

the Gauss-Newton algorithm. The author not only forms the normal equations at each step, but he even solves the system by inverting the Gram matrix he should not have formed. The algorithm thus seems inefficient in general and inaccurate for ill-conditioned problems.

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26 [2.05.1].—CHARLES L. LAWSON & RICHARD J. HANSON, *Solving Least Squares Problems*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1974, 340 pp., 24 cm. Price \$16.00.

This book is intended both as a text and a reference on solving linear least squares problems. It is written from the numerical analyst's point of view and not only brings together a lot of information previously scattered in research papers, but also contains some original contributions.

The authors evidently have a great deal of hard earned experience from solving least squares problems. The strongest feature of the book is that it covers all aspects of the solution up to a set of field tested portable Fortran programs. For a reader whose immediate concern is with solving problems, it is possible to bypass the first half of the book and pass directly to the last two chapters where the practical aspects are discussed.

The first half of the book develops basic theory and algorithms both for under- and overdetermined systems. Detailed perturbation bounds for the pseudoinverse and the least squares solution are given here. Algorithms based on Householder transformations and the singular value decomposition are then described thoroughly. An algorithm based on sequential Householder reduction for the case when A has a banded structure, is given in a later chapter. Problems when A is more generally sparse are not specially treated.

Two other methods for solving linear least squares problems (normal equations and modified Gram-Schmidt) are briefly described. A more extensive coverage of these and other alternative methods (e.g. the method of Peters and Wilkinson) would have been appropriate and made the book more useful as a textbook. Another topic, which this reviewer thinks should have been included is iterative refinement of a solution.

Linear least squares problems with linear equality or inequality constraints are, however, exhaustively treated. A solution of the problem to minimize $\|Ex - f\|$ subject to $Gx \geq h$ is given, which depends on transforming this problem in two steps into a nonnegative least squares problem. This solution gives an elegant modularity in the algorithms for different constrained problems. Unfortunately the transformation described in Chapter 23, Section 5, contains an error, and does not work when the matrix E is rank deficient. Recently in an ICASE report A. K. Cline has shown how to perform a corresponding reduction in the general case.

The last part of the book contains descriptions and ANSI Fortran listings of subroutines for most of the algorithms described earlier in the book. This includes the Householder method, the singular value analysis, the sequential solution of a problem with a banded matrix, the nonnegative least squares solution and the least distance problem. A set of six main programs are also given for validation of these subroutines. The codes can now also be obtained in machine readable form from IMSL.

This is a very useful book, which also sets a new style for books in numerical analysis. Similar books are needed for many other problem areas.

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27 [2.05.03].—HERBERT E. SALZER, NORMAN LEVINE & SAUL SERBEN, *Hundred-Point Lagrange Interpolation Coefficients for Chebyshev Nodes*, 47 computer print-out sheets, 1969, deposited in the UMT file.

Tables of Lagrange interpolation coefficients $L_i^{(100)}(x)$, where

$$L_i^{(100)}(x) = \prod_{j=1, j \neq i}^{100} (x - x_j) / \prod_{j=1, j \neq i}^{100} (x_i - x_j),$$

are given for the Chebyshev nodes

$$x_i = -\cos[(2i - 1)\pi/200], \quad i = 1(1)100,$$

for $x = 0(0.01)1.00$, to 26S. For negative arguments, we have

$$L_i^{(100)}(-x) = L_{101-i}^{(100)}(x).$$

$L_i^{(100)}(x)$ is tabulated so that there is a separate block of four columns for each i , and is read horizontally. The argument x is not printed, and the 2nd through 26th digits are unseparated.

Three functional checks,

$$\sum_{i=1}^{100} L_i^{(100)}(x) = 1, \quad \sum_{i=1}^{100} x_i L_i^{(100)}(x) = x \quad \text{and} \quad \sum_{i=1}^{100} x_i^2 L_i^{(100)}(x) = x^2,$$

for $x = 0(0.01)1.00$, were performed upon the entries on tape before final printout, the greatest relative deviation from a true answer being $< \frac{1}{4} \cdot 10^{-21}$. The user is cautioned that these checks upon the 26S entries, prior to printout, cannot guarantee the correctness of digits on tape which occur beyond the twenty-first decimal place, or the accuracy of the printout in any place. However, it appears likely that all entries are correct to around 23S.

It was not noticed until 1975 that the printout was defective in that minus signs were not printed in all the first columns, making uncertain twenty-five percent of the entries. As the means and opportunity for reproducing a corrected version of the printout were no longer available, a careful determination was made of the locations of the missing minus signs, which were then inserted by hand.

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28 [2.10].—PHILIP J. DAVIS & PHILIP RABINOWITZ, *Methods of Numerical Integration*, Academic Press, New York, 1975, xii + 459 pp., 24 cm. Price \$34.50.

This book is an expanded and updated successor to the previous works on this subject by the same authors, *Numerical Integration*, Blaisdell Publishing Co., Waltham, Mass., 1967 (see *Math. Comp.*, v. 22, 1968, pp. 459–460; *Math. Reviews*, v. 35, 1968, #2482). The new version is almost exactly twice the size of the old, yet retains the sparkle of the original version. The overall organization is the same, with about sixty-four new sections and subsections added, some of the latter being interpolated two