The chapter headings are as follows: 1. Introduction, 2. Basic analysis, 3. Taylor's polynomial series, 4. The interpolating polynomial, 5. "Best" approximation, 6. Numerical differentiation and integration, 7. Solution of algebraic equations of one variable, 8. Linear equations, 9. Matrix norms and applications, 10. Systems of non-linear equations, 11. Ordinary differential equations, 12. Boundary value and other methods for ordinary differential equations. Appendix: Computer arithmetic.

Each chapter has a collection of problems. Solutions to selected problems are given at the end of the book. As is appropriate for a book on this level, there are no references to the literature; instead, there is a list of some key texts.

W. G.

11 [2.05].– V. F. DEMYANOV & V. N. MALOZEMOV, *Introduction to Minimax*, John Wiley & Sons, New York, 1974, vii + 307pp., 25cm. Price $20.00.

This book is a thorough introduction to mathematical optimization and is intended for electrical engineers in Russia. The content is outlined.

I. *Tchebycheff approximation by polynomials–discrete case.* The problem is motivated by a data analysis application, formulated precisely and the basic mathematical results (existence, uniqueness and altervation) developed. Two computational methods and the linear programming interpretation are given.

II. *Tchebycheff approximation by polynomials–continuous case.* The development is similar to Chapter I along with various convergence results. The Remes algorithm and discretization method are analyzed in detail.

III. *The discrete minimax problem.* The problem is formulated precisely and various elementary properties developed. The necessary condition (derivative equal zero) and several sufficient conditions for a solution are given. The coordinate direction and steepest descent methods are presented and then three successive approximation methods are analyzed. This is the key chapter of the book.

IV. *The discrete minimax problem with constraints.* The complications introduced by constraints are examined in an analysis somewhat in parallel with Chapter III.

V. *The generalized problem of nonlinear programming.* The generalization of the previous problems is developed along with basic results. Lagrange multipliers and the Kuhn-Tucker theorem are presented. The generalization of the descent and successive approximation methods are presented along with the penalty function method.

VI. *The continuous minimax problem.* The final level of generality and abstraction is reached and developed. Discretization is analyzed and the final two sections return to polynomial approximation.

VII. *Appendices and notes.* There are 60 pages of mathematical material and a short set of notes.

The style of the book is definitely tutorial. It goes from the concrete to the abstract and there are numerous detailed examples. New notation is frequently introduced. A student who covers this material will have a solid background in mathematical optimization.

The principal weakness of the book is that it is not up-to-date. In some areas the mathematical aspects have developed considerably beyond that presented here. For example, the Remes algorithm is shown to be linearly convergent but over 10 years ago H. Werner showed it to be quadratically convergent. The newer and more effective methods such as Davidon, variable metric, Fletcher-Powell, etc. are not mentioned for the nonlinear programming problems. The influence of high speed computers is not seen; the aim of this book is the treatment of small problems.

The translation is of high quality and no misprints were noted. The references are alphabetized according to the Russian spellings.

J. R.

12 [4.00, 5.00, 6.00].—RICHARD BELLMAN & MILTON G. WING, *An Introduction to Minimax*, John Wiley & Sons, Inc., New York, 1975, 250 pp., 23 cm. Price $18.95.

For over twenty years Bellman and Wing have devoted much effort to developing and popularizing a mathematical technique which they call the method of invariant imbedding. They have now collaborated on a textbook/monograph which gives a wide ranging exposition of what might be called the classical method of invariant imbedding. The authors view their approach as a perturbation method for general mathematical systems where the structure of the system is varied and a functional relationship is derived which describes the behavior of the system under such perturbations. For example, in the context of two-point boundary value problems for ordinary differential equations the solution is considered to be a function not only of the independent variable but also of the length of the interval of integration and of the boundary values. The related invariant imbedding equation then describes the behavior of the solution as these variables are changed. However, unlike some earlier books on invariant imbedding, this book is not restricted to boundary value problems for ordinary differential equations; instead, it is the authors' intent to provide a "toolchest of invariant imbedding methods" which will allow the reader to find the invariant imbedding formulation for a variety of applications.

The book consists of twelve chapters; nine of them deal with two-point boundary value problems for ordinary and partial differential equations. Others explore invariant imbedding for random walk problems, wave propagation and integral equations. Throughout, whenever possible the terminology of particle transport theory is used and much effort is devoted to obtaining the imbedding equations for various Boltzmann transport equations. An extensive collection of problems is provided at the end of each chapter.

The level of presentation throughout the book is fairly elementary; the emphasis is on deriving, and occasionally solving, the invariant imbedding equations through formal manipulation or on physical grounds. Indeed, it is the authors' expressed intent to avoid all "mathematical pseudosophistication" so that the book be accessible to a variety of readers.

The overall impression is not of a book with a concise new mathematical technique but of a compendium of novel applications of one and multiparameter (operator) continuation methods (with the range of integration as the key imbedding parameter). Numerical analysts, however, will likely find the book to be of limited value since the authors by choice do not explore the computational aspects of their method. Throughout, the claim is implicit that the invariant imbedding equations, which typically are of evolution type, are easier to solve than alternate formulations. This is neither true in general since the continuation may terminate prematurely nor helpful in those cases where the equations have classical solutions since numerical stability and machine memory limitations abound. There is computational and theoretical merit to the initial value formulation now associated with the name of invariant imbedding. However, the authors' uncritical exposition is not likely to dispel the reservations widely held against this method.

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