*Algorithms for Curves and Surfaces* has been published in 1974 by Utilitas Mathematica Publ. Inc., Winnipeg, Manitoba (viii + 198pp).

C. D.

**20 [2.05]**.—SAMUEL KARLIN, CHARLES A. MICCHELLI, ALLAN PINKUS & I. J. SCHOENBERG, *Studies in Spline Functions and Approximation Theory,* Academic Press, New York, 1976, xii + 500 pp., 23.5cm. Price $19.50.

This book is a collection of 15 research papers written by the authors individually, and in various coauthorship combinations. The papers are related in content, and by the fact that each of the authors spent some time at the Weizmann Institute of Science, Rehovot, Israel, between September, 1970 and June, 1974. The papers are arranged into four categories, and we briefly describe each of them below.

Part I of the book deals with best approximations, optimal quadrature, and monosplines. The papers are:

(1) *On a class of best non-linear approximation problems and extended monosplines,* by S. Karlin. This paper deals with existence, uniqueness, and characterization of best (nonlinear) approximations to 0 from the class of extended monosplines with free knots. The results are applied to optimal quadrature.

(2) *A global improvement theorem for polynomial monosplines,* by S. Karlin. The main result is that given a polynomial monospline with odd multiple knots and with a maximum set of zeros, there exists another monospline with simple knots and the same set of zeros which has a smaller absolute value at all points.

(3) *Applications of representation theorems to problems of Chebyshev approximation with constraints,* by A. Pinkus. The author develops representation theorems for Chebyshev systems satisfying side constraints, and applies them to constrained best approximation problems.

(4) *Gaussian quadrature formulae with multiple knots,* by S. Karlin and A. Pinkus. It is shown that there is a Gaussian quadrature rule using derivatives to (odd) order $\mu_i - 1$ at $\tau_i$, $i = 1, 2, \ldots, k$, which is exact for a given ECT-system of $n = k + \Sigma_1^k \mu_i$ functions.

(5) *An extremal property of multiple Gaussian nodes,* by S. Karlin and A. Pinkus. If $\{u_i\}_1^{n+1}$ is a Chebyshev system, it is shown that there is a unique $u = u_{n+1} + \Sigma_1^n a_i u_i$ with prescribed $n$ zeros (each of odd multiplicity) which minimizes $\int_a^b u$. The connections with best quadrature are explored.

Part II of the book is devoted to cardinal splines and related matters. The papers are:

(6) *Oscillation matrices and cardinal spline interpolation,* by Ch. Micchelli. Interpolation of an infinite set of periodic data by spline functions with periodic knots is studied in terms of a generalized eigenvalue problem involving an oscillation matrix.

(7) *Cardinal L-splines,* by Ch. Micchelli. Interpolation of (bounded) data given

at the integers by splines which are piecewise in the null-space of a differential operator with constant coefficients is examined.

(8) *On Micchelli's theory of cardinal L-splines,* by I. J. Schoenberg, The author's methods for polynomial cardinal spline interpolation are used to provide a new development of cardinal $L$-spline interpolation of data of power growth.

(9) *On the remainders and the convergence of cardinal spline interpolation for almost periodic functions,* by I. J. Schoenberg. The convergence of cardinal spline interpolation at the integers to an almost periodic function with frequencies in $[-\pi, \pi]$ is examined with the help of an integral expression for the error.

Part III of the book is titled Interpolation with splines. It contains the following papers:

(10) *Interpolation by splines with mixed boundary conditions,* by S. Karlin and A. Pinkus. The question of when it is possible to find a unique spline interpolating given data and satisfying mixed boundary conditions (involving combinations of the derivatives at both ends) is discussed via determinants.

(11) *Divided differences and other non-linear existence problems at extremal points,* by S. Karlin and A. Pinkus. Given $1 \leqslant k \leqslant n-1$, and appropriately alternating data $\{e_i\}_k^n$, it is shown that there exists a polynomial of degree $n$ and points $\{t_i\}_k^n$ so that $p^{(k)}(t_i) = 0, i = k, \ldots, n-1$, and $[t_{i-k}, \ldots, t_i]p = e_i, i = k, \ldots, n.$

(12) *Notes on spline function* VI. *Extremum problems of the Landau-type for the differential operators $D^2 \neq 1$,* by I. J. Schoenberg. Best bounds on $\|f'\|_\infty$ given bounds on $\|f'' + \alpha^2 f\|_\infty$ (or $\|f'' - {}^2 f\|_\infty$) and on $\|f\|_\infty$ are derived. Landau's inequality is recovered when $\alpha \longrightarrow 0$.

Part IV of the book deals with generalized Landau and Markov type inequalities and with generalized perfect splines. The papers are:

(13) *Oscillatory perfect splines and related extremal problems,* by S. Karlin. Certain perfect splines generalizing the Zolotareff polynomials are constructed, and their extremal properties discussed.

(14) *Generalized Markov Bernstein type inequalities for spline functions,* by S. Karlin. Using the perfect splines of the previous paper, Markov Bernstein type inequalities are established for certain cardinal $L$-splines.

(15) *Some one-sided numerical differentiation formulae and applications,* by S. Karlin. Differentiation formulae of the form $f'(0) = \Sigma_1^{n+1} c_i f(z_i) + \int_0^1 M(t) f^{(n)}(t)\, dt$ are examined.

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