19 [7].—JEFFREY SHALLIT, *Calculation of $\sqrt{5}$ and ϕ (the Golden Ratio) to* 10,000 *Decimal Places*, ms. of 12 typewritten sheets deposited in the UMT file.

In a two-page introduction the author briefly describes his method of calculating these related numbers to 10015D on an IBM 360/75 system. He states that he successfully compared the first 4599D of his approximation to ϕ with the value given to that precision by Berg [1].

Following the tabulation of $\sqrt{5}$ and ϕ to 10000D, there appear tables of the frequency distribution of the decimal digits in each number.

As a further check on this calculation, this reviewer has successfully compared the present approximation to $\sqrt{5}$ with more extended, unpublished values of Jones [2] and of Beyer, Metropolis and Neergaard [3], which were carried to 22900D and 24576D, respectively.

J. W. W.

1. MURRAY BERG, "Phi, the golden ratio (to 4599 decimal places) and Fibonacci numbers," *Fibonacci Quart.*, v. 4, 1966, pp. 157–162.

2. M. F. JONES, 22900D *Approximations to the Square Roots of the Primes less than* 100, reviewed in *Math. Comp.*, v. 22, 1968, pp. 234–235, UMT 22.

3. W. A. BEYER, N. METROPOLIS & J. R. NEERGAARD, *Square Roots of Integers* 2 *to* 15 *in Various Bases* 2 *to* 10: 88062 *Binary Digits or Equivalent*, reviewed in *Math. Comp.*, v. 23, 1969, p. 679, UMT 45.

20 [9].—P. BARRUCAND, H. C. WILLIAMS & L. BANIUK, *Table of Pure Cubic Fields $Q(\sqrt[3]{D})$ for $D < 10^4$*, University of Manitoba, 1974, 133 pages computer output deposited in the UMT file.

There are 8122 distinct pure cubic fields $Q(\sqrt[3]{D})$ for $1 < D < 10^4$. They are listed here in order of D, not in order of their discriminants $-3k^2$. For the calculation of k, see the paper [1] in this issue for which this table was computed. There are listed here D; k; J, the period length of Voronoi's algorithm for computing the fundamental unit; R, the regulator to 10S; h, the class number; and $\Phi(1) = 2\pi hR/\sqrt{3}\,k$, Artin's function, to 10D.

Concerning Tables 1–5 in [1], the following comments may be of interest. In Table 1, for every natural number $n < 53$, there is at least one D for which $n|h$. But there are none here for $n = 53, 55, 59, \ldots$. In analogy with the results of Yamamoto [2] and Weinberger [3] for real quadratic fields, it is reasonable to conjecture that every n will be a divisor as $D \longrightarrow \infty$. One finds no less than 142 D here with $81|h$, but since the class groups are not computed in [1], nor even the 3-rank r_1 (see Section 7), it is left open whether $r_1 = 4$ or 5 occurs for $D < 10^4$.

Table 2 shows that the density of D with $h = 1$ declines as D increases. Of course, the density must $\longrightarrow 0$ since almost all D will have $3|h$ (and even $3^n|h$) as $D \longrightarrow \infty$. But if one restricts D to the primes $q \equiv 2 \pmod 3$, then $3\nmid h$, and it is reasonable to ask if the number of $Q(\sqrt[3]{q})$ having $h = 1$ has an asymptotic density as $q \longrightarrow \infty$. That is plausible. I find that 294 of the $617q$ here have $h = 1$ and the density remains close to 48%. It would be of interest to extend the table of such $Q(\sqrt[3]{q})$ having $h = 1$ for $q > 10^4$ to study this further. Since the Euler product method (see Section 5) should be able to distinguish $h = 1$ and $h \geqslant 2$ with a modest value of Q, this extension could be done very efficiently.

Tables 3 and 4 are analogous to the *lochamps* and *hichamps* of [4] for quadratic fields. Note that all D in Table 3 are $\equiv \pm 2, \pm 4$, or $\pm 6 \pmod{18}$. That guarantees that 2 and 3 ramify completely and thereby contribute the minimal factor 1 to $\Phi(1)$. In Table 4 all $D > 29$ are $\equiv \pm 1 \pmod{18}$, and now 2 and 3 contribute the maximal factor

2. Note that the largest and smallest $\Phi(1)$ here have the very modest ratio 3.81191/0.61997 = 6.14850. That is much smaller than the ratios obtainable in quadratic fields with comparable discriminants, cf. [4]. The reason is that all primes $\equiv 2 \pmod 3$ split the same way in every $Q(\sqrt[3]{D})$, unless they divide D, and so the variation possible in $\Phi(1)$ is much diminished.

Note that one cannot assure an exceptionally large $\Phi(1)$ merely by selecting D that are cubic residues of all small $p \equiv 1 \pmod 3$. Thus, $(D/p)_3 = 1$ for $D = 1546$ and $p = 7, 13, 19, 31, 37, 43$. No D in Table 4 has such a long run, but $1546 \equiv -2$ (mod 18), its $\Phi(1)$ loses a factor of 2 as above, and so $D = 1546$ does not appear in Table 4. The $\Phi(1)$ in Table 4 are also somewhat restrained by the competition of their D with perfect cubes, cf. [4, p. 275].

In contrast to *pure* cubic fields, *cyclic* cubic fields have discriminants d that are perfect squares and all primes either split completely in the field or are inert. Thus, [5], one finds

$$\Phi(1) = 0.17377 \quad \text{and} \quad \Phi(1) = 0.16850$$

for $d = 139^2$ and 2557^2, respectively. These $\Phi(1)$ are *even smaller* than occur in comparable quadratic fields. Correspondingly, the polynomial $f(x) = x^3 - 49x^2 - 52x - 1$, having $d = 2557^2$, has a very high density of primes considering the fact that $f(x)$ is cubic. At the other extreme, $Q(x)$ for $x^3 + x^2 - 1332x + 15840 = 0$ having $d = (7 \cdot 571)^2$ has the astonishingly large $\Phi(1) = 11.63136$. This is far larger than occurs in comparable quadratic fields. As H. Stark pointed out to me, this can occur since cyclic cubic fields have Artin functions $\Phi(s)$ that are the products of *two L* functions. It would be desirable for someone to extend Littlewood's analysis [6], [4] to such cyclic (and other) algebraic fields and thereby determine bounds on their $\Phi(1)$ when the Riemann hypothesis holds.

Table 5 gives the D having champion values of R. All $D > 15$ there have $h = 1$ and one notes that the ratio R/J always remains close to 1.12 when R and J are large. For quadratic fields the analogous ratio is [7] Lévy's constant: $\pi^2/12 \ln 2 = 1.18657$. It would be interesting to obtain an analytic expression for the ratio (≈ 1.12) here, but Voronoi's algorithm is quite intricate. That makes any such analysis quite complicated relative to the quadratic case which is based upon regular continued fractions.

As stated in [1], this table was computed using a formula of Barrucand for $\Phi(1)$; and this method is said to be much faster than Dedekind's method based upon Epstein zeta functions. But there are different ways of doing the latter: if the quadratic forms and their weights are determined by trial and error factorizations, then Dedekind's method is certainly very slow for large D. But if one used *group-theoretic* methods of generating the forms and determining their weights [8, pp. 278, 281], it may go much faster. Nonetheless, it would take time: these are large discriminants and the number of forms needed goes as $O(k)$.

D. S.

1. P. BARRUCAND, H. C. WILLIAMS & L. BANIUK, "A computational technique for determining the class number of a pure cubic field," *Math. Comp.*, v. 30, 1976, pp. 312–323.

2. Y. YAMAMOTO, "On unramified Galois extensions of quadratic number fields", *Osaka J. Math.*, v. 7, 1970, pp. 57–76.

3. P. J. WEINBERGER, "Real quadratic fields with class numbers divisible by n", *J. Number Theory*, v. 5, 1973, pp. 237–241.

4. DANIEL SHANKS, *Systematic Examination of Littlewood's Bounds on* $L(1, \chi)$, Proc. Sympos. Pure Math., vol. 24, Amer. Math. Soc., Providence, R.I., 1973, pp. 267–283.

5. DANIEL SHANKS, "The simplest cubic fields", *Math. Comp.*, v. 28, 1974, pp. 1137–1152.

6. J. E. LITTLEWOOD, "On the class-number of the corpus $P(\sqrt{-k})$", *Proc. London Math. Soc.*, v. 28, 1928, pp. 358–372.

7. P. LÉVY, "Sur le développement en fraction continue d'un nombre choisi au hasard", *Compositio Math.*, v. 3, 1936, pp. 286–303.

8. DANIEL SHANKS, "Calculation and applications of Epstein zeta functions", *Math. Comp.*, v. 29, 1975, pp. 271–287.

21 [9].—RICHARD P. BRENT, *Tables Concerning Irregularities in the Distribution of Primes and Twin Primes to* 10^{11}, Computer Centre, Australian National University, Canberra, August 1975, 2 pp. + 12 computer sheets deposited in the UMT file.

These tables supersede the author's earlier incomplete UMT [1], which one can see for further detail. The previous Tables 1 and 2 are here extended to $n = 10^{11}$, and the author thereby also completes two tables in his paper [2] as follows. To Table 1, page 45, add a final row:

8×10^{10}	10^{11}	8176	16088	−5618	3037	−9881	1786

and to Table 4, page 51, add two more rows:

9×10^{10}	203710414	−6872	1.797468808649	1.90216053
10^{11}	224376048	−7183	1.797904310955	1.90216054

While these tables required a great amount of machine time, the author expresses confidence in their accuracy since the counts of $\pi(n)$ obtained here for $n = 10^{10}(10^{10})10^{11}$ agree with earlier values computed by Lehmer's method. In the extension here, from $n = 8 \times 10^{10}$ to $n = 10^{11}$, of $r_1(n) = \langle L(n)\rangle - \pi(n)$, nothing extraordinary occurs, it being a melancholy feature of these computations that computation time goes as $O(n)$ while points of interest occur as $O(\log n)$.

The downward trend of $s_3(q)$ in Fig. 3 of [2] that began at $\log_{10}(q) \approx 10.6$ continues throughout this extension with one consequence that the estimate for Brun's constant is now up to 1.9021605. But the earlier value 1.9021604 may really be more accurate according to the discussion in the previous review [1]. Of course, it still is "unknown" that there are infinitely many twin primes; there are only 224376048 pairs here. Perhaps in all mathematics there is no conjecture for which there is more supporting data. Further, this data makes it almost certain that the Hardy-Littlewood conjecture is true. On the other hand, the second-order fluctuations, observed in Fig. 3, are a complete mystery; to my knowledge they have no rational interpretation whatsoever. It is a highly repetitive feature in the history of physics that the investigation of very small second-order effects (the perihelion of Mercury, the fine-structure of the hydrogen spectrum, etc.) have repeatedly led to a radically new understanding of the main phenomenon. If that is relevant here, let the reader draw the proper inference.

D. S.

1. RICHARD P. BRENT, UMT 4, *Math. Comp.*, v. 29, 1975, p. 331.

2. RICHARD P. BRENT, "Irregularities in the distribution of primes and twin primes", *ibid.*, pp. 43–55.

22 [9].—WILLIAM J. LEVEQUE, Editor, *Reviews in Number Theory,* Amer. Math. Soc., Providence, R. I., 6 vols., 2931 pp. Price $76.00 for individual AMS members.

This collection contains all reviews of papers of an arithmetical nature which have appeared in Volumes 1–44 (1940–1972) of *Mathematical Reviews.* As such, its value to anyone interested in recent research in number theory is hard to overestimate.

The reviews are classified by a modification of the 1970 MOS classification