esting introduction to algebraic number theory; the deepest result in the book on Fermat's Last Theorem is that it is true for regular primes. The author describes his approach as genetic. This means that we do not just receive a polished version with slick proofs. Rather, when a question is raised, we are shown through explicit calculations where the difficulties lie and why new ideas must be introduced. Concepts therefore arise naturally, in contrast to the usual approach in which it sometimes seems that some almighty power dictates what must be studied.

Using Fermat's Last Theorem as a guide, the author first leads us through the work of some of the early number theorists: Fermat, Euler, Sophie Germain, Dirichlet, and Legendre. Then we come to the main part of the book, dealing with Kummer's results. Through many explicit calculations, we are shown how Kummer was led to develop divisor theory for cyclotomic fields. This is then applied to prove Fermat's Last Theorem for regular primes. The relationship between the class number and Bernoulli numbers is then treated, via the explicit formula for $L(1, \chi)$. The book concludes with three chapters on quadratic fields and binary quadratic forms, including Gauss' theory of composition and Dirichlet's class number formula.

The prerequisites for reading the book are minimal (there is an appendix on congruences). However, the reader who knows some algebraic number theory, but only as a collection of abstract theorems, will profit the most. Those who read this book first should subsequently consult a standard treatment of algebraic number theory since the present book does not consider a general number fields or commutative algebra. For this reason the book probably should not be used by itself for a text, but would be excellent when used in conjunction with another book.

Finally, there is an excellent set of exercises, many of them interesting and difficult. Fortunately, there are answers at the end of the book.

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16[9].—HERMAN J. J. TE RIELE, *Unitary Aliquot Sequences,* Report MR 139/72, Mathematisch Centrum, Amsterdam, Sept. 1972, iv + 44 pp., 26.5 cm.

A divisor d of n is called *unitary* if $(d, n/d) = 1$; $s^*(n)$ denotes the sum of the unitary divisors of n, apart from n itself, e.g. $s^*(72) = 1 + 8 + 9 = 18$. A *unitary aliquot sequence* (UAS) is obtained by iterating the function s^*; if a UAS is periodic, its members form a *unitary t-cycle.* The first 40, 000 UAS's, were found to comprise 35701 terminating; 728 of which ultimately cycled on a unitary perfect number, 6, 60 or 90; 966 which ultimately cycled on one of six unitary amicable pairs (114, 126), (1140, 1260), . . . ; and 2605 which ultimately cycled on the triple (30, 42, 54), on the quintuple (1482, 1878, 1890, 2442, 2178) or on one of three groups of order 14.

All sequences < 40000 which are ultimately periodic are tabulated and many of them are displayed as digraphs. Only two sequences were found with maxima $> 10^9$; the UAS for 38370 and 31170 had lengths of 1154 and 879.

The existence of arbitrarily long UAS's is proved after the method of H. W.

Lenstra, and methods of constructing unitary t-cycles are given, illustrated by a unitary 4-cycle.

RICHARD K. GUY

17[9].—HERMAN J. J. TE RIELE, *Further Results on Unitary Aliquot Sequences*, Report NW 2/73, Mathematisch Centrum, Amsterdam, Mar. 1973, iv + 59 pp., 26.5 cm.

The work of the previous report is extended to 10^5 and the range $(10^6, 10^6 + 10^3)$ is also examined. The number of known unitary t-cycles is reported as $5 + 1186 + 1 + 8 + 1 + 1 + 3 + 1$ for $t = 1, 2, 3, 4, 5, 6, 14, 25$. The first 10^5 UAS's comprise 88590 terminating; $1697 + 3005 + 4722 + 1 + 1586 + 398$ which are ultimately periodic with $t = 1, 2, 3, 4, 5, 14$; and 1 unknown sequence (89610) which was abandoned at its 541st term, a number of 21 digits. The proportions of sequences in the range $(10^6, 10^6 + 10^3)$ were remarkably similar; no new unknown sequences were discovered.

The construction of unitary amicable pairs (2-cycles) is discussed and 1079 new pairs are listed. The tabulation of all ultimately periodic sequences is extended from 4×10^4 to 10^5.

An appendix gives three theorems of Walter Borho on the number of different prime factors of unitary t-cycles.

RICHARD K. GUY

18[9].—HERMAN J. J. TE RIELE, *A Theoretical and Computational Study of Generalized Aliquot Sequences*, Mathematical Centre Tracts 74, Mathematisch Centrum, Amsterdam, 1976, x + 76 pp., 24 cm. This is a slight revision of the author's 1975 doctoral thesis.

f is defined as a multiplicative arithmetic function with $f(p^e)$ a polynomial of degree e in p, with coefficients 0 and 1, the leading coefficient and at least one other being 1. E.g. if all coefficients are 1, $f(x) = \sigma(x)$, the sum of the divisors of x; if only the first and last are 1, $f(x) = \sigma^*(x)$, the sum of the unitary divisors of x. A *generalized aliquot n-sequence* is defined by $n_0 = n$, $n_{i+1} = f(n_i) - n_i$. These include ordinary and unitary aliquot sequences. An n-sequence is *terminating*, of *length l*, if $\exists l \ni n_l = 1$; and *periodic* with *cycle-length c* if $\exists l, c \ni n_{l+c} = n_l$. For ordinary aliquot sequences the Catalan-Dickson conjecture is that this classification is exhaustive. Many now believe that there are sequences which are unbounded. Most of the corresponding questions for generalized aliquot sequences are also open. The author generalizes a theorem of H. W. Lenstra [v. reviewer in *Number Theory and Algebra*, Academic Press, 1977, pp. 111–118] to show that there are generalized sequences of arbitrary length.

The author investigates the distribution of the values of f, and the mean value of $f(n)/n$. He also computed 15 different types of n-sequences for $n \leqslant 1000$ until they terminated, became periodic or exceeded 10^8. Some sequences, associated with a sum of divisors d for which n/d is $(k + 1)$-free (the first $k + 1$ coefficients in the polynomial are 1) were shown to be unbounded, e.g. $k = 1$, $n = 318$.