

formation of a balanced ${}_4F_3$ to a very well poised ${}_7F_6$. These are the formulas which lie behind many of the explicit sums that are known.

The remark on page 243 is correct, and it would have been helpful if some indication of its importance had been given. It is the reason for the existence of Hahn polynomials as orthogonal polynomials, and it has been used to prove deep inequalities for some ${}_3F_2$'s. As it stands, it looks like just another random comment, while it is actually a very important remark.

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4 [14].—PETER HENRICI, *Applied and Computational Complex Analysis*, Vol. 1, John Wiley & Sons, Inc., New York, 1974, xv + 682 pp., 24cm. Price \$24.95.

The importance of complex function theory in applied mathematics both from a qualitative and from a numerical standpoint is indisputable. Thus, the appearance of a modern treatment emphasizing the manipulative and computational aspects of the subject will be welcomed by those interested in applying mathematics in many areas, and by numerical mathematicians in particular.

This volume, the first of three, is devoted to power series, analytic continuation, complex integration, elementary conformal mapping, polynomials, and partial fractions. The second, scheduled to appear shortly, will include material pertinent to the analysis of ordinary differential equations, special functions, integral transforms, and continued fractions, while the last volume will treat topics bearing on the study of partial differential equations.

The treatment of power series is divided into two chapters. The first, on formal power series, discusses the formal manipulation of series in considerably more detail than usual, including composition and reversion as well as formal differentiation and algebraic operations. Convergence is not introduced until the second chapter in which, anticipating the need for analytic functions of matrices in discussing systems of differential equations, the variable is taken as an element of a general Banach algebra. Most of the standard results on convergence, analyticity, composition and inversion carry through with this generalization, and so do many of the properties of the elementary transcendental functions.

A distinctive feature of the next chapter, on analytic continuation, is the discussion of the techniques necessary to make the Weierstrass process a constructive one by reducing the number of terms included in the successive power series expansions.

The fourth chapter, of over 100 pages, is primarily devoted to complex integration and its many applications, but also includes an analytic treatment of the Laurent series, and of the principle of the argument.

The discussion of conformal mapping in Chapter 5 (to be extended in the next volume) begins with an exposition of the geometric approach to complex analysis. A thorough treatment of the Moebius transformation is followed by a brief development of the theory of holomorphic functions and their equivalence to analytic functions. Applications of the techniques developed so far to problems of two-dimensional electrostatics, fluid dynamics and elasticity are given before presenting the general mapping theorem, the symmetry principle, and the Schwarz-Christoffel mapping function.

Although there are interesting variations and extensions, and a refreshing selection of new illustrative examples, these first chapters are primarily standard material. The next two are less usual. Chapter 6, on polynomials, begins with material such as the

Horner algorithm, Descartes' rule of signs, and Sturm sequences, which used to be presented under the title "Theory of Equations," and may now be found in some numerical analysis texts. It then proceeds to more sophisticated topics including the geometry of zeros, the numbers of zeros in discs and half-planes (including the stability problem), and "circular arithmetic." The chapter concludes with a careful treatment of the problem of actually finding zeros numerically, both by refining inclusion regions, and by the more classical iterative techniques.

Polynomial zeros also play an important role in the final chapter on partial fractions; they are needed to construct partial fraction representations for rational functions; but conversely, the Hankel determinants, and the quotient-difference (qd) algorithm, which are developed in this chapter, are valuable in finding zeros of polynomials. In fact, the discussion of the qd algorithm in this chapter is entirely directed toward the location of poles and zeros. Applications to convergence acceleration and the construction of corresponding continued fractions will presumably be considered in the next volume.

As might be expected, there are a number of misprints, and some annoying indexing omissions. Most of these will undoubtedly be corrected in the later printings which this work clearly deserves.

In summary, all the standard material appears, supplemented by applications to problems to which the author has himself contributed significantly.

The treatment is modern in the sense that there is no hesitation in using the generalizations of abstract algebra when they can contribute to extending the domain of application of the results. It is applied both in the sense that there are many illustrations of the use of complex analysis in physics, engineering, and even mathematics, and also in the wide range of mathematics which is brought to bear on the problems of complex analysis, including Banach algebras, groups, fields, and integral domains from abstract algebra, and matrix algebra, the Jordan normal form, Gershgorin theory, and classical determinant results from linear algebra. The necessary definitions and results in these areas are included, however, so that the reader without broad training in mathematics should still be able to follow, and may be helped to realize that few branches of mathematics are irrelevant to the applied mathematician.

The exposition is supplemented by many worked examples, by a large number of exercises (unfortunately without solutions or hints), and each chapter includes a set of suggestions for somewhat more extensive investigations to be used as seminar assignments. Both the examples and the exercises extend the scope of the treatment by presenting useful supplementary results.

The work which this volume begins has been expected at least since 1963, when Henrici deferred proofs of most of the results in his definitive review of the quotient-difference algorithm to a forthcoming book. Most readers will agree that it has been worth waiting for, and will eagerly anticipate the appearance of the last two volumes. It may well be that Henrici will replace Whittaker and Watson as the standard source for results in applied complex analysis.

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