1. A. R. EDMONDS, *Angular Momentum in Quantum Mechanics,* Princeton Univ. Press, Princeton, N. J., 1960.

2. B. KROHN, Private communication, 1975.

30 [7].—RICHARD P. BRENT, *Knuth's Constants to* 1000 *Decimal and* 1100 *Octal Places,* Technical Report no. 47, Computer Centre, The Australian National Univ., Canberra, A.C.T. 2600, Australia, 1975, 25 pp., 30 cm.

In appendices to the three volumes published to date of *The Art of Computer Programming* [1], Knuth lists 33 mathematical constants to 40D and 44 octal places, and suggests in Volume 2 (Exercise 4.3.1.36) that it would be worthwhile to compute them to much higher precision.

The present author has followed this suggestion by extending the precision to that stated in the title, using his Fortran multiple-precision arithmetic package on a UNIVAC 1108 computer. Each constant was computed twice, once with base 10000 and 260 floating-point digits, and once with base 11701 and 250 digits. Each run required approximately 25 minutes of computer time, and both runs for each constant produced identical results. The results were also checked by comparison with available published values, cited in the appended list of 17 references.

Specifically, the constants are the square roots of 2, 3, 5, and 10; the cube roots of 2 and 3; the fourth root of 2; the natural logarithms of 2, 3, 10, π, and ϕ (the golden ratio); the reciprocals of ln 2, ln 10, and ln ϕ; π; $\pi/180$; π^{-1}; π^2; $\pi^{1/2}$; $\Gamma(1/3)$; $\Gamma(2/3)$; e; e^{-1}; e^2; γ; e^γ; ϕ; $e^{\pi/4}$; sin 1; cos 1; $\zeta(3)$; and ln ln 2.

J. W. W.

1. D. E. KNUTH, *The Art of Computer Programming,* v. 1, *Fundamental Algorithms*; v. 2, *Seminumerical Algorithms*; v. 3, *Sorting and Searching,* Addison-Wesley, Reading, Mass., 1968–1973.

31 [8].—THE INSTITUTE OF MATHEMATICAL STATISTICS, Editors, and H. L. HARTER & D. B. OWEN, Coeditors, *Selected Tables in Mathematical Statistics,* Vol. II, Amer. Math. Soc., Providence, R. I., 1974, viii + 388 pp., 26 cm. Price $16.40.

In a discussion of the contents of the first volume [1] of this series of statistical tables this reviewer directed his remarks to their applications and their importance to the practicing statistician. Although attention was drawn to the adequacy of the background explanation provided by the authors for specific mathematical procedures followed in developing the tables, the important questions regarding convergence properties of the relevant mathematical approaches were not addressed. The present review is written in the same vein.

As in the first volume, the tables herein relate to real problems that somehow have been neglected in the main stream of statistical literature. Perhaps the best example of this is the fixed-effect analysis-of-variance model usually discussed in the literature. It is generally assumed that the denominators of the F ratios are valid $\chi^2(\sigma^2)/f$ statistics (f being the number of degrees of freedom), and therefore, under the null hypothesis of no fixed effects, the F statistic is the correct one. Most practicing statisticians, in reality, feel very uncomfortable about this assumption; they are usually aware that the assumed model is not correct in that all the effects have *not* been accounted for, thereby truly making the denominator of the F ratio a multiple of a *noncentral* χ^2. The tables herein of Doubly Noncentral F Distribution, by M. L. Tiku, and one of the accompanying examples directly address this extremely important point. The other examples accompanying these particular tables also address problems that require more realistic models than those usually presented in the literature.

Tables 1 and 2 of the doubly noncentral F distribution give to 4D the values of the probability $P(f_1/2, f_2/2, \lambda_1, \lambda_2, u_0)$ for values of u_0 for which type I error of the

F-test equals 0.05 and 0.01 and for the ranges $f_1 = 1(1)8, 10, 12, 24, f_2 = 2(2)12, 16, 20, 24, 30, 40, 60, \phi_1 = 0(0.5)3$, and $\phi_2 = 0(1)8$. Table 3 gives 4D values of $P(f_1/2, f_2/2, \lambda_1, \lambda_2, u_0)$ for the same values of ϕ_1 and ϕ_2 and for $f_1 = f_2 = 4(2)12$, $u_0 = 0.02(.08)0.50, 0.60, 0.75, 0.95$.

While the same points could have been made about the examples accompanying the tables herein of the Doubly Noncentral t-Distribution, by William G. Bulgren, it seemed to this reviewer that a very serious problem in terminology occurs, for which there is inadequate background explanation. In particular, the exact meaning of the symbol $\bar{\mu}_i$, as contrasted to the symbol μ_i, is not made perfectly clear. As a consequence, this reviewer believes that the very important accompanying examples will not provide proper guidance for the potential user of the tables. It is hoped that this fault can be corrected in later editions because these tables can be extremely important in solving problems where the customary Student t-distribution cannot be realistically applied.

The probability integral to 6D of the doubly noncentral t-distribution with degrees of freedom n and non-centrality parameters δ and λ is tabulated over the following ranges of the parameters:

$$t = 0, \quad \delta = -4(1)5, \quad \text{any } n \text{ and } \lambda,$$
$$t = 0.1, \quad 0.2(0.2)9.0, \quad \delta = -4(1)5, \quad \lambda = 0(1)2(2)8, \quad n = 2(1)20.$$

The importance to the practicing statistician of Tables of Expected Sample Size for Curtailed Fixed Sample Size Tests of a Bernoulli Parameter, by Colin R. Blyth and David Hutchinson cannot be overemphasized. The direct benefits of these tables in attribute acceptance sampling and in reliability problems are quite obvious. They provide an entire class of sampling plans with a highly desirable minimax property (namely, that of minimizing the maximum expected sample size subject to known producer and consumer risks), and then provide an extensive tabulation of the expected sample size of these plans as functions of percent defects, sample size, and number of rejects.

The tabulation of Zonal Polynomials of Order 1 Through 12, by A. M. Parkhurst and A. T. James is an exceedingly praiseworthy undertaking and provides the means of solving a large class of multivariate problems where the distribution function or moments of the distribution function can be expressed as symmetric functions of the latent roots involved in the expression.

For the convenience of the user, two alternative sets of tables have been tabulated for evaluating the zonal polynomials. Table I gives the coefficients of the zonal polynomials in terms of the sum of the powers of the latent roots, while Table II gives the coefficients of the zonal polynomials in terms of the elementary symmetric functions of the latent roots.

It is the opinion of this reviewer, however, that the authors did not do themselves justice; their explanations of the use of these tables seemed a bit too concise and therefore may not appeal to those who would most need to use them. The authors do indicate that most expressions involving zonal polynomials are extremely complicated and it is therefore difficult to illustrate the use of the tables without burdening the reader with secondary calculations. Nevertheless, it seems to this reviewer that a middle ground could have been accomplished that would be more satisfying to those who want to use these tables. If these tables are to have a more general appeal, more examples relatable to the more familiar literature in multivariate analysis (Anderson's *Introduction to Multivariate Statistical Analysis,* for example) will have to be provided.

In summary, it can be stated that the tables in this volume, as those in the first, are addressed to a number of extremely pertinent problems confronting the practicing statistician for which tables were not previously available. However, as mentioned earlier, it is important that the terminology relating to the noncentral t tables be fully clarified so that these valuable tables can be properly understood and applied. In addition, more familiar examples are recommended to illustrate the use of the zonal polynomials, so that they will appeal to a wider class of users.

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1. THE INSTITUTE OF MATHEMATICAL STATISTICS, Editors, and H. L. HARTER & D. B. OWEN, Coeditors, *Selected Tables in Mathematical Statistics*, Vol. I, American Mathematical Society, Providence, R. I., second printing, 1973. (See *Math. Comp.*, v. 29, 1975, p. 661, RMT 32.)

32 [9].—I. O. ANGELL, *A Table of Totally Real Cubic Fields*, Royal Holloway College, Univ. of London, Surrey, England, 1975. 80 computer sheets deposited in the UMT file.

This is the table referred to in Angell's paper [1]. The 4794 nonconjugate totally real cubic fields $Q(x)$ having discriminants $D < 10^5$ are listed here in the format

$$D \quad I \quad A \quad B \quad C \quad H \quad P \quad Q \quad R \quad S \quad U \quad V \quad W \quad T.$$

Here, H is the class number and $(Px^2 + Qx + R)/S$, $(Ux^2 + Vx + W)/T$ is a fundamental pair of units. (In thirty-five fields here, one or both units have coefficients that are too large for this format and they are given in an appendix at the end of the table.) The three conjugate fields are generated by the three real roots of the polynomial

$$f(x) = x^3 - Ax^2 + Bx - C = 0 \tag{1}$$

which has index I and discriminant I^2D. The fifty-one self-conjugate (cyclic) fields included here are, of course, generated by any of the three roots.

The reader is referred to my longish review [2] of Angell's complex cubic fields for comparison with the discussion that follows. The class numbers tend to be very small here since the existence of two units implies that the regulators are relatively large. The number # of fields with class number H are as follows:

H	1	2	3	4	5	6	7	8	9
#	4184	287	268	20	19	7	7	1	1

Note the curious two-step wherein each even H has about the same population as the subsequent odd H.

The polynomial (1) follows Godwin's convention [3]; A, B and C are positive and the three roots satisfy

$$0 < x_0 < 1, \quad x_0 < x_1 < x_2, \quad 2x_1 > x_0 + x_2.$$

In the reviewer's opinion, the altered polynomial $g(x') = -f([x_2] + 1 - x')$, which has $2x'_1 < x'_0 + x'_2$ instead, is preferable. Since the polynomial coefficients are symmetric functions of the three roots, the smaller x'_1, instead of the larger x_1, implies that the coefficients of $g(x')$ will generally be smaller than those of $f(x)$ (and sometimes they will be much smaller). In Table 2 below, we follow this AG (anti-Godwin) convention.

As in [2], the index I is not always minimized here. Of the first eight cases of $I = 2$ listed, $f(x)$ for $D = 1304, 1772, 2292, 2589$ and 2920 can be easily transformed into