5 [9].—A. O. L. ATKIN, *The Number of Subgroups of the Classical Modular Group of Index N,* University of Illinois, Chicago Circle, 1976, 61 sheets of computer output deposited in the UMT file.

Newman's earlier table of this same function $M(N)$ was reviewed in [1]. It had the range $N = 1(1)255$ and was computed to check C. R. Johnson's conjecture (see [1]) which asserted that $M(N)$ is odd iff $N = 2^n - 3$ or $2(2^n - 3)$ for $n = 2, 3, \ldots$. It did check the conjecture in that range and so preprints of [1] were sent to several investigators in this field. There were two responses.

The table deposited here by Atkin goes further: $N = 1(1)1024$. It agrees with Newman's table at $N = 255$ and verifies that $M(N)$ is odd in the continuation only for $N = 506, 509, 1018$ and 1021. Subsequently, Atkin proved the conjecture. In the meantime, a proof was sent by W. Wilson Stothers of the University of Glasgow [2]. It uses results in his dissertation [3]. The whole episode is a nice example of the interplay of table computation, thoughtful examination of tables, conjectures, and new theory.

These large numbers $M(N)$ are printed here in blocks of five decimals and, as is so common in number-theoretic tables, the high-order digits in each block are suppressed if they equal zero. I have been arguing for years, cf. [4], against this easily eliminated inelegancy, but to little avail.

The value of $M(1024)$ here is

$$21655 \; {}^*(449)^* \; 38688$$

where $^*(k)^*$ means that k digits are not shown. Newman's asymptotic formula [5, Theorem 4] is

$$M(N) \sim K \exp\left(\frac{N \log N - N}{6} + N^{1/2} + N^{1/3} + \frac{\log N}{2}\right) \tag{1}$$

where $K = (12\pi e^{1/2})^{-1/2}$. For $N = 1024$ the right side of (1) gives 3.3229×10^{458}, an error of 53%. Of course, $\log M(N)$ is much more accurate, then the error is only 0.023%. Perhaps an interested reader may wish to determine the second-order term for (1).

D. S.

1. DANIEL SHANKS, "Review of Newman's Table," UMT 6, *Math. Comp.*, v. 31, 1977, p. 612.

2. W. W. STOTHERS, Letter to M. Newman, Sept. 9, 1976.

3. W. W. STOTHERS, *On Some Discrete Triangle Groups,* Dissertation, Cambridge, 1971.

4. DANIEL SHANKS, "Review of Kortum and McNiel's Table," UMT 30, *Math. Comp.*, v. 16, 1962, pp. 377–379.

5. MORRIS NEWMAN, "Asymptotic formulas related to free products of cyclic groups," *Math. Comp.*, v. 30, 1976, pp. 838–846.