CORRIGENDA

D. M. GAY, "Modifying singular values: Existence of solutions to systems of nonlinear equations having a possibly singular Jacobian matrix," *Math. Comp.*, v. 31, 1977, pp. 962–973.

This note corrects an error pointed out by K. Tanabe [1978]. Theorem (5) of this paper should have been stated as:

(5) THEOREM. *If* $F: \mathbf{R}^n \longrightarrow \mathbf{R}^n$ *is continuous, then for each* $x \in \mathbf{R}^n$ *and* $t_0 \in \mathbf{R}$ *there exist* $a \in [-\infty, t_0)$, $b \in (t_0, +\infty]$, *and a continuously differentiable function* x: $(a, b) \longrightarrow \mathbf{R}^n$ *such that*

$$x(t_0) = x_0 \quad \textit{and} \tag{6a}$$

$$x'(t) = F(x(t)) \quad \textit{for all } t \in (a, b). \tag{6b}$$

If $\|F(x)\| \leqslant c$ *for* $\|x - x_0\| \leqslant d$, *then* $a < t_0 - d/c$ *and* $b > t_0 + d/c$. *Moreover, if* F *is locally Lipschitz continuous, then the solution* $x(t)$ *of* (6) *is unique.*

In [Gay, 1977] it was erroneously asserted that $a = -\infty$ and $b = +\infty$. This has no effect on the rest of the paper, except that the proof of Theorem (23) must be revised to show that $b = +\infty$ for the F of interest. The revised proof may be stated as follows:

Proof. Fix x_0. As already remarked, the existence of $x(t)$ on some interval $[0, b)$ follows easily from Theorems (13) and (5). We first show for $s, t \in [0, b)$ that

$$\|f(x(t))\| \leqslant \|f(x_0)\| e^{-\theta t} \qquad \textit{and} \tag{24.1}$$

$$\|x(s) - x(t)\| \leqslant [\|f(x_0)\|/(\theta\epsilon)]\, |e^{-\theta s} - e^{-\theta t}|. \tag{24.2}$$

Indeed, let $\phi(t) = \|f(x(t))\|^2$. Then $\phi'(t) = -2f^T J\hat{J}^+ f$, so (22) implies $\phi'(t) \leqslant -2\theta\|f(x(t))\|^2 = -2\theta\phi(t)$. Hence, $\psi(t) \equiv \ln \phi(t)$ has $\psi'(t) \leqslant -2\theta$, so (for $t \geqslant 0$)

$$\psi(t) = \psi(0) + \int_0^t \psi'(\tau)\, d\tau \leqslant \psi(0) - 2\theta t$$

and

$$\|f(x(t))\|^2 = \phi(t) = e^{\psi(t)} \leqslant \|f(x_0)\|^2 e^{-2\theta t},$$

which establishes (24.1). Because of (9a), we have

$$\|x'(t)\| = \|\hat{J}^+ f(x(t))\| \leqslant \|f(x(t))\|/\epsilon \leqslant (\|f(x_0)\|/\epsilon) e^{-\theta t},$$

whence

$$\|x(s) - x(t)\| = \left\| \int_s^t x'(\tau)\, d\tau \right\| \leqslant \left| \int_s^t \|x'(\tau)\|\, d\tau \right| \leqslant \frac{\|f(x_0)\|}{\epsilon} \left| \int_s^t e^{-\theta\tau} d\tau \right|,$$

which gives (24.2).

Now let $d = \|f(x_0)\|/(\theta\epsilon)$, $c = \max\{\|f(x)\|: x \in \overline{B}(x_0, d)\}/\epsilon$, and $b_0 = 0$. By (9a), (24.2), Theorem (5), and induction on k we find:

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$$b > b_k,$$
$$\|x(b_k) - x_0\| \leqslant [1 - \exp(-\theta b_k)]\, d,$$
$$\|\hat{J}^+ f(x)\| \leqslant c \quad \text{for } x \in \overline{B}(x(b_k), \exp(-\theta b_k)d),$$
$$b > b_{k+1} \equiv b_k + \exp(-\theta b_k)d/c = \frac{d}{c}(1 + e^{-\theta b_1} + e^{-\theta b_2} + \cdots + e^{-\theta b_k}).$$

From this it follows that $b = +\infty$, for if b were finite, then we would have $b > d(1 + ke^{-\theta b})/c$ for all k, which is impossible.

From (24.2) it follows that the sequence $x(t_1), x(t_2), x(t_3), \ldots$ is a Cauchy sequence for any choice of $t_1, t_2, \ldots$ with $\lim_{i\to\infty} t_i = +\infty$, whence $x^* = \lim_{t\to\infty} x(t)$ exists. By the continuity of f and (24.1), $f(x^*) = \lim_{t\to\infty} f(x(t)) = 0$. □

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K. TANABE, (1978), "Global analysis of continuous analogues of the Levenberg-Marquardt and Newton-Raphson methods for solving nonlinear equations." (Preprint.)

I. S. GRADSHTEYN & I. M. RYZHIK, *Table of Integrals, Series, and Products*, 4th ed., Academic Press, New York, 1965.

On p. 906 of MTE **428** (*Math. Comp.*, v. 22, 1968, pp. 903–907) listing corrections in this set of tables there appears a typographical error in the correction of Formula 8.521(4). The emended correction should read

$$-\frac{1}{\sqrt{(2ki\pi - z)^2 + x^2 + y^2}}\,.$$

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REIJO ERNVALL & TAUNO METSÄNKYLÄ, "Cyclotomic invariants and E-irregular primes," *Math. Comp.*, v. 32, 1978, pp. 617–629.

On p. 619, the three first lines of the first table should read as follows:

x	π_B	π_E	π_{BE}	π_B/π	π_E/π	π_{BE}/π
2000	121	113	56	0.399	0.373	0.18
4000	218	213	91	0.396	0.387	0.17

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