The first column lists all the primes from 5 to 10000. The stars in the second column indicate the E-irregular ones. In the third column one finds the primitive root r, for which either r or $r - p$ is the least in its absolute value. These primitive roots have been checked from [5].

The next column gives the residue class mod 4 of the prime. It is known that $E_{p-1} \equiv 0$ or $2 \bmod p$ if $p \equiv 1$ or $3 \bmod 4$, respectively. In the fifth column $E_{p-1}/p \bmod p$ is given in the case $p \equiv 1 \bmod 4$. It turns out that in our range E_{p-1} never vanishes mod p^2. Cf. [3, Theorem 3].

In the next column there is the value of the Fermat quotient q_2 for those primes p that are either congruent to 1 mod 4 or E-irregular. This was printed in order to check our computations and was compared with the tables of [4]. Our value of q_2 was different from that of [4] for eleven primes, namely 2437, 4049, 4733, 4969, 5689, 6113, 6997, 7121, 7321, 8089, and 8093. A comparison with [1] and [2] showed that in these cases q_2 is incorrectly given in [4].

Similarly, for the primes p congruent to 1 mod 4 or E-irregular, we computed the value mod p of the sum

$$-6 \sum_{k=1}^{(p-1)/2} (2k-1)^2 q_{2k-1}$$

which is given in the next column. This value is always 1, as it should be.

The last three columns are associated with the E-irregular primes. First, the indices $2n$ ($2n \leqslant p - 3$) are given for which $E_{2n} \equiv 0 \bmod p$, i.e. the pair $(p, 2n)$ is E-irregular. The last two columns give the values of E_{2n}/p and $(E_{2n+p-1} - E_{2n})/2p \bmod p$. We observe that in our range E_{2n} and $E_{2n+p-1} - E_{2n}$ never vanish mod p^2. Cf. [3, Theorem 5].

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A *perfect rational cuboid* is a rectangular parallelepiped whose three edges, three face diagonals and body diagonal all have integer lengths. None is known. The present tables relate to cuboids of which six of these seven lengths are integers. For a general discussion see [5].

1. *Body diagonal irrational.* The dimensions satisfy

$$x_2^2 + x_3^2 = y_1^2, \quad x_3^2 + x_1^2 = y_2^2, \quad x_1^2 + x_2^2 = y_3^2.$$

Table 1 lists 769 solutions of the equation

$$\frac{a_1^2 - b_1^2}{2a_1b_1} \cdot \frac{a_2^2 - b_2^2}{2a_2b_2} = \frac{a_3^2 - b_3^2}{2a_3b_3},$$

in which each pair of integers a_i, b_i are of opposite parity with $a_i > b_i > 0$. The table is complete for solutions in which two of the a_i do not exceed 376. To each solution of this equation there correspond two cuboids, with

$$\frac{x_1}{x_2}, \frac{x_2}{x_3} = \frac{a_1^2 - b_1^2}{2a_1b_1}, \frac{a_2^2 - b_2^2}{2a_2b_2}$$

in respective or reverse order. Dimensions not exceeding 10^6 are given to facilitate comparison with published lists. These previous tables are those of Kraitchik [1], which lists dimensions and generators for 241 cuboids whose odd dimension does not exceed 10^6, Kraitchik [2], which supplements this list with 18 cuboids whose odd dimension does not exceed 10^5, and Lal and Blundon [3], which lists cuboids corresponding to a_1, $a_3 \leqslant 70$ (but often only one of each pair). Table 2 reproduces Kraitchik's cuboids as the original publications are not readily available. For the results of comparisons see the Table Errata in this issue.

2. *One face diagonal irrational.* The dimensions satisfy

$$x_1^2 + x_2^2 = y_3^2, \quad x_1^2 + x_3^2 = y_2^2, \quad x_1^2 + x_2^2 + x_3^2 = z^2.$$

Solutions are in cycles of five [4], [5]. Table 3 lists 560 cycles of five integer pairs a_i, b_i, satisfying the equation

$$\frac{a_{i-1}^2 - b_{i-1}^2}{2a_{i-1}b_{i-1}} \cdot \frac{a_{i+1}^2 - b_{i+1}^2}{2a_{i+1}b_{i+1}} = \frac{a_i^2 + b_i^2}{2a_ib_i},$$

where the subscripts are cyclically reduced modulo 5 and $a_i > b_i > 0$. The table is complete for cycles in which two of the a_i do not exceed 376. To each cycle there correspond five cuboids, having

$$\frac{x_2}{x_1} = \frac{a_i^2 - b_i^2}{2a_ib_i}, \quad \frac{x_3}{x_1} = \frac{a_{i+1}^2 - b_{i+1}^2}{2a_{i+1}b_{i+1}}.$$

Dimensions are not listed. An asterisk is placed between a_i, b_i and a_{i+1}, b_{i+1} in each solution satisfying the additional condition that $(a_ia_{i+1})^2 + (b_ib_{i+1})^2$ and $(a_ib_{i+1})^2 + (a_{i+1}b_i)^2$ are both perfect squares (their product is always square). This table extends the short table of 35 cycles in [4]. Table 4 lists the dimensions of the 130 cuboids with $z < 250000$, with the corresponding cycles of generators.

3. *One edge irrational.* The dimensions satisfy

$$x_1^2 + x_2^2 = y_3^2, \quad x_1^2 + y_1^2 = x_2^2 + y_2^2 = t + y_3^2 = z^2,$$

where t is the square of the irrational edge. The generators are integers a_1, b_1, a_2, b_2, α, β such that

$$\frac{z}{x_1} = \frac{a_1^2 + b_1^2}{2a_1b_1}, \quad \frac{z}{x_2} = \frac{a_2^2 + b_2^2}{2a_2b_2}, \quad \frac{x_2}{x_1} = \frac{\alpha^2 - \beta^2}{2\alpha\beta};$$

satisfying

$$\frac{a_1^2 + b_1^2}{2a_1b_1} \cdot \frac{2a_2b_2}{a_2^2 + b_2^2} = \frac{\alpha^2 - \beta^2}{2\alpha\beta}.$$

Since any ratio x_1/x_2 can occur in solutions [5], it is of less interest to list solutions according to their generators. Table 5 lists the generators and dimensions of the 160 solutions with $z < 250000$. Of these 78 have $t > 0$ and correspond to real cuboids; the other 82 have $t < 0$. There are no previous tables.

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