Lenstra, and methods of constructing unitary t-cycles are given, illustrated by a unitary 4-cycle.

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17[9].—HERMAN J. J. TE RIELE, *Further Results on Unitary Aliquot Sequences,* Report NW 2/73, Mathematisch Centrum, Amsterdam, Mar. 1973, iv + 59 pp., 26.5 cm.

The work of the previous report is extended to 10^5 and the range $(10^6, 10^6 + 10^3)$ is also examined. The number of known unitary t-cycles is reported as $5 + 1186 + 1 + 8 + 1 + 1 + 3 + 1$ for $t = 1, 2, 3, 4, 5, 6, 14, 25$. The first 10^5 UAS's comprise 88590 terminating; $1697 + 3005 + 4722 + 1 + 1586 + 398$ which are ultimately periodic with $t = 1, 2, 3, 4, 5, 14$; and 1 unknown sequence (89610) which was abandoned at its 541st term, a number of 21 digits. The proportions of sequences in the range $(10^6, 10^6 + 10^3)$ were remarkably similar; no new unknown sequences were discovered.

The construction of unitary amicable pairs (2-cycles) is discussed and 1079 new pairs are listed. The tabulation of all ultimately periodic sequences is extended from 4×10^4 to 10^5.

An appendix gives three theorems of Walter Borho on the number of different prime factors of unitary t-cycles.

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18[9].—HERMAN J. J. TE RIELE, *A Theoretical and Computational Study of Generalized Aliquot Sequences,* Mathematical Centre Tracts 74, Mathematisch Centrum, Amsterdam, 1976, x + 76 pp., 24 cm. This is a slight revision of the author's 1975 doctoral thesis.

f is defined as a multiplicative arithmetic function with $f(p^e)$ a polynomial of degree e in p, with coefficients 0 and 1, the leading coefficient and at least one other being 1. E.g. if all coefficients are 1, $f(x) = \sigma(x)$, the sum of the divisors of x; if only the first and last are 1, $f(x) = \sigma^*(x)$, the sum of the unitary divisors of x. A *generalized aliquot n-sequence* is defined by $n_0 = n$, $n_{i+1} = f(n_i) - n_i$. These include ordinary and unitary aliquot sequences. An n-sequence is *terminating,* of *length l,* if $\exists l \ni n_l = 1$; and *periodic* with *cycle-length c* if $\exists l, c \ni n_{l+c} = n_l$. For ordinary aliquot sequences the Catalan-Dickson conjecture is that this classification is exhaustive. Many now believe that there are sequences which are unbounded. Most of the corresponding questions for generalized aliquot sequences are also open. The author generalizes a theorem of H. W. Lenstra [v. reviewer in *Number Theory and Algebra,* Academic Press, 1977, pp. 111–118] to show that there are generalized sequences of arbitrary length.

The author investigates the distribution of the values of f, and the mean value of $f(n)/n$. He also computed 15 different types of n-sequences for $n \leqslant 1000$ until they terminated, became periodic or exceeded 10^8. Some sequences, associated with a sum of divisors d for which n/d is $(k + 1)$-free (the first $k + 1$ coefficients in the polynomial are 1) were shown to be unbounded, e.g. $k = 1$, $n = 318$.

f-aliquot k-cycles are also investigated. The infôrmation on p. 60 is not as up-to-date as his earlier work or that quoted [Lal et al., Proc. 2nd Manitoba Conf. Numerical Math., 1972, pp. 211–216]. The inverse problem of solving $f(x) - x = m$ and continuing sequences backwards, is discussed; it is shown for example that the set of m for which $\sigma(x) - x = m$ has no solution has lower density > 0.0324.

The bibliography contains 41 items.

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