROBERT N. HIGGINS, *Table of $\phi(n) = \phi(n + 1)$*, UMT 2, *Math. Comp.*, v. 29, 1975, pp. 329–330.

2. DANIEL SHANKS, *Solved and Unsolved Problems in Number Theory*. Vol. I, Spartan, Washington, D. C., 1962.

7 [10].—LOUIS COMTET, *Advanced Combinatorics*, D. Reidel Publishing Co., Dordrecht, Holland; Boston, Mass., 1974, translated from the French by J. W. Nienhuys, xi + 343 pp. Price \$34; \$19.50 paperback.

The original French edition, *Analyse Combinatoire*, appeared in 1970 in two pocketsize paper-covered volumes of modest price; a review, by the present reviewer, is published in *Math. Rev.*, v. 41, 1971, #6697. The current edition, as advertised, is revised and enlarged; the most evident revision is the absence of footnotes, now absorbed in the text, and it is also apparent that there are many additional "supplements and exercises", the author's variation on the conventional problem section. While much of my earlier review is still relevant, the book comes to me now in a new light, probably because I am more at home in English than in French.

Incidentally, the translation has a Dutch accent and seems to have lost the author's French elegance; one oddity is the use of figured for figurate (numbers).

In the new light, the book appears as a continuation of traditional combinatorial analysis, made current by greater use of the vocabulary of set theory. To some combinatorialists this is a great step forward; it supplies a sorely lacking mathematical respectability. To me, the prescribed use of any particular vocabulary is a Procrustean bed, the more irrelevant as combinatorial mappings proliferate.

However, the author's uses of his vocabulary are both elegant and uninhibited, with lively appreciation of the protean aspects of his subject. One nice example is his treatment of bracketing problems (Chapter I, p. 52) where the familiar Catalan problem is exposed both in the traditional manner and in the equivalent form of trivalent plane trees, the latter derived *ab ovo.* Moreover, two other representations, triangulations of a convex polygon, and majority paths (also known as weak-lead ballots) are mentioned, strangely without reference to H. G. Forder's elegant mappings in a paper included in the book's bibliography. The new settings discovered in the last five years, many unpublished, convey the strong feeling that the last word on this subject will be a long time coming.

In content, the emphasis is on variety rather than depth; paraphrasing a remark in the introduction, the book is fairly described as various questions of elementary combinatorial analysis. The variety is impressive; it extends from the inevitable material on permutations and combinations to necklaces, set coverings, Steiner triple systems, Ramsey numbers, Sperner systems, postage stamp foldings, polynomials, permanents, tournaments, full sets, geometries, T_0-topologies, and so on. Most of these appear as exercises, and in compressed exposition, though with references to more expanded treatments. It should be noted that a number of exercises are devoted to enumerations in which the structure is so elusive that no general numerical results are known. A celebrated example is postage stamp folding. The reference list is extensive, often up to the minute, but for me especially valuable in spelling out the abbreviated early references in Netto. One remaining abbreviation bothers me. Authors are identified only by surnames, which seems dangerous. Maybe it works; I have not found a single case where different authors are confounded. However, the Newman appearing alone is Morris, the one with collaborators is Donald J.

I omit further consideration of combinatorial aspects of these problems, in order to give attention to the profuse numerical tables, which may be of greater interest to readers of this journal.

The following are of general interest: Fibonacci numbers, 0 (1) 25; Lucas numbers, 1 (1) 12; Bernoulli, Euler and Genocchi numbers, trinomial and quadrinomial numbers, generated by $(1 + t + t^2)^n$ and $(1 + t + t^2 + t^3)^n$; respectively; partitions with distinct parts 1 (1) 22; sums of multinomial coefficients; D'Arcais numbers, generated by powers of the generating function for partitions of numbers; Euler's totient function, 1 (1) 28; tangent numbers, and numbers $T(n, k)$ generated by $(\tan t)^k/k!$; Salie's numbers, generated by $\cosh t/\cos t$; Leibniz numbers, associated with the harmonic triangular array, defined by $(n + 1)\binom{n}{k}L(n, k) = 1$ (the author uses a Gothic L); the number of terms in expressions for derivatives of implicit functions; postage stamp foldings 2 (1) 28; coefficients in the expansion of gamma functions of large arguments (given a combinatorial setting) 1 (1) 7, borrowed from [1]; Cauchy numbers (integrals over the unit interval of falling and rising factorials) 0 (1) 10.

Of course, in most cases, these tables may seem of trivial interest to the experts in their computation.

A few remarks on the section, Fundamental Numerical Tables, which concludes the book, may be helpful. First, the Bell polynomials $Y_n(x_1, x_2, \ldots, x_n)$ are exhibited as the sum $Y_n = B_{n1} + B_{n2} + \cdots + B_{nn}$, with B_{nk} the collection of terms corresponding to partitions with k parts; B_{nk} is called a partial Bell polynomial. This terminology seems to me unfortunate; apparently, it is dictated by the need to write inverse Bell polynomials, which are called logarithmic polynomials, as

$$L_n(x_1, \ldots, x_n) = \sum f_k B_{nk}, \quad f_k = (-1)^{k-1}(k-1)!$$

and so avoid the compressed notation (proscribed in France?):

$$L_n(x_1, \ldots, x_n) = Y_n(fx_1, \ldots, fx_n), \qquad f^k \equiv f_k = (-1)^{k-1}(k-1)!$$

which I have been using for years.

The third kind of Bell polynomial, with the nondescriptive title Partial Ordinary Bell polynomial, may be written

$$\hat{B}_n(x_1, \ldots, x_n) = \sum [k; k_1, \ldots, k_n] x_1^{k_1} \ldots x_n^{k_n},$$

with summation over partitions of $n = k_1 + 2k_2 + \cdots + nk_n$, $k = k_1 + \cdots + k_n$ and $[k; k_1, \ldots, k_n]$ a multinomial coefficient. It stays in the memory as a consequence of the generating function identity, dropping the arguments in $\hat{B}_n$:

$$1 = (1 - x_1 y - x_2 y^2 - \cdots)(1 + \hat{B}_1 y + \hat{B}_2 y^2 + \cdots).$$

Finally, I notice that the definition of coefficients a_{ms} in Exercise 27 of Chapter III (p. 166) does not agree with the table on p. 167. Indeed, the table gives values of $\binom{m}{s} a_s a_{m-s}$, with a_n the double factorial (for odd factors): $a_0 = 1$, $a_n = (2n-1)a_{n-1}$. The simplest recurrence seems to be $(s+1)a_{m,s} = m(2s+1)a_{m-1,s}$.

Also, many of the number sequences in the first edition appear in [2].

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1. J. W. WRENCH, JR., "Concerning two series for the gamma function," *Math. Comp.*, v. 22, 1968, pp. 617–626.

2. NEIL J. A. SLOANE, *A Handbook of Integer Sequences*, Academic Press, New York, 1973.

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8 [8].—E. S. PEARSON & H. O. HARTLEY, Editors, *Biometrika Tables for Statisticians*, Vol. 2, Cambridge Univ. Press, 1972, xvii + 385 pp., 29 cm. Price $17.50.

Abramowitz [*MTAC*, v. 9, 1955, pp. 205–211; see Savage, *Math. Comp.*, v. 21, 1967, pp. 271–273] reviewed *Biometrika Tables for Statisticians*, Volume 1, with remarks which in large part remain appropriate to Volume 2: this is a major continuing project of fine table making. Again there is an extensive introduction of 149 pages to the tables, 230 pages for 69 tables. The authors point out that Volume 2 "is one of many possible companions" to Volume 1. The main difference between Volume 1 and Volume 2 is the computer revolution. The rationalization for Volume 2 deserves full quotation and careful consideration:

"It seems appropriate to comment briefly on the relevance of statistical tables vis à vis the advent of high-speed computers. Indeed it has been argued by some that there is no need for a new volume of statistical tables since any desired numerical value of the mathematical functions involved can be readily computed with the help of fast subroutines loaded into a high-speed computer. Tables, it is argued, will in due course be superseded by a library of algorithms for mathematical functions.

"Whilst we do not wish to underrate the growing importance of the latter, we believe the need for printed tables will be with us for a good time to come, both in the area of (a) data analysis and (b) research in statistical methodology. With regard to (a) there is a real danger that automated, stereotype 'processing' of data may discourage intelligent examination of observations for unexpected features which may suggest new results and interpretations. Such intelligent inspection, besides being often assisted by graphical means, will generally be accompanied by the computation of test criteria, the need to apply which evolves in the course of the examination of the observations; this