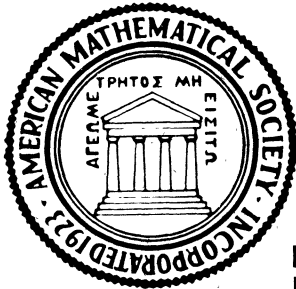


# Mathematics of Computation



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April 1979

Volume 33, Number 146, Pages 435–848

**Published by the American Mathematical Society  
Providence, Rhode Island USA**

ISSN 0025-5718

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