the Fortran programs must be carefully tailored for the particular computing environment where they are to be used.

The first project of the National Activity to Test Software (NATS) was to produce these programs. One may view this effort as one of technology transfer. A knowledgeable person starting with the Wilkinson-Reinsch procedures can expect to spend 30 days to produce a high quality program for a particular eigensystem calculation. The same person starting with EISPACK can expect to spend 30 minutes to obtain a similar program, one which is probably better than his own product. Even more significant is that a naive user can start with EISPACK and a simple interface routine and obtain the same high quality programs. Thus NATS has placed the eigensystem computation expertise of Wilkinson and Reinsch at the finger tips of the whole scientific community. One should note that NATS also made various improvements in the algorithms as well as corrected a few errors.

People who have not seen a full cost accounting of top quality software are amazed at its cost. The EISPACK project cost something less than a million dollars to produce about 12,000 lines of Fortran code. This cost of \$80 per Fortran line is unusually high for several reasons, but \$25 per Fortran statement is to be expected for top quality, reliable and documented programs.

This book has three distinct audiences: the naive user (he knows what eigenvalues are, but has little idea of how to compute them), the sophisticated user (he has a broad knowledge of modern methods for eigensystem computations and some familiarity with the EISPACK terminology) and the eigensystem software expert. There are about 50 pages of basics on "How to use EISPACK" which will challenge the naive user. He would be better off with 5 pages describing how to use a simple interface. The second release of EISPACK recognized the naive user's problem and provides an interface for IBM users which is described among these 50 pages.

The 120 pages of "How to use EISPACK" (50 pages of basics plus 70 pages of special considerations) are exactly aimed at the sophisticated user. Once one is familiar with the EISPACK terminology and organization, one can readily obtain the right programs for almost any eigensystem calculation.

The remaining 430 pages are for the eigensystem software expert. About 350 pages give the actual Fortran programs and their documentation. The validation and certification of EISPACK are described only briefly (5 pages) and references are given to other publications. It is unfortunate that these topics are slighted since they are crucial to the scientific merit of EISPACK. Another 20 pages of text would have been well spent to substantiate the claims of quality and reliability for EISPACK. There are 57 pages of tables which substantiate the efficiency claims for EISPACK. Somewhat random comparisons with other programs (not reported in this book) indicate that other programs are usually significantly less efficient or less reliable and often both.

This book is a must for the library of a mathematical software expert or a numerical analyst who does eigensystem calculations. People who occasionally do straightforward eigensystem calculations will prefer a short and simple description of how to use EISPACK.

J. R. R.

4 [5, 6.30].—R. Glowinski, J. L. Lions, & R. Trémolières, *Analyse Numérique de Inéquations Variationnelles,* Vol. 1; *Applications aux Phénomènes Stationnaires et d'Évolution,* Vol. 2, Dunod, Paris, 1976, xiv + 290 pp., 25 cm. Price: Vol. 1 \$36.00, Vol. 2 \$42.00.

This two volume set is the long-awaited work dealing with the numerical analysis of problems formulated in *Les inéquations en mécanique et en physique* by G. Duvaut and J. L. Lions.

The first volume deals with the general theory of stationary variational inequalities (Chapter 1), discusses algorithms for solving the finite-dimensional optimization problems which result from the approximation schemes (Chapter 2), and then considers in detail the specific model problem of elasto-plastic torsion of a cylindrical bar (Chapter 3).

To illustrate the type of problems dealt with in the books, the elasto-plastic torsion problem can be formulated as the variational inequality:

Find $u \in K$ such that

$$\int_\Omega \nabla u \cdot \nabla(v-u)\,dx \geqslant C\int_\Omega (v-u)\,dx \;\forall v \in K,$$

where $K = \{v \in H_0^1(\Omega): |\text{grad } v| \leqslant 1 \text{ a.e. in } \Omega\}$,

Ω is the cross section of the bar, and u represents the stress potential. It also has the equivalent formulation:

Find $u \in K$ minimizing over K the functional

$$J(v) = \frac{1}{2}\int_\Omega |\nabla v|^2\,dx - C\int_\Omega v\,dx.$$

Basically, one obtains an approximation scheme by replacing the infinite-dimensional minimization problem by a finite-dimensional one through the use of finite differences or finite elements.

As pointed out in the introduction to Volume 1, this model contains many of the essential difficulties of the general theory (e.g., how should the convex set K be approximated). Hence, by studying this concrete problem in detail, the authors are able to lend a great deal of insight into the special difficulties encountered in the approximation of variational inequalities. The discussion of this problem in Chapter 3 includes the formulation of various approximation schemes, the proof of convergence of the methods, a discussion of algorithms for solving the resulting finite-dimensional optimization problems, and a presentation of numerical computations.

Other applications that can be formulated as stationary variational inequalities appear in Volume 2. In particular, the problem of a nondifferentiable cost functional is studied in the context of problems of temperature control and of diffusion of fluids through semipermeable walls in Chapter 4 and the stationary flow of a Bingham fluid in a cylindrical pipe in Chapter 5. As an illustration, the cost functional for the last problem is similar to the $J(v)$ described previously with the addition of the nondifferentiable term $\int_\Omega |\text{grad } v|\,dx$. Finally, Chapter 6 contains a discussion of some general approximation schemes for variational inequalities of evolution type. Once again the theory is illustrated by several examples and the results of numerical computations are given.

As evidenced by the book of Duvaut and Lions, many problems in mechanics and physics can be formulated in the context of variational inequalities. The work of Glowinski, Lions, and Trémolières should provide a valuable reference for the numerical solution of such problems, especially for the reader already acquainted with finite-difference and finite-element approximations for unconstrained problems.

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