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Complex Roots of $\sin z = az$, $\cos z = az$, and $\cosh z = az$

By Henry E. Fettis

Abstract. Values of the first five complex roots of the equations $\sin z = az$, $\cos z = az$ and $\cosh z = az$ are given to 10S, for $a = 10, 5, 2, 1.6, 1.2, 1(.1).3$, and selected values below.

Introduction. If "a" is a real parameter, the equations

$$\sin z = az, \tag{1}$$

$$\cos z = az, \tag{2}$$

$$\cosh z = az, \tag{3}$$

all have a finite number of real roots and an infinite number of pairs of conjugate complex ones. Since Eq. (1) is unchanged if z is replaced by $-z$, the complex roots of (1) are symmetrically situated in each of the four quadrants of the complex z-plane. Replacing z by $-z$ in Eqs. (2) and (3) is equivalent to replacing "a" by "$-a$". Thus the roots of these equations which lie in the first and fourth quadrants for positive "a" are identical to those in the second and fourth quadrants for "a" negative.

The first ten complex roots of Eq. (1) for $a = \pm 1$ were first given by Fadle [2] to 5D and later by Hillman and Salzer [1] to 10D, by Mittelman and Hillman [3] to 7D and by Robbins and Smith [4] to 6D. The most extensive tabulation is that of Ling [7], who computed the first 100 roots of Eqs. (1), (2) and (3) to 11D. Tabulations of the roots of Eq. (1) for values of "a" other than ± 1 are few and difficult to obtain. Notable are those of Ricci [5] which appear in *La Rivesta di Ingegneria*, No. 2 (Milan), and those of Coghlan and Smith [6] which have been deposited in the UMT file of the present journal. Another table by Sidney Johnson which gives only the real roots is mentioned in the UMT section of *Mathematical Tables and Other Aids to Computation* (v. 4, no. 31, July, 1950) but is virtually unattainable.

As far as the author has been able to determine, no tables of the roots of either (2) or (3) other than for $a = \pm 1$ have been published. Miller [8] gives six roots of Eq. (2) for $a = 1$, lying in the first quadrant, but only three of these, namely the second, fourth and sixth are correct, due to an oversight which introduced three extraneous roots belonging in actuality to the equation $\cos z = \bar{z}$. Later Silberstein [9] gave three roots of this equation lying in the second and third quadrants which were overlooked by Miller, but failed to note those in the first and fourth quadrants. Both Miller's and Silberstein's results may be found in [11, p. 96].

The most complete tabulation of the roots of Eqs. (1), (2) and (3) for $a = \pm 1$ is that of Ling [7].

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1. The Equation $\sin z = az$. It is evident that $z = 0$ is always a simple root of this equation and that for $a = 1$, it is a threefold root. It is also clear from graphical considerations that for $a > 1$, no other real roots exist, so that for this range of a, all zeros of Eq. (1) occur in complex conjugate pairs, lying symmetrically in the four quadrants of the z-plane. For this last reason, future discussions will be restricted to roots in the first quadrant.

The original complex equation (1) can be rewritten as two real equations:

$$\text{(1a)} \qquad \left\{ \sin x \cosh y = ax, \right.$$

$$\text{(1b)} \qquad \left\{ \cos x \sinh y = ay, \right.$$

where $z = x + iy$. If $x = 0$, Eq. (1a) is automatically satisfied, while Eq. (1b) will also be satisfied provided

$$\text{(4)} \qquad \sinh y = ay.$$

For any $a > 1$, this equation has a single pair of nonzero real roots which are equal and opposite in sign, and which coincide with the root $z = 0$ when $a = 1$. Thus Eq. (1) has, for any $a > 1$, a pair of roots located symmetrically on the imaginary axis, in addition to the complex roots noted earlier.

As a is decreased below unity, the triple root becomes again a simple zero, and two real roots appear (equal and opposite in sign) lying in the interval $0 < x < \pi$, all subsequent roots remaining complex. As a continues to decrease, a value is reached where the imaginary part of the first complex pair vanishes. This value of a is determined by

$$\text{(5)} \qquad a = \cos \alpha,$$

where α is the smallest positive root of the equation

$$\text{(6)} \qquad \theta = \tan \theta.$$

The nonzero real part of this double root has the value

$$\text{(7)} \qquad x = \cos^{-1} a + 2\pi,$$

where the principal value of the inverse cosine is understood. As a is decreased further, two additional real roots appear, lying in the interval $2\pi \leqslant x \leqslant 3\pi$, the remaining roots being complex. This continues until a attains the value corresponding to the second root of Eq. (6) for which $\cos \alpha > 0$ (the third root of this equation in order of increasing magnitude) at which value there will be another double root. This continues indefinitely, with more real roots being added as each successive zero of Eq. (6) with $\cos \alpha > 0$ is reached, until finally, for $a = 0$, all roots are real. The real parts of the double roots are given by

$$\text{(8)} \qquad x = \cos^{-1} a + 2n\pi, \qquad n = 1, 2, \ldots .$$

For negative values of a, all roots are complex, until a reaches the value determined by the first root of Eq. (6) for which $\cos \alpha < 0$ (the second root of (6) in

order of increasing magnitude). As before, for this value of a, the first pair of complex roots coincide at a double root determined by

$$x = \cos^{-1}|a| + \pi; \tag{9}$$

and, as previously, there is an infinite sequence of such double roots, given by

$$x = \cos^{-1}|a| + (2n - 1)\pi, \qquad n = 1, 2, \ldots . \tag{10}$$

2. The Equation $\cos z = az$. The roots of this equation in the right half-plane are identical to those of the same equation in the left half-plane with "a" replaced by "$-a$". Thus, again, it suffices to restrict the discussion to positive x. For $a > 0$, there is always one real root, lying in the interval $0 < x < \pi/2$. The remaining ones are all complex until a reaches the value $-\sin \alpha$, where α is the first root of

$$\cot \theta + \theta = 0 \tag{11}$$

for which $\sin \alpha$ is negative. Other double roots are found at the higher roots of (11) for which $\sin \alpha < 0$, and are given by

$$x = -\sin^{-1}|a| + 2n\pi, \qquad n = 1, 2, \ldots . \tag{12}$$

As each double root is passed, a pair of real roots is added, until, for $a = 0$, all roots are real.

Double roots for negative "a" are found when "a" takes on the values corresponding to the roots of (11) for which $\sin \alpha > 0$. The appropriate values of x in this case are

$$x = -\sin^{-1}|a| + (2n - 1)\pi, \qquad n = 1, 2, \ldots . \tag{13}$$

3. The Equation $\cosh z = az$. For $a > 0$, this equation has one double root which occurs when "a" has the value $\sinh \alpha$, α being the positive root of

$$\coth \theta = \theta. \tag{14}$$

For values of "a" above this value, there are two real roots of (3), the remaining ones being complex. For $a < 0$, all of the roots are complex. Also, because of symmetry, the roots in the left half-plane for $a > 0$ are identical to those in the right half-plane when $a < 0$.

4. Method of Computation. The roots of Eq. (4) may be found, e.g., by successive Newtonian approximations:

$$y_{k+1} = \frac{y_k \cosh y_k - \sinh y_k}{\cosh y_k - a}. \tag{15}$$

For the complex roots, Eqs. (1a) and (1b) are solved by successive approximations as follows:

$$y_{k+1} = \cosh^{-1}\left[\frac{a(n\pi + \theta_k)}{\sin \theta_k}\right], \qquad n = 1, 2, \ldots, \tag{16}$$

where

$$\theta_k = \cos^{-1}\left[\frac{ay_k}{\sinh y_k}\right], \qquad 0 \leqslant \theta_k \leqslant \pi/2. \tag{17}$$

In the above, even values of n gives the roots for $a > 0$ and odd values the roots for $a < 0$. As an initial assumption, the value

$$y_0 = \cosh^{-1}[a(n + \tfrac{1}{2})\pi] \tag{18}$$

may be used. However, a somewhat better starting value may be obtained by linear interpolation of cosh y between the already computed results for $a = 1$ as given in [7] and the value of "a" for which $y = 0$, as given by Eq. (5). When convergence has been obtained for y, the corresponding value of x is given by

$$x = \theta + n\pi. \tag{19}$$

For Eq. (2), a similar sequence is formed:

$$y_{k+1} = \cosh^{-1}\left[\frac{a(n\pi - \theta_k)}{\cos \theta_k}\right], \qquad n = 1, 2, \ldots, \tag{20}$$

$$\theta_k = \sin^{-1}\left[\frac{ay_k}{\sinh y_k}\right]. \tag{21}$$

As before, either the value

$$y_0 = \cosh^{-1}(n\pi a) \tag{22}$$

or that obtained by linear interpolation of (cosh y) may be used as an initial assumption. The final value of x is found by

$$x = n\pi - \theta. \tag{23}$$

The corresponding sequences for Eq. (3) are

$$x_{k+1} = \sinh^{-1}\left[\frac{a(n\pi + \theta_k)}{\sin \theta_k}\right], \tag{24}$$

$$\theta_k = \cos^{-1}\left[\frac{ax_k}{\cosh x_k}\right], \qquad 0 < \theta_k < \pi/2, \tag{25}$$

with

$$x_0 = \sinh^{-1}[a(n + \tfrac{1}{2})\pi] \tag{26}$$

and, after convergence,

$$y = n\pi + \theta. \tag{27}$$

5. Numerical Results. Tables of numerical values of the roots of Eqs. (1), (2) and (3) are to be found in the microfiche supplement of this issue. Table 1 gives the values of the parameter "a" for which these equations have double roots, together with

the real parts of the corresponding roots. In computing these, use was made of the appropriate roots of Eqs. (6), (11) and (14) as given in [10].

Table 2 contains the values of the pure imaginary roots of Eq. (1), for selected values of "a" between 1 and 10.

Tables 3 through 22 give the complex roots of Eqs. (1) and (2) for the following values of "a":

$$10,\ 5,\ 2,\ 1.6,\ 1.2,\ 1.1;\ 1.0(-.1).3$$

and for selected values below the last value in the neighborhood of the double root, which in each table constitutes the final entry. The final value of "a" (indicated by * in the tables) is given to only 3S, since more accurate values are contained in Table 1.

Tables 23 through 32 give the complex roots of Eq. (3) for $a = 10, 5, 2, 1.6, 1.2, 1.1(-.1).1, .05.0$, with the exception of the first root for which "a" ranges from 0 to 1.5, and terminates with the upper limiting value 1.508879561... for which the imaginary part of the complex pair vanishes.

The calculations were made by the author on a Hewlett-Packard HP-35 pocket calculator which displays only 10S. For this reason the terminal digits are subject to verification.

Tables 3 through 32 also contain the quantities $\cosh y$ (Eqs. (1) and (2)) and $\sinh x$ (Eq. (3)) which are very nearly linear functions of "a".

1885 California, Apt. 62
Mountain View, California 94041

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COMPLEX ROOTS OF sin z = az, cos z = az, and cosh z = az

by Henry E. Fettis

Table 1

Values of "a" for which the Equations Sin z = a z, Cos z = a z, and Cosh z = a z have Double Roots

sin z = a z		cos z = a z	
a	$x = n\pi + \cos^{-1}\lvert a\rvert$	a	$x = n\pi - \sin^{-1}\lvert a\rvert$
-.21723 36326	4.49340 94579	-.33650 84174	2.79838 60458
.12837 45536	7.72525 18369	.16122 80343	6.12125 04669
-.09132 5208	10.90412 1659	-.10660 79478	9.31786 64618
.07091 3459	14.06619 3913	.07983 1181	12.48645 4395
-.05797 1802	17.22075 5272	-.06379 1553	15.64412 8370
.04902 9624	20.37130 2959	.05312 6533	18.79640 4366
-.04247 9617	23.51945 2499	-.04551 9960	21.94561 2880
.03747 4520	26.66605 4259	.03982 0286	25.09291 0412
-.03352 5141	29.81159 8791	-.03538 9916	28.23893 6575
.03032 9171	32.95638 9040	.03184 7132	31.38407 4018.
x = tan x a = cos x		x = -cot x a = -sin x	

cosh z = a z	
a	$x = \sinh^{-1} a$
1.50887 9561	1.19967 9640
x = coth x a = sinh x	

Table 2

Pure Imaginary Root of $\sin z = a z$

a	y
10	4.49991 3995
9	4.36389 9926
8	4.21035 6274
7	4.03417 1676
6	3.82762 4756
5	3.57823 4808
4	3.26379 6101
3	2.83844 6380
2	2.17731 8985
1.9	2.08562 1234
1.8	1.98613 4311
1.7	1.87729 6469
1.6	1.75697 6715
1.5	1.62213 1217
1.4	1.46813 5745
1.3	1.28730 2891
1.2	1.06486 8549
1.1	.76340 07970
1.05	.54369 12294
1.025	.38585 96709
1.	0

Table 3

First Complex Root of $\sin z = a z$, for Selected Negative Values of "a"

-a	x	y	cosh y
10	3.81162 8689	4.81012 3762	61.37747 737
5	3.91206 6803	4.02805 6395	28.08473 868
2	4.07410 8875	3.00775 7826	10.14568 267
1.6	4.11773 2689	2.76280 8255	7.95369 4789
1.2	4.17540 8155	2.44893 3246	5.83118 8546
1.1	4.19304 0041	2.35430 5919	5.31288 8235
1.0	4.21239 2231	2.25072 8612	4.79998 6841
.9	4.23378 3202	2.13622 2783	4.29274 7029
.8	4.25762 4369	2.00803 1128	3.79144 5114
.7	4.28445 9600	1.86212 2754	3.29636 4896
.6	4.31502 8219	1.69220 7052	2.80778 3859
.5	4.35037 0777	1.48748 4359	2.32594 3867
.4	4.39201 8368	1.22630 1712	1.85098 7887
.3	4.44195 6021	.84925 27317	1.38281 6803
.25	4.47195 6021	.54303 12688	1.15110 0446
.217	4.49340 9458	0	1.00000 0000

Table 4

Second Complex Root of Sin z = a z, for Selected Negative Values of "a"

-a	x	y	cosh y
10	10.51604 456	5.46828 7954	118.52910 83
5	10.57305 861	4.75286 9017	57.96250 041
2.	10.65159 590	3.81188 0922	22.62844 489
1.6	10.67112 761	3.58350 4225	18.01362 426
1.2	10.69644 750	3.28941 386	13.43220 514
1.1	10.70412 436	3.20051 4800	12.29295 201
1.0	10.71253 7397	3.10314 8746	11.15643 237
.9	10.72183 735	2.99551 4657	10.02282 951
.8	10.73222 586	2 87197 5676	8.89235 5814
.7	10.74398 103	2.73863 5149	7.76526 0312
.6	10.75750 1350	2.58082 0131	6.64183 8931
.5	10.77338 603	2.39366 8376	5.52244 8214
.4	10.79259 510	2.16333 5058	4.40752 2913
.3	10.81680 738	1.86251 3866	3.29759 3639
.2	10.84936 225	1.42197 8261	2.19327 4581
.1	10.89847 298	.43252 30849	1.09500 5463
.0913*	10.90412 166	0	1.00000 0000

Table 5

Third Complex Root of Sin z = a z, for Selected Negative Values of "a"

-a	x	y	cosh y
10	16.94460 785	5.88257 3830	179.36707 78
5	16.98273 182	5.17922 1963	88.77512 663
2	17.03407 079	4.25154 6936	35.11409 351
1.6	17.04669 418	4.02596 1058	28.02599 060
1.2	17.06301 081	3.73526 8887	20.96157 849
1.1	17.06795 160	3.64736 7575	19.19978 861
1.0	17.07336 4853	3.55108 7347	17.43994 283
.9	17.07924 866	3.44465 4134	15.68218 027
.8	17.08603 589	3.32566 0995	13.92666 423
.7	17.09360 986	3.19072 4748	12.17358 942
.6	17.10233 708	3.03487 1442	10.42319 239
.5	17.11262 381	2.85034 2837	8.67576 7118
.4	17.12513 439	2.62400 6563	6.93168 9387
.3	17.14106 656	2.33075 3492	5.19145 5632
.2	17.16292 503	1.91156 0716	3.45574 3067
.1	17.19748 680	1.14153 0730	1.72544 4054
.075	17.21028 415	.74892 90599	1.29380 3375
.0580	17.22075 5272	0	1.00000 0000

Table 6

Fourth Complex Root of Sin z = a z, for Selected Negative Values of "a"

-a	x	y	cosh y
10	23.30277 546	6.17826 7689	241.07903 12
5	23.33127 316	5.47920 6405	119.83030 72
2	23.36935 12	4.55627 1447	47.61912 826
1.6	23.37867 680	4.33169 0844	28.04297 522
1.2	23.39071 808	4.04223 1283	28.48541 913
1.1	23.39436 258	3.95469 3366	26.09942 797
1.0	23.39835 523	3.85880 8993	23.71497 328
.9	23.40276 886	3.75281 3545	21.33216 856
.8	23.40770 175	3.63431 4853	18.95114 4773
.7	23.41329 096	3.49995 4471	16.57207 151
.6	23.41973 579	3.34480 3618	14.19513 692
.5	23.42734 421	3.16119 6220	11.82059 172
.4	23.43661 430	2.93621 8445	9.44875 8049
.3	23.46489 429	2.64540 6453	7.00807 3504
.2	23.46489 429	2.23249 2341	4.71516 7095
.1	23.49147 766	1.50121 2155	2.35499 2361
.05	23.51468 385	.58661 32784	1.17704 8481
.0425*	23.51945 250	0	1.00000 0000

Table 7

Fifth Complex Root of Sin z = a z, for Selected Negative Values of "a"

-a	x	y	cosh y
10	29.63217 521	6.40743 8728	303.17017 08
5	29.65489 259	5.71040 6639	150.99857 86
2	29.68513 623	4.78972 5704	60.13834 485
1.6	29.69252 912	4.56562 7076	48.06662 742
1.2	29.70207 004	4.27676 0837	36.01035 379
1.1	29.70495 711	4.18939 823	32.99911 388
1.0	29.70811 9825	4.09370 4925	29.98915 569
.9	29.71161 604	3.98792 0765	26 98057 577
.8	29.71552 386	3.86966 0479	23.97348 842
.7	29.71995 241	3.73557 7035	20.96803 139
.6	29.72506 075	3.58076 2198	17.96437 423
.5	29.73109 368	3.39759 1203	14.96273 162
.4	29.73845 679	4.17324 6583	11.96338 519
.3	29.74789 664	2.88354 3482	8.96672 2956
.2	29.76103 176	2.47336 8167	5.97331 7198
.1	29.78259 344	1.75710 6216	2.98409 2608
.05	29.80197 649	.95477 05029	1.49148 7366
.0335*	29.81159 8791	0	1.00000 0000

Table 8

First Complex Root of Sin z = a z, for Selected Positive Values of "a"

a	x	y	cosh y
10	7.23228 4959	5.18141 3400	88.96987 280
5	7.30693 9983	4.44901 8566	42.77731 902
2.	7.41337 8468	3.48902 7910	16.39231 142
1.6	7.44030 3139	3.25670 2981	13.00115 396
1.2	7.47535 6295	2.95784 001	9.65419 9274
1.1	7.48600 3743	2.86755 6505	8.82540 6246
1.0	7.49767 6278	2.76867 8283	8.00015 0015
.9	7.51057 9147	2.65937 1596	7.17865 0135
.8	7.52498 4606	2.53172 1024	6.36115 6392
.7	7.54126 4273	2.39835 3526	5.54795 4688
.6	7.55994 3433	2.23773 1541	4.73937 3661
.5	7.58179 7956	2.04671 4647	3.93579 0610
.4	7.60804 1096	1.81019 5337	3.13763 1658
.3	7.64072 2091	1.49667 7274	2.34534 7577
.2	7.68370 7505	1.01367 0928	1.55929 1426
.15	7.71139 7298	.57277 29962	1.16856 8151
.128 *	7.72525 1837	0	1.00000 0000

Table 9

Second Complex Root of Sin z = a z, for Selected Positive Values of "a"

a	x	y	cosh y
10	13.7443 0402	5.69558 6939	148.77737 37
5	13.79007 063	4.98797 6201	73.32301 364
2	13.85218 768	4.05560 5855	28.86872 370
1.6	13.86752 196	3.82903 1743	23.01984 505
1.2	13.88736 292	3.53713 2796	17.19866 684
1.1	13.89337 364	3.44887 6545	15.74840 212
1.0	13.89995 971	3.35220 9885	14.30039 899
.9	13.90724 017	3.24534 9309	12.85481 522
.8	13.91537 481	3.12587 294	11.41183 520
.7	13.92458 522	2.99037 2238	9.97167 7400
.6	13.93519 098	2.83382 6265	8.53460 5169
.5	13.94767 704	2.64837 9364	7.10094 2243
.4	13.96283 029	2.42066 3966	5.67109 5626
.3	13.98250 5304	2.12486 0285	4.24558 8703
.2	14.00822 2100	1.69878 3148	2.82509 8211
.1	14.04882 017	.87753 2163	1.41038 2564
.0709*	14.06619 3913	0	1.00000 0000

Table 10

Third Complex Root of $\sin z = a z$, for Selected Positive Values of "a"

a	x	y	cosh y
10	20.12878 459	6.04099 9589	210.15767 21
5	20.16141 265	5.34023 8953	104.28366 78
2.	20.20514 384	4.41523 759	41.36448 745
1.6	20.21587 084	4.19044 3274	33.03360 152
1.2	20.22922 742	3.90049 2695	24.72351 368
1.1	20.23392 213	3.81280 9900	22.64978 577
1.0	20.23851 771	3.71676 7680	20.57777 107
.9	20.24359 784	3.61059 7675	18.50759 439
.8	20.24772 7531	3.49190 1897	16.43940 272
.7	20.25570 721	3.35731 2200	14.37337 197
.6	20.26312 161	3.20188 2181	12.30971 695
.5	20.27186 761	3.01791 0007	10.24870 653
.4	20.28251 900	2.79239 7576	8.19068 7871
.3	20.29611 84	2.50063 4125	6.13612 7296
.2	20.31487 561	2.08531 1055	4.08568 1321
.1	20.34493 801	1.33991 4732	2.04029 2939
.075	20.35626 385	.98866 66883	1.52986 0529
.0490*	20.37130 2959	0	1.00000 0000

Table 11

Fourth Complex Root of Sin z = a z, for Selected Positive Values of "a"

a	x	y	cosh y
10	26.46990 283	6.29928 1805	272.09138 96
5	26.49518 796	5.60139 8239	135.40424 27
2	26.52890 172	4.67976 9979	53.87728 356
1.6	26.53714 942	4.45546 8383	43.05404 121
1.2	26.54779 580	4.16635 2063	32.24765 650
1.1	26.55101 769	4.07891 5527	29.54914 439
1.0	26.55454 727	3.98314 1640	26.85202 845
.9	26.55844 901	3.87726 8371	24.15641 295
.8	26.56280 996	3.75890 756	21.46242 111
.7	26.56775 166	3.75890 756	18.77020 116
.6	26.57345 103	3.46975 1135	16.07883 524
.5	26.58018 007	3.28639 6828	13.39185 328
.4	26.58838 853	3.06178 184	10.70625 574
.3	26.59890 163	2.77162 249	8.02355 4082
.2	26.61349 840	2.36031 675	5.34434 8497
.1	26.63731 360	1.63798 5523	2.66958 3100
.05	26.65844 159	.79643 63976	1.33427 8575
.0375*	26.66605 4259	0	1.00000 0000

Table 12

Fifth Complex Root of Sin z = a z, for Selected Positive Values of "a"

a	x	y	cosh y
10	32.79088 091	6.50518 1381	334.29921 24
5	32.81150 019	5.80878 5852	166.60865 48
2.	32.83891 920	4.88881 4998	66.40182 371
1.6	32.84561 757	4.66486 8960	53.08054 733
1.2	32.85426 085	4.37619 0977	39.77354 006
1.1	32.85687 610	4.28888 4062	36.44940 403
1.0	32.85971 005	4.19325 1470	33.12645 429
.9	32.86290 799	4.08753 4448	29.80478 092
.8	32.86644 793	3.96934 9877	26.48449 064
.7	32.87045 981	3.83535 4264	23.16571 218
.6	32.87508 807	3.68064 5700	19.84860 436
.5	32.88055 519	3.49761 2034	16.53336 866
.4	32.88723 042	3.27346 4973	13.22027 026
.3	32.89579 503	2.98410 3301	9.90967 6251
.2	32.90773 267	2.57475 4423	6.60213 2661
.1	32.92742 518	1.86218 4987	3.29843 0008
.05	32.94532 321	1.08500 6117	1.64867 8820
.0303 *	32.95638 904	0	1.00000 0000

Table 13

First Complex Root of Cos z = a z, for Selected Negative Values of "a"

-a	x	y	cosh y
10	1.97578 8887	4.60797 4704	50.14540 871
5	2.07448 0252	3.76020 6671	21.29029 100
2.	2.27868 7132	2.63519 0181	7.00834 901
1.6	2.34232 7894	2.36616 2924	5.37513 2165
1.2	2.42981 5232	2.02410 6061	3.85072 6639
1.1	2.45698 3963	1.92151 2187	3.48883 3136
1.0	2.48688 5699	1.80936 1341	3.13515 2438
.9	2.51993 4835	1.68532 093	2.78989 0576
.8	2.55663 6269	1.54613 6002	2.45318 4940
.7	2.59761 6786	1.38633 1127	2.12506 8937
.6	2.64367 3982	1.19652 5162	1.80542 1354
.5	2.69585 6766	.95693 94622	1.49389 1018
.4	2.75560 6326	.60673 08999	1.18977 7356
.35	2.78893 7273	.28239 58226	1.04013 9391
.337 *	2.79838 6046	0	1.00000 0000

Table 14

Second Complex Root of Cos z = a z, for Selected Negative Values of "a"

-a	x	y	cosh y
10	8.88405 1382	5.33389 3341	103.62404 97
5	8.94884 2424	4.61182 1107	50.33862 122
2.	9.03926 9587	3.66344 0662	19.51046 302
1.6	9.06190 4699	3.43352 5397	15.50989 012
1.2	9.09129 5857	3.13757 2884	11.54562 354
1.1	9.10021 3421	3.04812 9136	10.56166 251
1.0	9.10998 7454	2.95070 8617	9.58077 4615
.9	9.12079 1814	2.84188 1748	8.60315 9625
.8	9.13285 8295	2.72078 6868	7.62904 7394
.7	9.14650 5282	2.58338 5452	6.65870 5030
.6	9.16218 6501	2.42448 1255	5.69244 5993
.5	9.18057 9295	2.23684 4756	4.73064 1236
.4	9.20275 8201	2.00217 5199	3.77373 0681
.3	9.23057 5323	1.69769 5151	2.82222 5182
.2	9.26763 3201	1.24261 8370	1.87665 0445
.15	9.29188 3371	.87329 54771	1.40618 1508
.107 *	9.31786 6462	0	1.00000 0000

Table 15

Third Complex Root of $\cos z = a z$, for Selected Negative Values of "a"

-a	x	y	cosh y
10	15.34701 037	5.79325 2906	164.04077 35
5	15.38861 808	5.08805 6859	81.04039 602
2	15.44483 835	4.15834 920	31.99073 695
1.6	15.45868 572	3.93233 4694	25.52278 329
1.2	15.47659 231	3.64111 8575	19.08034 606
1.1	15.48201 563	3.55306 3059	17.47437 663
1.0	15.48795 7789	3.45661 4935	15.87049 621
.9	15.49452 645	3.34999 6039	14.26885 261
.8	15.50186 637	3.23-79 3002	12.66961 894
.7	15.51017 857	3.09561 1762	11.07300 106
.6	15.51975 364	3.93945 8285	9.47924 8233
.5	15.53013 380	2.75453 2861	7.88866 8728
.4	15.54473 974	2.52760 3476	6.30165 3769
.3	15.56216 410	2.23326 1833	4.71871 4241
.2	15.58598 631	1.81116 9836	3.14053 1317
.1	15.62332 962	1.02087 0138	1.56794 5092
.0638 *	15.64412 837	0	1.00000 0000

Table 16

Fourth Complex Root of Cos z = a z, for Selected Negative Values of "a"

-a	x	y	cosh y
10	21.71680 462	6.11192 7792	225.60496 41
5	21.74723 018	5.41209 9423	112.05302 02
2	21.78794 059	4.48831 5097	44.49132 543
1.6	21.79791 798	4.26355 3641	35.35806 828
1.2	21.81080 331	3.97387 1887	26.60443 955
1.1	21.81470 360	3.88626 8389	24.37461 600
1.0	21.81897 654	3.79031 2559	22.14641 185
.9	21.82370 002	6.68423 8090	19.91994 597
.8	21.82897 907	3.56560 191	17.69535 843
.7	21.83496 009	3.43118 6054	15.47281 719
.6	21.84185 581	3.27590 8988	13.25252 774
.5	21.84999 231	3.09213 6902	11.03474 793
.4	21.85990 652	2.86691 8309	8.81981 1970
.3	21.87257 706	2.57567 9352	6.60817 1537
.2	21.89008 893	2.16169 0573	4.40046 9783
.1	21.91830 705	1.42422 1194	2.10765 8393
.05	21.94270 806	.44011 45434	1.09842 3868
.0455	21.94561 288	0	1.00000 0000

Table 17

Fifth Complex Root of Cos z = a z, for Selected Negative Values of "a"

-a	x	y	cosh y
10	28.05155 289	6.35479 6960	287.62363 46
5	28.07548 620	5.65737 1598	143.19919 16
2.	28.10737 118	4.73625 2804	57.00748 407
1.6	28.11516 815	4.51206 0284	45.56015 609
1.2	28.12523 157	4.22307 8314	34.12894 441
1.1	28.12827 688	4.13568 1488	28.27409 161
1.0	28.13161 297	4.03995 0882	28.42057 499
.9	28.13530 084	3.93412 5474	25.56849 484
.8	28.13942 282	3.81581 866	22.71796 971
.7	28.14409 391	3.68168 1212	19.86914 236
.6	28.14948 162	5.52680 0956	17.02218 839
.5	28.15584 366	3.34354 525	14.17732 983
.4	28.16360 657	3.11907 8336	11.33485 739
.3	28.17355 433	2.28916 272	8.49517 0478
.2	28.18738 210	2.41846 8763	5.65885 5149
.1	28.21001 553	1.69944 4656	2.82684 6659
.05	28.23023 454	.88004 41363	1.41288 5361
.0354	28.23893 657	0	1.00000 0000

Table 18

First Complex Root of Cos z = a z, for Selected Positive Values of "a"

a	x	y	cosh y
10	5.54902 6904	5.00718 4130	74.74495 366
5	5.63604 6567	4.25744 9643	35.32189 059
2.	5.76502 8056	3.27740 1200	13.27226 170
1.6	5.79828 8122	3.04091 2951	10.48606 433
1.2	5.84179 5828	2.73707 2042	7.75323 2930
1.1	5.85503 7946	2.64533 4381	7.07956 8358
1.0	5.86956 0377	2.54488 5767	6.41012 7245
.9	5.88561 3274	2.43384 1840	5.74515 2140
.8	5.90352 5604	2.30961 2982	5.08491 3049
.7	5.92374 1624	2.16849 0132	4.42971 0153
.6	5.94688 0950	2.00482 158	3.77987 4601
.5	5.97384 3289	1.80956 6976	3.13576 3525
.4	6.00600 4448	1.56581 1797	2.49773 8700
.3	6.04561 8945	1.23594 6904	1.86609 7688
.2	6.09676 1510	.68082 37486	1.24085 2110
.161	6.12125 0467	0	1.00000 0000

Table 19

Second Complex Root of Cos z = a z, for Selected Positive Values of "a"

a	x	y	cosh y
10	12.13481 709	5.58795 8837	133.59671 38
5	12.18562 321	4.87709 322	65.62791 840
2.	12.25500 273	3.94113 0415	25.74809 237
1.6	12.27218 233	3.71380 3843	20.51694 426
1.2	12.29442 845	3.42098 8719	15.31616 732
1.1	12.30117 012	3.35246 3471	14.02147 844
1.0	12.30855 760	3.23550 4069	12.72926 097
.9	12.31672 393	3.12831 9733	11.43868 437
.8	12.32584 742	3.00847 6967	10.15294 594
.7	12.33617 480	2.87254 7308	8.86927 8534
.6	12.34806 091	2.71542 488	7.58896 1092
.5	12.36204 207	2.52931 8515	6.31233 3676
.4	12.37898 387	2.30052 6065	5.03981 8510
.3	12.40041 616	2.00268 4928	3.77194 7121
.2	12.42943 452	1.57088 2783	2.50937 7450
.1	12.47389 485	.69679 24310	1.25274 2249
.0798	12.48645 4395	0	1.00000 0000

Table 20

Third Complex Root of Cos z = a z, for Selected Positive Values of "a"

a	x	y	cosh y
10	18.53825 587	5.96481 9456	194.74263 53
5	18.57342 293	5.26290 9242	96.52373 105
2	18.62065 565	4.33682 4514	38.23870 999
1.6	18.63225 390	4.11157 4907	30.52958 027
1.2	18.64724 013	3.82129 4420	22.84258 848
1.1	18.65177 738	3.73351 4373	20.92487 522
1.0	18.65674 8350	3.63736 6216	19.00898 231
.9	18.66224 345	3.53107 058	17.09504 127
.8	18.66838 443	3.41225 0952	15.18430 675
.7	18.67534 076	3.27750 7116	13.27336 354
.6	18.68335 817	3.12188 9027	11.36663 676
.5	18.69281 224	2.93767 0098	9.46240 7294
.4	18.70431 882	2.71179 3503	7.56133 6163
.3	18.71899 302	2.41937 4946	5.66390 4727
.2	18.73918 382	2.00236 4325	3.77078 1291
.1	18.77134 253	1.24653 8699	1.88289 0454
.0531 *	18.79640 4366	0	1.00000 0000

Table 21

Fourth Complex Root of Cos z = a z, for Selected Positive Values of "a"

a	x	y	cosh y
10	24.88704 976	6.24056 6877	256.57562 47
5	24.91384 645	5.54214 4737	123.61436 05
2	24.92961 036	4.61991 851	50.74780 764
1.6	24.95836 400	4.39548 911	40.54830 651
1.2	24.96966 498	4.10621 549	30.36648 499
1.1	24.97308 518	4.01873 2482	27.82426 206
1.0	24.97683 204	3.92290 7946	25.28350 167
.9	24.98097 397	3.81697 8668	22.74431 248
.8	24.98560 330	3.69855 4656	20.20682 281
.7	24.99084 886	3.56428 1083	17.67118 687
.6	24.99689 811	3.40923 570	15.13759 391
.5	25.00403 898	3.22576 5611	12.60628 237
.4	25.01274 705	3.00098 7866	10.07756 322
.3	25.02389 322	2.71052 8434	7.55186 0621
.2	25.03934 861	2.29849 3565	5.02978 9150
.1	25.06447 324	1.57215 1464	2.51229 9360
.05	25.08659 308	.70065 54424	1.25566 6483
.0398 *	25.09291 412	0	1.00000 0000

Table 22

Fifth Complex Root of Cos z = a z, for Selected Positive Values of "a"

a	x	y	cosh y
10	31.21191 221	6.24748 6548	318.72919 65
5	31.23353 021	5.76079 4653	158.80187 98
2	31.26229 261	4.84049 2979	63.26980 847
1.6	31.26932 117	4.61647 5798	50.57343 191
1.2	31.27839 113	4.32771 0015	37.89188 602
1.1	31.28113.558	4.24037 7122	34.72421 707
1.0	31.28414 204	4.14471 6209	31.55778 005
.9	31.28746 552	4.03896 7868	28.39266 820
.8	31.29118 031	3.92074 798	25.22899 182
.7	31.29539 026	3.78671 1406	22.06688 401
.6	31.30024 674	3.63195 330	18.90650 875
.5	31.30598 287	3.44885 5661	15.74807 389
.4	31.31298 524	3.22461 6679	12.59185 257
.3	31.32196 630	2.93509 6447	9.43822 2050
.2	31.33447 433	2.52536 4846	6.28774 1231
.1	31.35505 974	1.81143 5764	3.14132 3114
.05	31.37367 150	1.02264 0084	1.57008 5050
.0319 *	31.38407 4018	0	1.00000 0000

Table 23

First Complex Root of Cosh z = a z, for Selected Positive Values of "a"

a	x	y	sinh x
1.50888*	1.19967 8640	0	1.50887 9561
1.5	1.19639 9615	.10859 11310	1.50295 2068
1.4	1.15820 4982	.39439 7961	1.43508 1513
1.3	1.11746 9760	.53841 72411	1.36500 1392
1.2	1.07383 8525	.66234 60504	1.29244 8684
1.1	1.02687 8113	.77119 19079	1.21710 5810
1.0	.97605 45183	.87095 14658	1.13858 4792
.9	.92070 14627	.96472 25967	1.05640 5794
.8	.85997 75554	1.05430 0993	.96996 80369
.7	.79280 86538	1.14073 3094	.87851 09067
.6	.71781 36694	1.22449 2532	.78106 44127
.5	.63322 10855	1.30544 0655	.67639 46221
.4	.53681 61377	1.38257 4403	.56297 27105
.3	.42605 79309	1.45349 4052	.43906 54853
.2	.29874 04309	1.51358 5929	.30321 32634
.1	.15494 60212	1.55548 5283	.15556 67625
0	0	1.57079 6327	0

Table 24

Second Complex Root of Cosh z = a z, for Selected Positive Values of "a"

a	x	y	sinh x
10	5.18144 1134	7.23234 6249	88.96672 022
5	4.44917 7816	7.30718 154	42.77244 181
2	3.49043 6419	7.41472 771	16.38488 584
1.6	3.25905 3259	7.44231 2156	12.99323 094
1.2	2.96234 9590	7.47866 2475	9.64583 5041
1.1	2.87302 5733	7.48983 3203	8.81696 8146
1.0	2.77543 8474	7.50216 4621	7.99166 9460
.9	2.66790 4811	7.51591 3458	7.17017 4462
.8	2.54817 3966	7.53143 0409	6.35275 7294
.7	2.41314 3660	7.54921 1157	5.53974 2110
.6	2.25837 8091	7.56998 5944	4.73151 9498
.5	2.07724 1140	7.59488 6248	3.92857 0393
.4	1.85922 9625	7.62577 3567	3.13149 8693
.3	1.58652 6055	7.66590 8667	2.34105 3929
.2	1.22648 6643	7.72108 9061	1.55795 4657
.1	.71775 41612	7.79738 3481	.78098 89049
.05	.38247 70866	7.83617 7517	.39187 08822
0	0	7.85398 1635	0

Table 25

Third Complex Root of Cosh z = a z, for Selected Positive Values of "a"

a	x	y	sinh x
10	5.69560 4302	13.74431 931	148.77659 63
5	4.98805 2049	13.79012 649	73.32182 906
2.	4.05614 1043	13.85248 141	28.86685 306
1.6	3.82988 6624	13.86795 596	23.01780 203
1.2	3.53869 2589	13.88807 865	17.19641 747
1.1	3.45074 6461	13.89420 401	15.74609 648
1.0	3.35449 0037	13.90093 568	14.29803 627
.9	3.24818 7478	13.90840 519	12.85239 607
.8	3.12949 6496	13.91679 197	11.40936 274
.7	2.99514 9507	13.92634 990	9.96915 9459
.6	2.84039 4512	13.93745 490	8.53205 8308
.5	2.65794 1657	13.95069 765	7.09840 0174
.4	2.43578 5123	13.96708 341	5.66862 8086
.3	2.15210 3518	13.98852 567	4.24334 8208
.2	1.76108 6585	14.01930 454	2.82344 9254
.1	1.14388 8013	14.07095 058	1.41018 5484
.05	.65738 71190	14.11030 908	.70576 99904
0	0	14.13716 694	0

Table 26

Fourth Complex Root of Cosh z = a z, for Selected Positive Values of "a"

a	x	y	sinh x
10	6.04100 9441	20.12879 045	210.15736 34
5	5.34028 0388	20.16143 375	104.28319 42
2.	4.41569 7373	20.20525 395	41.36371 748
1.6	4.19087 5723	20.21603 408	33.03275 047
1.2	3.90127 2075	20.22999 692	24.72255 826
1.1	3.81374 1039	20.23423 528	22.64879 964
1.0	3.71789 9016	20.23888 865	20.57675 218
.9	3.61200 0549	20.24403 922	18.50654 071
.8	3.49368 5813	20.24981 406	16.43831 245
.7	3.35965 4132	20.25638 120	14.37224 404
.6	3.20508 758	20.26399 189	12.30855 204
.5	3.02255 4108	20.27303 975	10.24750 941
.4	2.79970 3675	20.28419 368	8.18947 3228
.3	2.51372 3945	20.29873 202	6.13493 6102
.2	2.11503 7942	20.31959 183	4.08463 5792
.1	1.46130 4212	20.35598 294	2.03982 2744
,05	.89641 11662	20.38900 377	1.01995 1315
0	0	20.42035 225	0

Table 27

Fifth Complex Root of Cosh z = a z, for Selected Positive Values of "a"

a	x	y	sinh x
10	6.29928 7989	26.46990 568	272.09123 47
5	5.60143 741	26.49519 815	135.40400 31
2	4.67993 4842	26.42895 497	53.87688 555
1.6	4.45572 7827	26.53722 849	43.05359 789
1.2	4.16681 7261	26.54792 676	32.24715 282
1.1	4.07947 0508	26.55117 004	29.54862 234
1.0	3.98381 4919	26.55472 694	26.85148 641
.9	3.87810 1901	26.55866 443	24.15584 911
.8	3.75996 567	26.56307 351	21.46183 351
.7	3.62609 3944	26.56808 234	18.76958 781
.6	3.47164 5116	26.57387 970	16.07929 437
.5	3.28913 499	26.58076 062	13.39118 413
.4	3.06608 4706	26.58922 485	10.70556 071
.3	2.77930 1923	26.60022 486	8.02284 5366
.2	2.37770 2784	26.61595 279	5.34367 3365
.1	1.70828 4763	26.64356 877	2.66915 4920
.05	1.09916 4276	26.67057 122	1.33425 3515
0	0	26.70353 756	0

Table 28

First Complex Root of Cosh z = a z, for Selected Negative Values of "a"

-a	x	y	sinh x
10	4.81010 6404	3.81177 8854	61.36826 519
5	4.02813 6975	3.91728 2186	28.06919 295
2	3.01025 0205	4.07893 1248	10.12159 861
1.6	2.76758 2434	4.12509 7663	7.92864 2935
1.2	2.45941 7465	4.18772 5475	5.80625 5162
1.1	2.36747 5887	4.20731 7326	5.28875 3966
1.0	2.26755 5472	4.22099 1281	4.77610 1553
.9	2.15816 3116	4.25354 2654	4.26984 3625
.8	2.03735 7103	4.28127 5353	3.76996 8864
.7	1.90255 6785	4.31314 8111	3.27691 2008
.6	1.75024 7346	4.35033 4694	2.79114 7632
.5	1.57551 0287	4.39446 5824	2.31315 2683
.4	1.37126 3328	4.44775 1153	1.84326 9666
.3	1.12704 1323	4.51278 9035	1.38126 0283
.2	.82731 44723	4.59062 0440	.92497 31440
.1	.45198 77345	4.67143 2807	.46753 53497
.05	.23295 48911	4.70105 0053	.23506 76143
0	0	4.71238 8981	0

Table 29

Second Complex Root of Cosh z = a z, for Selected Negative Values of "a"

-a	x	y	ainh x
10	5.46831 1441	10.51607 305	118.52797 36
5	4.75298 0104	10.57316 495	57.96031 278
2	3.81270 7608	10.65216 368	22.62538 296
1.6	3.58484 1646	10.67196 911	18.00995 395
1.2	3.29188 9797	10.69783 045	13.42822 784
1.1	3.20349 4863	10.70572 740	12.28889 902
1.0	3.10679 7716	10.71441 909	11.15230 833
.9	3.00007 6335	10.72407 941	10.01864 347
.8	2.88101 2886	10.73494 603	8.88812 4102
.7	2.74638 4789	10.74735 562	7.76101 1584
.6	2.59152 7786	10.76180 704	6.63762 3341
.5	2.40933 734	10.77908 344	5.51835 5364
.4	2.18824 5688	10.80051 085	4.40371 9225
.3	1.90865 0226	10.82857 118	3.29440 5223
.2	1.52608 2526	10.86852 133	2.19136 7514
.1	.94725 09718	10.93166 580	1.09540 2792
.05	.52440 61811	10.97258 573	.54877 42865
0	0	10.99557 429	0

Table 30

Third Complex Root of Cosh z = a z, for Selected Negative Values of "a"

-a	x	y	sinh x
10	5.88258 6744	16.94461 697	179.36660 66
5	5.17927 7294	16.98276 478	88.77440 644
2	4.25191 9563	17.03424 314	35.11293 823
1.6	4.02655 2333	17.04694 955	28.02472 031
1.2	3.73633 9443	17.06343 161	20.96016 427
1.1	3.64864 8242	17.06844 020	19.19833 323
1.0	3.55264 5482	17.07393 976	17.43844 434
.9	3.44658 9025	17.08003 611	15.68063 710
.8	3.32812 519	17.08687 349	13.92507 571
.7	3.19396 5011	17.09465 561	12.17195 678
.6	3.03931 4092	17.10368 367	10.42152 068
.5	2.85679 1536	17.11443 031	8.67406 9620
.4	2.63417 2114	17.12769 981	6.92999 7702
.3	2.34900 6504	17.14502 918	5.18984 6787
.2	1.95312 3715	17.16992 497	3.45442 3541
.1	1.31344 2458	17.21283 3960	1.72503 0617
.05	.78098 72763	17.24919 014	.86283 66915
0	0	17.27875 960	0

Table 31

Fourth Complex Root of Cosh z = a z, for Selected Negative Values of "a"

-a	x	y	sinh x
10	6.17827 5404	23.30327 7946	241.07881 72
5	5.47923 8474	23.33128 749	119.82997 75
2	4.55648 0564	23.36942 601	47.61858 609
1.6	4.33202 0509	23.37878 779	38.04237 340
1.2	4.04282 3627	23.39090 163	28.48473 906
1.1	3.95540 0449	23.39457 599	26.09872 442
1.0	3.85966 7332	23.39860 675	23.71424 439
.9	3.75387 6887	23.40307 019	21.33141 233
.8	3.63566 5649	23.40807 001	18.95036 213
.7	3.50172 5885	23.41375 240	16.57125 473
.6	3.34722 5329	23.42033 292	14.18428 786
.5	3.16470 0465	23.42814 880	11.81971 135
.4	2.94172 3745	23.43777 218	9.44785 2918
.3	2.65525 493	23.45029 517	7.07916 5986
.2	2.25481 7891	23.46823 184	4.71433 2180
.1	1.59180 7191	23.49967 824	2.35453 0770
.05	1.00122 6595	23.52952 707	1.17709 4814
0	0	23.56194 491	0

Table 32

Fifth Complex Root of Cosh z = a z, for Selected Negative Values of "a"

-a	x	y	sinh x
10	6.40744 3785	29.63217 73	303.17005 47
5	5.71042 7374	29.65490 01	150.99839 82
2.	4.78985 8936	29.68517 555	60.13804 301
1.6	4.56583 6475	29.69258 754	48.06629 018
1.2	4.27713 5733	29.70216 692	36.00996 887
1.1	4.18984 5288	29.70506 987	32.99871 431
1.0	4.09424 7031	29.70825 290	29.98874 006
.9	3.98859 1576	29.71177 570	26.98014 249
.8	3.87051 1589	29.71571 936	23.97303 573
.7	3.73669 1698	29.72019 800	23.96755 731
.6	3.58228 3891	29.72537 959	17.96387 681
.5	3.39978 9688	29.73152 636	14.96220 930
.4	3.17669 4562	29.73908 197	11.96283 819
.3	2.88969 9770	29.74889 062	8.96615 7688
.2	2.48729 2360	29.76289 310	5.97276 3968
.1	1.81328 0951	29.78747 568	2.98370 5179
.05	1.18999 3565	29.81198 854	1.49141 8417
0	0	29.84513 021	0