understanding of the methods treated, and it should be a good help to physicists and engineers who already have some knowledge of practical problems.

BERTIL GUSTAFSSON

Department of Computer Science
Uppsala University
Uppsala, Sweden

2 [6.15, 12.05.1].—KENDALL E. ATKINSON, *A Survey of Numerical Methods for the Solution of Fredholm Integral Equations of the Second Kind,* Society for Industrial and Applied Mathematics, Philadelphia, 1976, vii + 230 pp., 25.5cm. Price $17.50.

This volume surveys some of the numerical methods available for solving, mainly, the equation

$$\lambda x(s) - \int_a^b K(s, t)x(t)\,dt = y(s), \qquad -\infty < a \leqslant s \leqslant b < +\infty.$$

Emphasis is placed on methods which allow rather general kernels $K$. The survey includes mathematically rigorous error analyses which are done with an eye toward their usefulness for a priori estimation. Computational aspects are also treated, and a number of illustrative numerical examples given, In the end, flowcharts and FORTRAN listings for implementations of two methods are reproduced.

I shall now briefly describe the contents.

The first thirty pages, Part I, are devoted to a review of basic results from functional analysis, necessary for the mathematical development.

In Part II, the first two chapters treat a host of different methods: Successive approximation, Degenerate kernel methods–including ways of obtaining the degenerate kernels, Projection methods–the collocation and Galerkin methods.

In Chapter 3 the author considers what he calles the Nyström method: Assume that for $n = 1, 2, \ldots$ we are given points $t_{j,n} \in [a, b]$, $j = 1, \ldots, n$, and an approximate integration procedure

$$(1) \qquad \int_a^b f(s)\,ds \sim \sum_{j=1}^{n} w_{j,n} f(t_{j,n}).$$

Then obtain $z_{i,n}$, which should approximate $x(t_{i,n})$, by solving the system of equations

$$\lambda z_{i,n} - \sum_{j=1}^{n} w_{j,n} K(t_{i,n}, t_{j,n}) z_{j,n} = y(t_{i,n}), \qquad i = 1, \ldots, n.$$

This method is analyzed for continuous kernels $K$, using a crucial observation of Nyström's. Extensions to the case of singular kernels are given (the product integration method). In the analysis, the notion of collectively compact operators is used, following Anselone and Moore.

The author then notes, in Chapter 4, that the Nyström method probably leads to larger linear systems of equations than the degenerate kernel or projection methods, equal accuracy being demanded. These systems are not sparse, but the author feels that the Nyström method is competitive if certain iterative procedures for solving the linear systems are employed. These procedures are analyzed.

Finally, in Chapter 5, the author discusses computer programs implementing the Nyström method, combined with an iteration procedure given by Brakhage. Two programs are given, the first with the numerical integration scheme (1) being Simpson's rule, the second with Gaussian quadrature.

The author succeeds in treating, and making interplay between, the mathematical and the computational aspects of the surveyed methods, while maintaining high mathematical rigor and giving useful numerical considerations. His value judgements, e.g., his choice of method to implement in Chapter 5, are well argued from the analysis.

The style of the book is clear and readable, and the misprints few. Interconnections between the methods treated are pointed out, and the text is well integrated.

All in all, this is a delightful volume.

LARS B. WAHLBIN

Department of Mathematics
Cornell University
Ithaca, New York 14853

3 [7.75].—YUDELL L. LUKE, *Mathematical Functions and their Approximations*, Academic Press, New York, 1975, xvii + 568 pp., 23.5cm. Price $14.50.

One of the best selling mathematics books of all time (discounting text books) is *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables* edited by M. Abramowitz and I. Stegun, National Bureau of Standards Applied Mathematics Series 55, U. S. Government Printing Office, Washington, D. C., 1964 (or AMS 55 for short). To quote from the preface of the book of Luke under review:

"The cutoff date for much of the material in AMS 55 is about 1960. In the past 15 years much valuable new information on the special functions has appeared. In some quarters, it has been suggested that a new AMS 55 should be produced. This is not presently feasible. The task would be gigantic and would consume much time. Most certainly the economics of the situation forbids such a program. A feasible approach is a handbook in the spirit of AMS 55, which, in the main, supplements the data given there.

The present volume can be conceived as an updated supplement to that portion of AMS 55 dealing with mathematical functions."

This extensive quote is given so that the reader will have some idea what the author purports to do. Actually, this is not an adequate summary of Luke's book, and the title is misleading. The book is about hypergeometric functions, some of the generalized hypergeometric functions and special cases. Hypergeometric functions are very important, and a number of books need to be written about them from various points of view. But they are not the only useful mathematical functions, so the title of this book is unfortunate.

However, the content of a book is much more important than the title. The main focus is the problem of computing the hypergeometric series

$$ {}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; x) = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n}{(b_1)_n \cdots (b_q)_n} \frac{x^n}{n!}, $$

where the shifted factorial $(a)_n$ is defined by $(a)_n = a(a+1)\cdots(a+n-1)$, $n = 1, 2, \ldots$, $(a)_0 = 1$. Among the special cases treated in some detail are the binomial function $(1+z)^a = {}_1F_0(-a; -; -z)$, $\ln(1+z) = z\,{}_2F_1(1, 1; 2; -z)$, $\exp(z) = {}_0F_0(-; -; z)$, $\arcsin z = z\,{}_2F_1(1/2, 1/2; 3/2; z^2)$, $\arctan z = z\,{}_2F_1(1/2, 1; 3/2; -z^2)$, the incomplete gamma function $\gamma(\nu, z) = \nu^{-1}z^{\nu}\,{}_1F_1(\nu, \nu+1; -z)$, and the error function $\mathrm{Erf}(z) = z\,{}_1F_1(1/2; 3/2; -z^2)$. The Bessel function $J_\nu(z) = [(z/2)^\nu/\Gamma(\nu+1)]\,{}_0F_1(-; \nu+1; -z^2/4)$, the confluent hypergeometric function ${}_1F_1(a, b; c; z)$, and the classical hypergeometric function ${}_2F_1(a, b; c; z)$ are each treated in separate chapters.

The longest chapter is "The generalized hypergeometric function ${}_pF_q$ and the $G$-function." The function ${}_3F_2(a, b, c; d, e: 1)$ is very important, and it was completely omitted from AMS 55. For example, the functions

$$ {}_3F_2(-n, n+\alpha+\beta+1, -x; \alpha+1, -N; 1) $$