REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

9 [**5.05.2.1, 5.05.3.1**].—P. BRENNER, B. THOMÉE & L. B. WAHLBIN, *Besov Spaces and Applications to Difference Methods for Initial Value Problems*, Springer-Verlag, Berlin Heidelberg and New York, 1975, 154 pp., 24 cm. Price $7.80.

The authors of this well-written volume have all made important contributions to the study of finite-difference methods for initial value problems of partial differential equations. The main question addressed by this book is the extent by which the accuracy of a finite-difference method suffers when the initial data is not smooth enough. A quite complete theory, including inverse results, is presented. A few model problems, rather than general parabolic systems, etc., are treated to simplify the presentation. The theory is developed in L_p for general p. The authors show how only a few, well-chosen, extra technical tools are required to extend the theory from L_2 to the general case.

The first two chapters contain introductory material on Fourier multipliers and Besov spaces. This highly useful material has not, to my knowledge, previously been presented in English with a comparable clarity.

Chapter 3 surveys the theory of well-posed initial value problems and stable finite-difference schemes with constant coefficients. Chapter 4 treats the heat equation in a very complete way. A discussion of the effects of smoothing of the initial data is included. The theory for hyperbolic problems is developed in the next chapter. Thomée's interesting application of Besov spaces to a semilinear problem is included. The last chapter, which includes previously unpublished material, develops a theory for the Schrödinger equation.

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10 [**2.00, 3.00, 4.00**].—G. M. PHILLIPS & P. J. TAYLOR, *Theory and Applications of Numerical Analysis*, Academic Press, London and New York, 1973, x + 380 pp., 23 cm. Price $14.95 paperbound.

This is an introductory text for use at the undergraduate level. It is organized in such a way that the only prerequisite is a one-year course in calculus. Background material in linear algebra, e.g., is provided in the appropriate chapters. Although the selection and treatment of topics is fairly conventional, the exposition is exceptionally clear and, to the extent possible, fully supported by mathematical theory. Among topics not included (or only briefly mentioned) are rational and spline approximation, Fourier analysis, polynomial equations, optimization, sparse matrices, overdetermined systems of linear equations, algebraic eigenvalue problems, and partial differential equations. Advanced topics, such as best (polynomial) approximation, systems of nonlinear equations, and boundary value problems for ordinary differential equations, on the other hand, are treated in some detail.

The chapter headings are as follows: 1. Introduction, 2. Basic analysis, 3. Taylor's polynomial series, 4. The interpolating polynomial, 5. "Best" approximation, 6. Numerical differentiation and integration, 7. Solution of algebraic equations of one variable, 8. Linear equations, 9. Matrix norms and applications, 10. Systems of non-linear equations, 11. Ordinary differential equations, 12. Boundary value and other methods for ordinary differential equations. Appendix: Computer arithmetic.

Each chapter has a collection of problems. Solutions to selected problems are given at the end of the book. As is appropriate for a book on this level, there are no references to the literature; instead, there is a list of some key texts.

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11 [2.05].– V. F. DEMYANOV & V. N. MALOZEMOV, *Introduction to Minimax*, John Wiley & Sons, New York, 1974, vii + 307pp., 25cm. Price $20.00.

This book is a thorough introduction to mathematical optimization and is intended for electrical engineers in Russia. The content is outlined.

I. *Tchebycheff approximation by polynomials–discrete case.* The problem is motivated by a data analysis application, formulated precisely and the basic mathematical results (existence, uniqueness and altervation) developed. Two computational methods and the linear programming interpretation are given.

II. *Tchebycheff approximation by polynomials–continuous case.* The development is similar to Chapter I along with various convergence results. The Remes algorithm and discretization method are analyzed in detail.

III. *The discrete minimax problem.* The problem is formulated precisely and various elementary properties developed. The necessary condition (derivative equal zero) and several sufficient conditions for a solution are given. The coordinate direction and steepest descent methods are presented and then three successive approximation methods are analyzed. This is the key chapter of the book.

IV. *The discrete minimax problem with constraints.* The complications introduced by constraints are examined in an analysis somewhat in parallel with Chapter III.

V. *The generalized problem of nonlinear programming.* The generalization of the previous problems is developed along with basic results. Lagrange multipliers and the Kuhn-Tucker theorem are presented. The generalization of the descent and successive approximation methods are presented along with the penalty function method.

VI. *The continuous minimax problem.* The final level of generality and abstraction is reached and developed. Discretization is analyzed and the final two sections return to polynomial approximation.

VII. *Appendices and notes.* There are 60 pages of mathematical material and a short set of notes.

The style of the book is definitely tutorial. It goes from the concrete to the abstract and there are numerous detailed examples. New notation is frequently introduced. A student who covers this material will have a solid background in mathematical optimization.

The principal weakness of the book is that it is not up-to-date. In some areas the mathematical aspects have developed considerably beyond that presented here. For example, the Remes algorithm is shown to be linearly convergent but over 10 years ago H. Werner showed it to be quadratically convergent. The newer and more effective methods such as Davidon, variable metric, Fletcher-Powell, etc. are not mentioned for the nonlinear programming problems. The influence of high speed computers is not seen; the aim of this book is the treatment of small problems.