REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

19 [2.05].— HELMUTH SPÄTH, *Spline-Algorithmen zur Konstruktion glatter Kurven und Flächen,* Oldenbourg Verlag, Munich, 1973, 134 pp., 24 cm. Price DM 40.

The algorithms and programs in this handy book deal almost exclusively with cubic splines.

A short introduction stresses the maximum "smoothness" of "natural" splines. The next chapter reviews the numerical solution of tridiagonal linear systems by Gauss elimination and of block tridiagonal linear systems by relaxation.

Chapter 3 offers programs for cubic spline interpolation with various (separated) boundary conditions. The next chapter deals with periodic splines and with the interpolation of curves by interpolation of their component functions with respect to some parametrization. Also, various "smoothing methods" are described, all of which turn out to be interpolation methods based on piecewise cubic Hermite interpolation with the slopes at the interpolation points estimated locally. The author's own construction of an area matching cubic spline approximation to histograms follows.

Quintic spline interpolation is the subject of Chapter 5, and block underrelaxation (!) is used for the solution of the appropriately ordered linear system for the determination of the interpolant. The interpolating quintic spline fares badly in the comparison with the cubic spline interpolant as given in five accompanying pictures.

The cubic smoothing spline of (Schoenberg and) Reinsch is introduced in the next chapter without any motivation at all as a cubic spline whose deviation at each data point (and knot) is proportional to the jump in its third derivative across the knot, with proportionality factor at each knot to be determined. Block underrelaxation is used in its construction, and the user is expected to choose the proportionality factors, perhaps interactively.

Chapter 7 gives a very nice summary of the author's efforts to obtain simple interpolating functions (and curves) which are less wiggly than the cubic spline is at times. Starting with Schweikert's spline in tension, the author discusses a general class of methods in which each cubic piece is replaced by a more flexible function, involving an exponential perhaps or a rational function, and offers programs for the construction of such "generalized cubic spline" interpolants. Again, much is left to interactive user efforts and the simple and attractive alternative of sticking to ordinary cubic splines and combatting extraneous inflection points by the suitable addition of knots instead is not mentioned at all.

The last chapter uses tensor products of the various univariate approximation schemes to produce interpolants or approximants to data on a rectangular grid.

An English translation, by W. D. Hoskins and H. W. Sager, with the title *Spline*

Algorithms for Curves and Surfaces has been published in 1974 by Utilitas Mathematica Publ. Inc., Winnipeg, Manitoba (viii + 198pp).

C. D.

20 [2.05].—SAMUEL KARLIN, CHARLES A. MICCHELLI, ALLAN PINKUS & I. J. SCHOENBERG, *Studies in Spline Functions and Approximation Theory,* Academic Press, New York, 1976, xii + 500 pp., 23.5cm. Price $19.50.

This book is a collection of 15 research papers written by the authors individually, and in various coauthorship combinations. The papers are related in content, and by the fact that each of the authors spent some time at the Weizmann Institute of Science, Rehovot, Israel, between September, 1970 and June, 1974. The papers are arranged into four categories, and we briefly describe each of them below.

Part I of the book deals with best approximations, optimal quadrature, and monosplines. The papers are:

(1) *On a class of best non-linear approximation problems and extended monosplines,* by S. Karlin. This paper deals with existence, uniqueness, and characterization of best (nonlinear) approximations to 0 from the class of extended monosplines with free knots. The results are applied to optimal quadrature.

(2) *A global improvement theorem for polynomial monosplines,* by S. Karlin. The main result is that given a polynomial monospline with odd multiple knots and with a maximum set of zeros, there exists another monospline with simple knots and the same set of zeros which has a smaller absolute value at all points.

(3) *Applications of representation theorems to problems of Chebyshev approximation with constraints,* by A. Pinkus. The author develops representation theorems for Chebyshev systems satisfying side constraints, and applies them to constrained best approximation problems.

(4) *Gaussian quadrature formulae with multiple knots,* by S. Karlin and A. Pinkus. It is shown that there is a Gaussian quadrature rule using derivatives to (odd) order $\mu_i - 1$ at τ_i, $i = 1, 2, \ldots, k$, which is exact for a given ECT-system of $n = k + \Sigma_1^k \mu_i$ functions.

(5) *An extremal property of multiple Gaussian nodes,* by S. Karlin and A. Pinkus. If $\{u_i\}_1^{n+1}$ is a Chebyshev system, it is shown that there is a unique $u = u_{n+1} + \Sigma_1^n a_i u_i$ with prescribed n zeros (each of odd multiplicity) which minimizes $\int_a^b u$. The connections with best quadrature are explored.

Part II of the book is devoted to cardinal splines and related matters. The papers are:

(6) *Oscillation matrices and cardinal spline interpolation,* by Ch. Micchelli. Interpolation of an infinite set of periodic data by spline functions with periodic knots is studied in terms of a generalized eigenvalue problem involving an oscillation matrix.

(7) *Cardinal L-splines,* by Ch. Micchelli. Interpolation of (bounded) data given