

Chapter 2 the author introduces random variables, expectation and variance. The important notion of a martingale is also given, and illustrated with a simple "stock market" model. Chapter 3 deals with limit theorems, but only by approximation to finite range experiments. The discussion includes the weak law of large numbers, central limit theorem and arc sine law. Each is illustrated with illuminating computer graphics and several simulations. Key ideas, e.g. Chebyshev's inequality and the reflection principle, are discussed, but details of proofs are often omitted. The final chapter gives the basic theory of finite Markov chains, culminating in the limit theorem for regular chains. The text is complemented by many problems, of greatly varying difficulty, often involving the writing of a BASIC program.

The book constitutes a novel approach to elementary probability theory, which should appeal to students and teachers interested in a computer oriented perspective. The tenor of the discussion is casual, with an emphasis on ideas rather than formalities. The computer simulations add a dimension of tangibility to the subject matter, a dimension often lacking in the modern, abstract approach to mathematics.

DAVID GRIFFEATH

Department of Mathematics  
Cornell University  
Ithaca, New York 14853

15 [4.10.4, 5.05.4, 5.10.3, 5.15.3, 5.20.4].—J. R. WHITEMAN, *A Bibliography for Finite Elements*, Academic Press, Inc., London, New York and San Francisco, 1975, 26 cm. Price \$9.25.

16 [4.00, 5.00].—R. ANSORGE, L. COLLATZ, G. HÄMMERLIN & W. TÖRNIG, Editors, *Numerische Behandlung von Differentialgleichungen*, International Series of Numerical Mathematics, Birkhäuser Verlag, Basel, Switzerland, 1975, 355 pp., 25 cm. Price approximately \$18.00.

The volume contains papers presented at a meeting organized by R. Ansorge, L. Collatz, G. Hämmerlin and W. Törnig. This meeting took place at the Mathematical Research Institute at Oberwolfach, Germany from June 9–June 14, 1974.

J. B.

17 [2.05].—L. COLLATZ & G. MEINARDUS, Editors, *Numerische Methoden der Approximationstheorie*, Band 2, International Series of Numerical Mathematics, Birkhäuser Verlag, Basel, Switzerland, 1975, 199 pp., 25 cm. Price approximately \$14.00.

This volume contains papers presented at a meeting organized by L. Collatz and G. Meinardus. This meeting took place at the Mathematical Research Institute at Oberwolfach, Germany from June 3–June 9, 1973.

J. B.

18 [9].—G. SCHRUTKA v. RECHTENSTAMM, *Tabelle der (Relativ)-Klassenzahlen der Kreiskörper deren  $\phi$ -Funktion des Wurzalexponenten (Grad) nicht grösser als 256 ist*, Deutschen Akad. Wiss. Berlin, Abhandlungen, K1. Math. Phys. Tech., 1964, No. 2, 64 pp.

This remarkable work, a labor of some twenty-eight years, has apparently gone unreviewed and unnoticed for more than a decade. It is an extension of a small table of H. Hasse [1] which in turn is an elaboration of an original work of E. Kummer [2] on cyclotomic fields.

As the title indicates, it covers fields and subfields generated by  $\exp(2\pi i/f)$  whenever Euler's  $\phi(f) \leq 256$ . The tables of Kummer and Hasse are for  $f \leq 100$ . Schrutka

gives data on three-hundred and thirty-eight different fields. His format is the same as that of Hasse except that he puts all those fields for which  $\phi(f)$  are the same under one heading.

For each such  $f$  and each appropriate subfield is given the generating character, the order, the degree, and relative class number of the subfield and finally the product  $h^*(f)$  of these class numbers, the so-called first factor of the field. This last is often a 30–50 digit integer. With each class number is given its factorization when known. Otherwise an indication “keine primzahl” after a number  $N$  means that  $2^{N-1} \not\equiv 1 \pmod{N}$ . Whenever  $2^{N-1} \equiv 1 \pmod{N}$ , Schrutka enters  $N$  as a prime although he admits in a footnote that the probability that  $N$  is composite is positive but of “the order of  $10^{-6}$  to  $10^{-10}$ ”. It would be useful to complete the proof of primality in all such cases, and some steps in this direction have already been taken.

Finally, there is a small table of  $\phi(n)$  for  $n = 1(1)1059$  to aid the user in entering the table.

Most of the computational effort in obtaining the huge class numbers is spent in the “norming” of character sums of the form  $M_1 = \sum k\chi(k)$ . Schrutka does not indicate precisely what method he used for norming. Metsankyla [3] and Spira [4] have suggested the use of multiprecise floating point approximations to  $M_1$  but apparently this is pretty expensive [5]. Recently, Newman [6], unaware at the time of Schrutka, published a table of  $h^*(p)$  for all primes under 200 in which he used a determinant method. A comparison of the Newman method with that of Schrutka seems to favor the latter since the class numbers appear already algebraically partially factored into numbers whose prime power divisors belong to predictable arithmetic progressions. But there is considerable room for improvement of Schrutka’s algorithm.

The tables seem to be quite accurate. Newman’s table is in complete agreement wherever it intersects the one under review. In testing an improved algorithm, the reviewer obtained Schrutka’s 55D value of

$$\begin{aligned} h^*(257) &= 54524\ 8502341923\ 0873223822\ 6255559644\ 6147642285\ 4662168321 \\ &= 257 \cdot 20738946049 \cdot \text{prime} \end{aligned}$$

whose factorization is due to John Selfridge.

Schrutka has attempted to fit a conjectured asymptotic formula of the form

$$\log h^*(mp) = \frac{1}{4}\phi(mp) \log(a_m + b_m p),$$

where  $m$  is small, to the data in his table. For  $m = 1$  he suggests  $a_1 = .56$  and  $b_1 = .0257$ . This gives for  $p = 257$  the value  $h^*(257) = 5.37 \cdot 10^{54}$ .

D. H. LEHMER

Department of Mathematics  
University of California  
Berkeley, California 94720

1. H. HASSE, *Über die Klassenzahl abelscher Zahlkörper*, Akademie-Verlag, Berlin, 1952, Tafel I, pp. 139–141.
2. E. KUMMER, “Über die Klassenzahl der aus  $n$ -ten Einheitswurzeln gebildeten komplexen Zahlen,” *Monatsh. Preuss. Akad. Wiss. Berlin*, 1861, pp. 1051–1053.
3. T. METSANKYLA, “Calculation of the first factor of the class number of the cyclotomic field,” *Math. Comp.*, v. 23, 1969, pp. 533–537.
4. R. SPIRA, “Calculation of the first factor of the cyclotomic class number,” *Computers in Number Theory*, Academic Press, New York and London, 1971, pp. 149–151.
5. R. SPIRA, Personal communication.
6. M. NEWMAN, “A table of the first factor for prime cyclotomic fields,” *Math. Comp.*, v. 24, 1970, pp. 215–219.