MATHEMATICS OF COMPUTATION, VOLUME 31, NUMBER 137
JANUARY 1977, PAGES 318–320

Angstl's Mechanism for Checking Wellformedness of Parenthesis-Free Formulae

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Abstract. In 1950, H. Angstl invented a mechanical device, the design of which was the functional basis of a later construction of a relay calculator for logical formulae in polish notation. The principles of this device are illustrated.

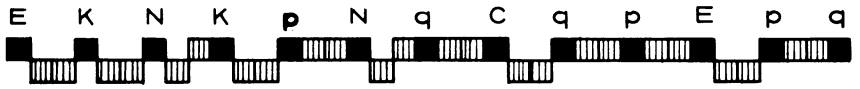
The formula-controlled logical computer STANISLAUS is based, as mentioned in [1], on a design invented by H. Angstl in 1950. In the following, a short description of the device is given, based on original drawings that Angstl kindly made available to us.

To start with, imagine a grid consisting of fixed bars (black) and between any two adjacent bars, two movable bars (red) that can be moved down (by pressing keys) so that gaps are opened:

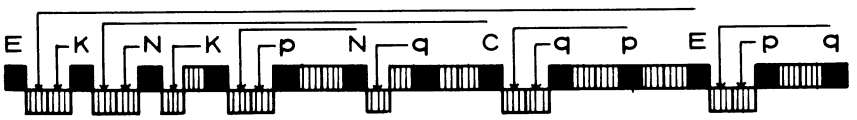
The formula in parenthesis-free (polish) form [2] is now written over the fixed bars, and from the two movable bars to the right of a fixed bar

- both bars are moved down in case of a dyadic operator like K, C, E*,
- one bar (the leftmost) is moved down in case of a monadic operator like N,
- no bar is moved down in case of a variable.

Thus, for example,



It is intuitively clear that the intermediate results are interconnected in the following way



Received December 11, 1975.
AMS (MOS) subject classifications (1970). Primary 01A65, 02B05, 68A05.

*Łukasiewicz [2] used K for "and", A for "or", C for "if-then", E for "if and only if", and N for "not".

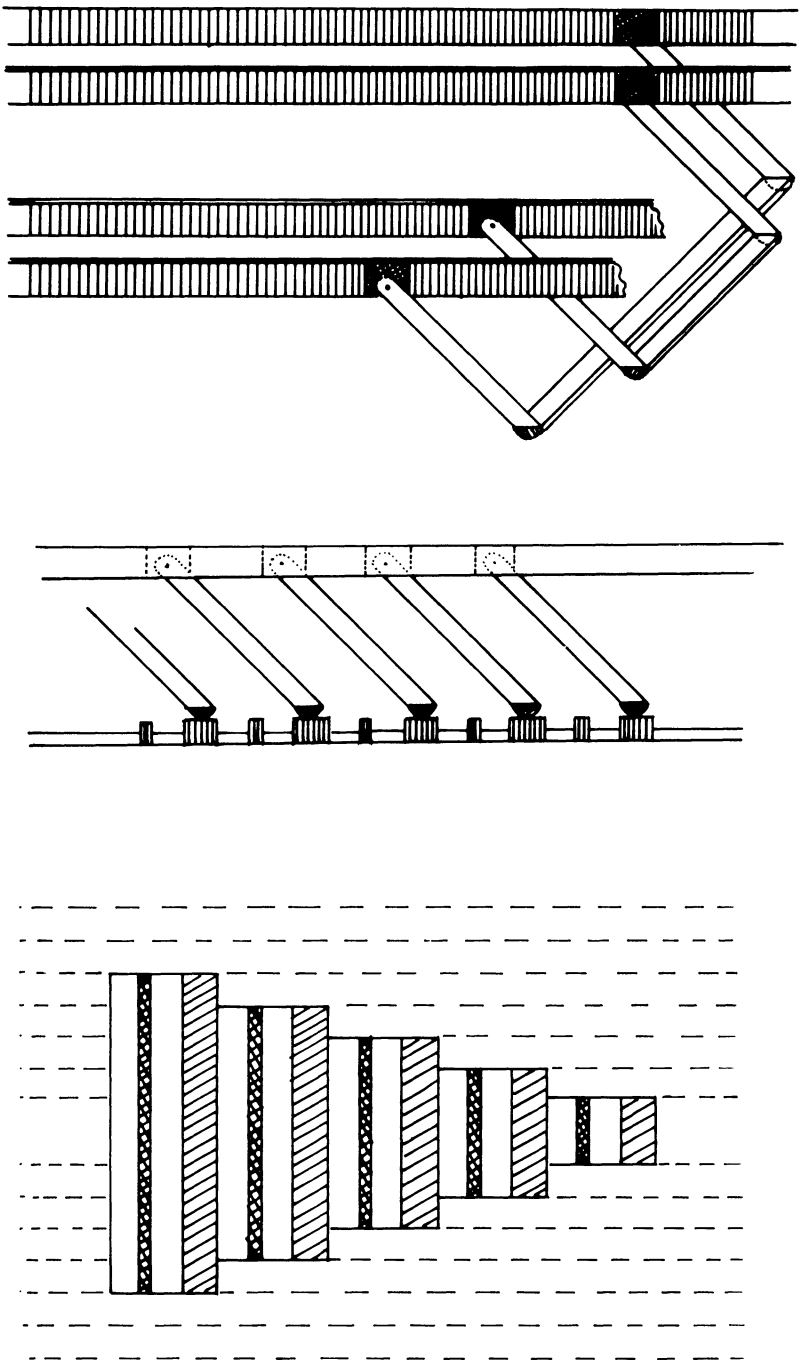
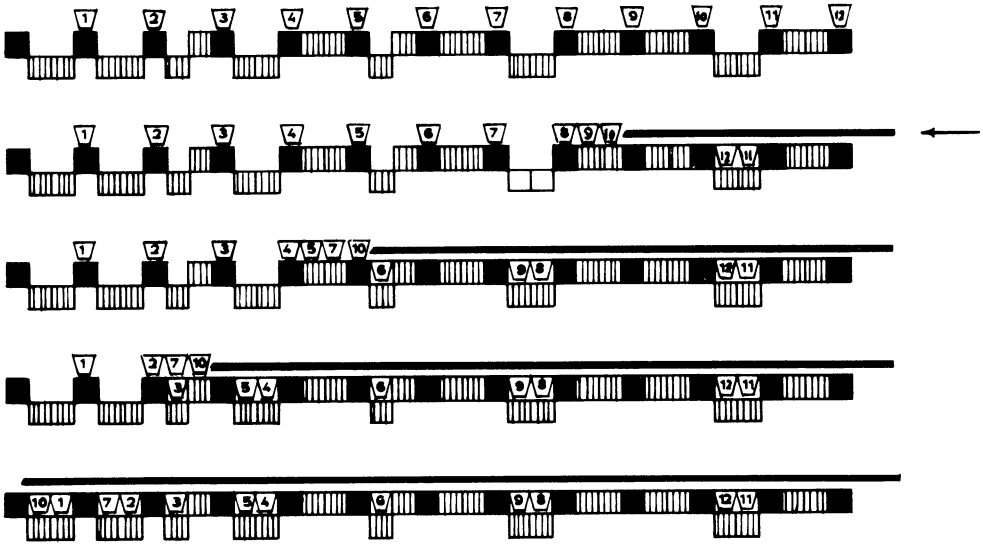


Fig. 1
Cut from Angstl's original drawing from September 1950.

In order to mechanize this interconnection Angstl assumed now that a skittle is put over each fixed bar. With a rod pushing from the right the skittles are moved and fall into gaps which they fill out so that other skittles can glide over them. In order to follow the process, we have numbered the skittles:



Again, it is intuitively clear that the formula is well-formed if and only if by the process described all skittles are dispersed to the gaps.

There are no formal proofs in Angstl's original work; in this respect Burks, Warren and Wright, in 1953 [3], went a step further—obviously unaware of Angstl's design.

From the description above, it is clear that Angstl's mechanical device could not only check for wellformedness, but could also connect the output of an operation with the corresponding input of the next operation. Angstl had indeed included this possibility (see Figure 1, a cut from the original drawing), but reliable operation could not be expected. Thus, a conventional solution with relays and a keyboard was favored when STANISLAUS, based on Angstl's idea, was designed at the turn of 1950/1951 and built later [1].

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2. JAN ŁUKASIEWICZ, *Elementy Logiki Matematycznej*, Warszawa, 1929.

3. ARTHUR W. BURKS, DON W. WARREN & JESSE B. WRIGHT, "An analysis of a logical machine using parenthesis-free notation," *MTAC*, v. 8, 1954, pp. 53–57. MR **15**, 833.