

The Nonabelian Simple Groups G , $|G| < 10^6$ — Minimal Generating Pairs

By John McKay and Kiang-Chuen Young

Abstract. Minimal (k, m, n) generating pairs and their associated presentations are defined for all nonabelian simple groups G , $|G| < 10^6$, excepting the family $\text{PSL}(2, q)$. A complete set of minimal $(2, m, n)$ generating permutations of minimal degree is tabulated for these G . The set is complete in the sense that any minimal generating pair for G will satisfy the same presentation as exactly one of the listed pairs.

Introduction. This paper is one of a series on the simple groups of order up to 10^6 . In another paper [1] we exhibit certain presentations, known as minimal (k, m, n) presentations, for all simple groups of order up to 10^5 excepting most members of a family $\text{PSL}(2, q)$. Here we give the permutations corresponding to these presentations for all simple groups (with the same exceptions) of order up to 10^6 .

Notation. G is a nonabelian simple group, which is identified with its group of inner automorphisms.

A is the group of automorphisms of G .

$|x|$ is an abbreviation for $|\langle x \rangle|$.

$C_A(x)$ is the centralizer of x in A .

$x \stackrel{A}{\sim} y$ means $x = y^t$, $t \in A$.

Definitions. Let $S = \{u \mid \langle u, v \rangle = G \text{ for some } v\}$, and let $k = k(G) = \min_{u \in S} \{|u|\}$. For $a \in S$ of order k , a *minimal (k, m, n) generating pair (for G) with respect to a* is an ordered pair (x, y) such that

- (1) $\langle x, y \rangle = G$,
- (2) $x \in a^A$,
- (3) if $\langle x, z \rangle = G$, then $|z| \geq |y| = m$,
- (4) $|xy| = n$.

A minimal (k, m, n) generating pair (x, y) satisfies a *minimal (k, m, n) presentation for G* :

$$\langle x, y; x^k = y^m = (xy)^n = 1, \{r_i(x, y)\}_{i \in I} = 1 \rangle.$$

Useful Results. A minimal generating pair with respect to a is a minimal generating pair with respect to any element of a^A , since the above properties are invariant under the action of automorphisms in A .

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Let A act on $G \times G$ as

$$(x, y)^t \mapsto (x^t, y^t), \quad t \in A.$$

The transitivity sets are the orbitals of A . Denote the orbital containing (x, y) by $O_{x,y}$.

PROPOSITION 1. $|O_{x,y}| = |A|/|C_A(x) \cap C_A(y)|$. If, further, $\langle x, y \rangle \cong G$ then $|O_{x,y}| = |A|$.

Let $P_G(x, y)$ mean that (x, y) satisfies a presentation P_G of G , where $\langle x, y \rangle = G$.

THEOREM 1. Suppose $P_G(a, b)$, then $P_G(x, y)$ if and only if $\exists t \in A$ such that $a^t = x$ and $b^t = y$.

Proof. Sufficiency is immediate. For necessity we may construct a map t' : $G \rightarrow G$ such that $a^{t'} = x$ and $b^{t'} = y$. The map t' is surjective and preserves the defining relations (and hence all relations); therefore, $t' \in A$. \square

LEMMA 1.

$$|\{(x^t, y) | P_G(x^t, y), t \in A\}| = |C_A(y)|.$$

Proof. Use Theorem 1 and Proposition 1, restricting the action of A to $C_A(y)$. \square

This result can be used in counting arguments [3], [4], [7] to count the number of orbitals of minimal generating pairs for G when the class structure constants [5] and the maximal subgroups [2] are known. We can also estimate the probability that a pair of elements chosen at random from their conjugacy classes should generate G or satisfy a given presentation [1].

THEOREM 2. The pair (x, y) , where $x \in a^A$ and $y \in b^A$, is a minimal generating pair for G with respect to a if and only if

$$(x, y) \stackrel{A}{\sim} (a^{u_i}, b),$$

where $A = \bigcup_i C_A(a)u_iC_A(b)$ and $\langle a^{u_i}, b \rangle = G$.

Proof. Let $(x, y) = (a^s, b^t)$, where $s, t \in A$. Then for some $\alpha \in C_A(a)$ and $\beta \in C_A(b)$, we have

$$(x, y) \stackrel{A}{\sim} (a^{s\alpha^{-1}}, b) = (a^{\alpha u_i \beta}, b) = (a^{u_i \beta}, b) \stackrel{A}{\sim} (a^{u_i}, b). \quad \square$$

We deduce from this result that for a complete set of minimal generating pairs we need choose for a and b only those representatives of conjugacy classes in A whose orders satisfy the minimality conditions. The set is complete in the sense that for any minimal generating pair (x, y) , there is some $t \in A$ such that $(x, y)^t = (a, b)$ for exactly one listed pair (a, b) . We have listed a representative pair (a, b) from each orbital containing a minimal (k, m, n) pair. We find, as conjectured for all finite non-abelian simple groups by Steinberg [6], that $k(G) = 2$ for all groups G listed. It is an old conjecture that all finite simple groups require at most two generators. This result is proved in [6] for groups of Lie type.

The Tables. The first line for each group G gives its name, order, minimal degree as a permutation group, and the order of its outer automorphism group. This is followed by the conjugacy classes and their cycle types and the orders of centralizers

of elements. When there is more than one conjugacy class of generators of a cyclic subgroup, the notation identifies the classes by letters following their period, e.g., $10AB$ denotes two classes of period 10.

The generating pairs of permutations are displayed in image form. Each pair is identified by number, e.g., 20.5 denotes the fifth generating pair of the twentieth group. The notation $(2, m, n; s)$ denotes the relations $a^2 = b^m = (ab)^n = (a^{-1}b^{-1}ab)^s = 1$; the names of the conjugacy classes of a and b are given as well. Additional relators sufficient to distinguish the generating pair uniquely are given on the same line where needed.

The permutations are given in pairs (a, b) ; when only a is given, the generator b is taken to be the one last printed in full. When (a, b^{-1}) is a distinct minimal generating pair, this is denoted by $b \rightarrow b^{-1}$.

One word of warning—for purposes of distinguishing minimal generating pairs it is the conjugacy class in $\text{Aut}(G)$ that matters, and it is possible for elements to have distinct cycle types and yet be conjugate in $\text{Aut}(G)$, e.g., classes $2A$ and $2B$ are conjugate in $\text{Aut}(\text{Sp}(4, 4))$ but have differing cycle types.

Although great care has been taken in preparing these tables, the authors will be grateful to hear of any errors found in them. (The tables are located in the microfiche section at the end of this issue.)

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MINIMAL GENERATING PAIRS FOR NON-ABELIAN SIMPLE GROUPS G , $|G| < 10^6$
 By John McKay and Kiang-Chuen Young, p. 812

1. A(5) ORDER = 60 DEGREE = 5 |OUT(G)| = 2

| | | | | |
|------------|-------|---------|-----------|-------|
| CLASS | 1 | 2 | 3 | 5AB |
| C(X) | 5 | 4 | 3 | 5 |
| CYCLE TYPE | 1^5 | $1^2 2$ | $1^2 3^1$ | 5^1 |

1.1 (2,3,5)

| | | | | | |
|----|---|---|---|---|---|
| A= | 3 | 2 | 1 | 5 | 4 |
| B= | 4 | 1 | 3 | 2 | 5 |

2. PSL(2,7) ORDER = 168 DEGREE = 7 |OUT(G)| = 2

| | | | | | |
|------------|-------|-----------|-----------|---------------|-------|
| CLASS | 1 | 2 | 3 | 4 | 7AB |
| C(X) | 7 | 8 | 3 | 4 | 7 |
| CYCLE TYPE | 1^7 | $1^3 2^2$ | $1^1 3^2$ | $1^1 2^1 4^1$ | 7^1 |

2.1 (2,3,7)

| | | | | | | | |
|----|---|---|---|---|---|---|---|
| A= | 2 | 1 | 5 | 4 | 3 | 6 | 7 |
| B= | 1 | 4 | 2 | 3 | 7 | 5 | 6 |

3. A(6) ORDER = 360 DEGREE = 6 |OUT(G)| = 4

| | | | | | | |
|------------|-------|-----------|-----------|-------|-----------|-----------|
| CLASS | 1 | 2 | 3A | 3B | 4 | 5AB |
| C(X) | 6 | 8 | 9 | 9 | 4 | 5 |
| CYCLE TYPE | 1^6 | $1^2 2^2$ | $1^3 3^1$ | 3^2 | $2^1 4^1$ | $1^1 5^1$ |

3.1 (2,4,5)

| | | | | | | |
|----|---|---|---|---|---|---|
| A= | 1 | 3 | 2 | 5 | 4 | 6 |
| B= | 4 | 1 | 2 | 3 | 6 | 5 |

4. A(7) ORDER = 2520 DEGREE = 7 |OUT(G)| = 2

| | | | | | | | | |
|------------|----------------|-------------------------------|-------------------------------|-------------------------------|--|-------------------------------|-------------------------------|----------------|
| CLASS | 1 | 2 | 3A | 3B | 4 | 5 | 6 | 7AB |
| C(X) | G | 24 | 36 | 9 | 4 | 5 | 12 | 7 |
| CYCLE TYPE | 1 ⁷ | 1 ³ 2 ² | 1 ⁴ 3 ¹ | 1 ¹ 3 ² | 1 ¹ 2 ¹ 4 ¹ | 1 ² 5 ¹ | 2 ² 3 ¹ | 7 ¹ |

4.1 (2,4,7;5)
A= 5 2 3 4 1 7 6
B= 2 3 4 1 6 5 7

4.2 (2,4,7;6)
A= 1 5 3 7 2 6 4

5. PSL(3,3) ORDER = 5616 DEGREE = 13 |OUT(G)| = 2

| | | | | | | | | |
|------------|-----------------|-------------------------------|-------------------------------|-------------------------------|--|---|--|-----------------|
| CLASS | 1 | 2 | 3A | 3B | 4 | 6 | 8AB | 13ABCD |
| C(X) | G | 48 | 54 | 9 | 8 | 6 | 8 | 13 |
| CYCLE TYPE | 1 ¹³ | 1 ⁵ 2 ⁴ | 1 ⁴ 3 ³ | 1 ¹ 3 ⁴ | 1 ¹ 2 ² 4 ² | 1 ² 2 ¹ 3 ¹ 6 ¹ | 1 ¹ 4 ¹ 8 ¹ | 13 ¹ |

5.1 (2,3B,13;4)
A= 2 1 4 3 6 5 8 7 9 10 11 12 13
B= 11 6 5 13 9 12 7 4 3 1 10 2 8

5.2 (2,3B,13;6)
A= 1 2 8 7 6 5 4 3 9 10 13 12 11

6. PSU(3,3) ORDER = 6048 DEGREE = 28 |OUT(G)| = 2

| | | | | | | | | | | |
|------------|-----------------|--------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|--|----------------|--|---|
| CLASS | 1 | 2 | 3A | 3B | 4AB | 4C | 6 | 7AB | 8AB | 12AB |
| C(X) | G | 96 | 108 | 9 | 96 | 16 | 12 | 7 | 8 | 12 |
| CYCLE TYPE | 1 ²⁸ | 1 ⁴ 2 ¹² | 1 ¹ 3 ⁹ | 1 ¹ 3 ⁹ | 1 ⁴ 4 ⁶ | 2 ² 4 ⁶ | 1 ¹ 3 ¹ 6 ⁴ | 7 ⁴ | 1 ² 2 ¹ 8 ³ | 1 ¹ 3 ¹ 12 ² |

6.1 (2,6,7)
A= 21 2 8 15 5 22 16 3 28 27 20 19 23 14 4 7 26 25 12 11 1 6 13 24 18 17 10 9
B= 3 5 16 22 23 18 10 4 9 14 7 17 6 12 26 24 11 13 1 25 20 15 21 27 2 28 19 8

6.2 (2,6,8)
A= 6 16 21 4 15 1 7 22 26 25 17 18 13 23 5 2 11 12 27 28 3 8 14 24 10 9 19 20

7. $M(11)$ ORDER = 7920 DEGREE = 11 |OUT(G)| = 1

| | | | | | | | | |
|------------|----------|-----------|-----------|-----------|-----------|---------------|---------------|--------|
| CLASS | 1 | 2 | 3 | 4 | 5 | 6 | 6AB | 11AB |
| C(X) | G | 48 | 18 | 8 | 5 | 6 | 8 | 11 |
| CYCLE TYPE | 1^{11} | $1^3 2^4$ | $1^2 3^3$ | $1^3 4^2$ | $1^1 5^2$ | $2^1 3^1 6^1$ | $1^1 2^1 6^1$ | 11^1 |

7.1 $(2,4,11)$ $(AB^{-1}ABAB^2)^3$

| | | | | | | | | | | | |
|----|----|----|----|---|---|---|---|---|---|---|----|
| A= | 10 | 8 | 11 | 4 | 7 | 6 | 5 | 2 | 9 | 1 | 3 |
| B= | 4 | 11 | 3 | 7 | 5 | 1 | 6 | 8 | 2 | 9 | 10 |

7.2 B \rightarrow B⁻¹

8. $A(8)$ ORDER = 20160 DEGREE = 8 |OUT(G)| = 2

| | | | | | | |
|------------|-------|-------|-----------|-----------|-----------|-------|
| CLASS | 1 | 2A | 2B | 3A | 3B | 4A |
| C(X) | G | 192 | 96 | 180 | 18 | 16 |
| CYCLE TYPE | 1^8 | 2^4 | $1^4 2^2$ | $1^5 3^1$ | $1^2 3^2$ | 4^2 |

| | | | | | | |
|------------|---------------|-----------|---------------|-----------|-----------|-----------|
| CLASS | 4B | 5 | 6A | 6B | 7AB | 15AB |
| C(X) | 8 | 15 | 12 | 6 | 7 | 15 |
| CYCLE TYPE | $1^2 2^1 4^1$ | $1^3 5^1$ | $1^1 2^2 3^1$ | $2^1 6^1$ | $1^1 7^1$ | $3^1 5^1$ |

8.1 $(2A,4B,15;4)$

| | | | | | | | | |
|----|---|---|---|---|---|---|---|---|
| A= | 4 | 7 | 5 | 1 | 3 | 8 | 2 | 6 |
| B= | 1 | 2 | 4 | 3 | 6 | 7 | 8 | 5 |

8.2 $(2B,4A,15;6)$

| | | | | | | | | |
|----|---|---|---|---|---|---|---|---|
| A= | 8 | 2 | 7 | 4 | 5 | 6 | 3 | 1 |
| B= | 2 | 3 | 4 | 1 | 6 | 7 | 8 | 5 |

9. $PSL(3,4)$ ORDER = 20160 DEGREE = 21 |OUT(G)| = 12

| | | | | | | | | |
|------------|----------|---------|---------|---------------|---------------|---------------|-----------|-------|
| CLASS | 1 | 2 | 3 | 4A | 4B | 4C | 5AB | 7AB |
| C(X) | G | 64 | 9 | 16 | 16 | 16 | 5 | 7 |
| CYCLE TYPE | 1^{21} | $1^5 8$ | $1^3 6$ | $1^1 2^2 4^4$ | $1^1 2^2 4^4$ | $1^1 2^2 4^4$ | $1^1 5^4$ | 7^3 |

9.1 $(2,4ABC,7)$

| | | | | | | | | | | | | | | | | | | | | |
|----|----|---|---|----|----|----|---|----|----|----|----|----|----|----|----|----|----|----|----|----|
| A= | 21 | 2 | 3 | 4 | 6 | 5 | 9 | 10 | 7 | 12 | 15 | 10 | 17 | 16 | 11 | 14 | 13 | 8 | 19 | 20 |
| B= | 16 | 9 | 6 | 21 | 14 | 15 | 3 | 8 | 17 | 1 | 12 | 18 | 10 | 5 | 7 | 13 | 20 | 19 | 11 | 2 |

10. PSU(4,2) ORDER = 25920 DEGREE = 27 |OUT(G)| = 2

| CLASS | 1 | 2A | 2B | 3AB | 3C | 3D | 4A | 4B |
|------------|-------------------------------|--------------------------------|--|-------------------------------|---|----------------|--------------------------------|--|
| C(X) | G | 576 | 96 | 648 | 108 | 54 | 48 | 8 |
| CYCLE TYPE | 1 ²⁷ | 1 ³ 2 ¹² | 1 ⁷ 2 ¹⁰ | 3 ⁹ | 1 ⁹ 3 ⁶ | 3 ⁹ | 1 ³ 4 ⁶ | 1 ¹ 2 ³ 4 ⁵ |
| CLASS | 5 | 6AB | 6CD | 6E | 6F | 9AB | 12AB | |
| C(X) | 5 | 72 | 36 | 18 | 12 | 9 | 12 | |
| CYCLE TYPE | 1 ² 5 ⁵ | 3 ¹ 6 ⁴ | 1 ³ 2 ³ 6 ³ | 3 ¹ 6 ⁴ | 1 ¹ 2 ⁴ 3 ² 6 ² | 9 ³ | 3 ¹ 12 ² | |

10.1 (2B,4A,9;5)
A= 1 27 25 22 18 6 12 26 24 21 17 7 13 14 15 19 11 5 16 20 10 4 23 9 3 8 2
B= 4 2 5 12 21 9 7 27 18 19 10 24 16 25 14 8 17 23 20 11 26 15 6 1 22 3 13

10.2 (2B,4B,9;4)
A= 4 22 18 1 5 6 14 15 19 26 12 11 25 7 8 16 17 3 9 20 21 2 24 23 13 10 27
B= 2 21 22 4 15 9 20 24 25 3 14 10 23 11 17 6 27 13 18 7 26 12 19 8 16 1 5

10.3 (2B,4B,9;6) (AB⁻¹ABAB²)⁵
A= 1 27 25 22 18 6 12 26 24 21 17 7 13 14 15 19 11 5 16 20 10 4 23 9 3 8 2

10.4 B --> B⁻¹

10.5 (2B,4B,12;4) (AB⁻¹ABAB²)⁴
A= 9 14 26 19 6 5 22 25 1 18 11 12 15 2 13 16 17 10 4 21 20 7 23 24 8 3 27

10.6 B --> B⁻¹

10.7 (2B,4B,12;5)
A= 11 16 20 7 25 26 4 21 17 10 1 14 13 12 15 2 9 18 19 3 8 22 27 24 5 6 23

10.8 (2A,5,9)
A= 1 17 21 24 26 13 7 18 22 25 27 12 6 23 20 19 2 8 16 15 3 9 14 4 10 5 11
B= 26 19 3 9 17 16 22 15 2 8 21 20 18 13 7 23 24 1 25 5 6 10 11 12 4 14 27

10.9 (2A,5,12)
A= 12 17 21 4 26 25 7 20 16 13 14 1 10 11 18 9 2 15 22 8 3 19 27 24 6 5 23

11. Sz(8) ORDER = 29120 DEGREE = 65 |OUT(G)| = 3

| CLASS | 1 | 2 | 4AB | 5 | 7ABC | 13ABC |
|------------|-----------------|--------------------------------|--------------------------------|-----------------|-------------------------------|-----------------|
| C(X) | G | 64 | 16 | 5 | 7 | 13 |
| CYCLE TYPE | 1 ⁶⁵ | 1 ¹ 2 ³² | 1 ¹ 4 ¹⁶ | 5 ¹³ | 1 ² 7 ⁹ | 13 ⁵ |

11.1 (2,4,5;7) (AB⁻¹ABAB²)¹³
A= 11 49 22 43 10 41 60 27 47 5 1 25 15 50 13 37 55 39 63 65 62 3 57 38 12 59 8 30 44 28
56 36 33 45 64 32 16 24 18 52 6 46 4 29 34 42 9 58 2 14 61 40 54 53 17 31 23 48 26 7
51 21 19 35 20
B= 16 52 11 63 17 15 49 38 39 62 35 31 7 55 34 42 29 22 14 40 21 43 37 1 56 8 5 3 27 60
46 9 18 54 28 50 47 51 65 64 23 24 33 20 25 61 41 45 58 30 26 4 19 6 53 48 10 13 57 36
12 59 2 44 32

11.2 B --> B⁻¹

11.3 (2,4,5;13) (AB⁻¹ABAB²)⁷
A= 10 61 59 26 16 27 56 28 33 1 38 29 17 36 39 5 13 44 34 64 24 32 60 21 55 4 6 8 12 51
53 22 9 19 40 14 62 11 15 35 49 47 50 18 52 46 42 57 41 43 30 45 31 58 25 7 48 54 3 23
2 37 65 20 63

11.4 B --> B⁻¹

11.5 (2,4,7;7) (ABAB²)¹³ (AB⁻¹ABAB²)⁷
A= 39 35 61 23 64 8 52 6 41 13 28 34 10 22 50 42 26 46 37 65 53 14 4 27 31 17 24 11 63 44
25 40 45 12 2 38 19 36 1 32 9 16 62 30 33 18 60 54 51 15 49 7 21 48 58 56 59 55 57 47
3 43 29 5 20

11.6 B --> B⁻¹

11.7 (2,4,7;7) (ABAB²)⁷ (AB⁻¹ABAB²)⁷
A= 31 29 48 44 5 50 13 35 65 12 22 10 7 15 14 51 60 28 49 39 40 11 61 43 33 27 26 18 2 42
1 34 25 32 8 57 45 63 20 21 55 30 24 4 37 47 46 3 19 6 16 64 56 58 41 53 36 54 62 17
23 59 38 52 9

11.8 B --> B⁻¹

11.9 (2,4,7;13) (AB²)⁵ (AB⁻¹ABAB²)⁷
A= 60 24 59 65 48 35 39 18 50 27 14 63 44 11 38 42 33 8 40 28 46 34 25 2 23 41 10 20 31 53
29 55 17 22 6 57 43 15 7 19 26 16 37 13 61 21 56 5 58 9 54 52 30 51 32 47 36 49 3 1
45 64 12 62 4

11.10 B --> B⁻¹

11.11 (2,4,7;13) (AB²)¹³ ((AB⁻¹AB²)²BAB)¹³
A= 14 54 34 47 42 24 31 29 13 18 41 27 9 1 20 58 25 10 50 15 43 56 53 6 17 51 12 59 8 65
7 44 49 3 55 60 40 62 39 37 11 5 21 32 52 57 4 64 33 19 26 45 23 2 35 22 46 16 28 36
63 38 61 48 30

11.12 B --> B⁻¹

11.13 (2,4,13;5) (AB⁻¹ABAB²)⁷
A= 35 47 43 4 56 12 30 65 11 28 9 6 21 34 50 59 27 48 38 39 13 60 49 32 26 25 17 10 41 7
22 34 38 64 51 60 19 35 12 45 59 11 52 62 40 26 20 31 49 15 5 43 63 17 36 23 28 37 41 6
33 24 31 14 1 44 62 19 20 54 29 63 3 36 46 45 2 18 23 15 64 55 57 40 52 5 53 61 16 22
58 37 42 51 8

11.14 B --> B⁻¹

11.15 (2,4,13;7) (AB⁻¹ABAB²)⁵
A= 22 34 38 64 51 60 19 35 3 12 45 59 11 52 62 40 26 20 31 49 15 5 43 63 17 36 23 28 37 41 6
48 25 27 2 8 55 58 3 12 45 59 11 52 62 40 26 20 31 49 15 5 43 63 17 36 23 28 37 41 6
30 44 53 4 14

11.16 B --> B⁻¹

| | | | | | | | | | |
|------------|-----------------|------------------------------|------------------------------|------------------------------|------------------------------|-----------------|--|-----------------|---|
| 12. | PSU(3,4) | ORDER = 62400 | DEGREE = 65 | OUT(G) = 4 | | | | | |
| CLASS | 1 | 2 | 3 | 4 | 5ABCD | 5EF | 10ABCD | 13ABCD | 15ABCD |
| C(X) | G | 320 | 15 | 16 | 300 | 25 | 20 | 13 | 15 |
| CYCLE TYPE | 1 ⁶⁵ | 1 ₂ ³² | 1 ₃ ²¹ | 1 ₄ ¹⁶ | 1 ₅ ¹² | 5 ¹³ | 1 ₂ ² ₁₀ ⁶ | 13 ⁵ | 1 ₃ ² ₁ ¹⁵ ₄ |

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|------|----------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 12.1 | (2,3,13) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A= | 46 | 35 | 24 | 13 | 57 | 6 | 8 | 7 | 10 | 9 | 31 | 47 | 4 | 34 | 39 | 62 | 48 | 27 | 65 | 55 | 30 | 42 | 58 | 3 | 45 | 50 | 18 | 59 | 38 | 21 |
| | 11 | 41 | 53 | 14 | 2 | 56 | 61 | 29 | 15 | 49 | 32 | 22 | 52 | 64 | 25 | 1 | 12 | 17 | 40 | 26 | 60 | 43 | 33 | 63 | 20 | 36 | 5 | 23 | 28 | 51 |
| B= | 37 | 16 | 54 | 44 | 19 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | 1 | 3 | 4 | 2 | 47 | 59 | 16 | 23 | 35 | 20 | 49 | 62 | 8 | 21 | 14 | 58 | 44 | 48 | 42 | 29 | 32 | 26 | 56 | 19 | 9 | 28 | 41 | 15 | 55 |
| | 60 | 54 | 39 | 40 | 38 | 13 | 31 | 10 | 61 | 53 | 27 | 11 | 12 | 51 | 6 | 52 | 45 | 25 | 43 | 24 | 18 | 65 | 34 | 22 | 63 | 50 | 7 | 64 | 57 | 37 |
| | 33 | 36 | 30 | 17 | 46 | | | | | | | | | | | | | | | | | | | | | | | | | |

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|------|----------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 12.2 | (2,3,15) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A= | 11 | 55 | 44 | 33 | 22 | 6 | 10 | 9 | 8 | 7 | 1 | 65 | 53 | 41 | 29 | 17 | 16 | 59 | 47 | 35 | 23 | 5 | 21 | 64 | 52 | 40 | 28 | 27 | 15 | 58 |
| | 46 | 34 | 4 | 32 | 20 | 63 | 51 | 39 | 38 | 26 | 14 | 57 | 45 | 3 | 43 | 31 | 19 | 62 | 50 | 49 | 37 | 25 | 13 | 56 | 2 | 54 | 42 | 30 | 18 | 61 |
| | 60 | 48 | 36 | 24 | 12 | | | | | | | | | | | | | | | | | | | | | | | | | |

| | | | | | | | |
|------------|--|----------------|---|--|--|---|---|
| 13. | M(12) | ORDER = 95040 | DEGREE = 12 | OUT(G) = 2 | | | |
| CLASS | 1 | 2A | 2B | 3A | 3B | 4A | 4B |
| C(X) | G | 240 | 192 | 54 | 36 | 32 | 32 |
| CYCLE TYPE | 1 ¹² | 2 ⁶ | 1 ⁴ ₂ ⁴ | 1 ³ ₃ ³ | 3 ⁴ | 2 ² ₄ ² | 1 ⁴ ₄ ² |
| CLASS | 5 | 6A | 6B | 8A | 8B | 10 | 11AB |
| C(X) | 10 | 12 | 6 | 8 | 8 | 10 | 11 |
| CYCLE TYPE | 1 ₂ ⁵ ₂ | 6 ² | 1 ₂ ¹ ₃ ¹ ₆ ¹ | 4 ₁ ⁸ ₁ | 1 ₂ ² ₁ ⁸ ₁ | 2 ₁ ¹⁰ ₁ | 1 ₁ ¹¹ ₁ |

| | | | | | | | | | | | | |
|------|------------|---|---|---|---|---|----|----|----|---|---|----|
| 13.1 | (2A,3A,11) | ((AB) ² AB ⁻¹) ¹¹ | | | | | | | | | | |
| A= | 11 | 2 | 5 | 8 | 3 | 6 | 10 | 4 | 9 | 7 | 1 | 12 |
| B= | 9 | 7 | 8 | 5 | 6 | 4 | 11 | 10 | 12 | 3 | 2 | 1 |

| | | | | | | | | | | | | |
|------|------------|----|----|---|---|---|---|---|---|----|---|---|
| 13.2 | (2A,3B,10) | | | | | | | | | | | |
| A= | 1 | 11 | 12 | 8 | 5 | 6 | 9 | 4 | 7 | 10 | 2 | 3 |

| | | | | | | | | | | | | |
|------|------------|---|---|---|----|---|----|----|---|----|---|----|
| 13.3 | (2A,3B,11) | ((AB) ² AB ⁻¹) ¹⁰ | | | | | | | | | | |
| A= | 9 | 6 | 4 | 3 | 10 | 2 | 12 | 11 | 1 | 5 | 8 | 7 |
| B= | 10 | 7 | 4 | 6 | 5 | 3 | 9 | 8 | 2 | 11 | 1 | 12 |

14. PSU(3,5) ORDER = 126000 DEGREE = 50 |OUT(G)| = 6

| | | | | | | |
|------------|-----------------|---------------------------------|--------------------------------|--------------------------------|-----------------|------------------|
| CLASS | 1 | 2 | 3 | 4 | 5A | 5B |
| C(X) | G | 240 | 36 | 8 | 250 | 25 |
| CYCLE TYPE | 1 ⁵⁰ | 1 ¹⁰ 2 ²⁰ | 1 ⁵ 3 ¹⁵ | 1 ² 4 ¹⁰ | 5 ¹⁰ | 1 ⁵ 9 |

| | | | | | | |
|------------|-----------------|-----------------|--|------------------|---------------------------------|--------------------------------|
| CLASS | 5C | 5D | 6 | 7AB | 8AB | 10 |
| C(X) | 25 | 25 | 12 | 7 | 8 | 10 |
| CYCLE TYPE | 5 ¹⁰ | 5 ¹⁰ | 1 ² 3 ³ 6 ⁶ | 1 ¹ 7 | 2 ¹ 4 ⁸ 5 | 5 ² 10 ⁴ |

14.1 (2,4,10)

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| A= | 36 | 38 | 9 | 4 | 49 | 6 | 40 | 43 | 3 | 23 | 46 | 35 | 34 | 33 | 15 | 17 | 16 | 18 | 21 | 20 | 19 | 22 | 10 | 37 | 39 | 42 | 27 | 47 | 29 | 45 |
| B= | 44 | 32 | 14 | 13 | 12 | 1 | 24 | 2 | 25 | 7 | 48 | 26 | 8 | 31 | 30 | 11 | 28 | 41 | 5 | 50 | | | | | | | | | | |
| | 3 | 4 | 13 | 12 | 9 | 1 | 14 | 7 | 5 | 8 | 11 | 50 | 6 | 10 | 25 | 27 | 35 | 43 | 16 | 24 | 40 | 34 | 29 | 26 | 45 | 44 | 42 | 39 | 36 | 38 |
| | 31 | 15 | 37 | 17 | 22 | 47 | 49 | 21 | 28 | 30 | 33 | 19 | 18 | 20 | 32 | 48 | 23 | 46 | 41 | 2 | | | | | | | | | | |

15. J(1) ORDER = 175560 DEGREE = 266 |OUT(G)| = 1

| | | | | | | | | | | |
|------------|------------------|---------------------------------|--------------------------------|--------------------------------|--|-----------------|--|---------------------------------|--|------------------|
| CLASS | 1 | 2 | 3 | 5AB | 6 | 7 | 10AB | 11 | 15AB | 19ABC |
| C(X) | G | 120 | 30 | 30 | 6 | 7 | 10 | 11 | 15 | 19 |
| CYCLE TYPE | 1 ²⁶⁶ | 1 ¹⁰ 1 ²⁸ | 1 ⁵ 3 ⁸⁷ | 1 ⁶ 5 ⁵² | 1 ¹ 2 ² 3 ³ 6 ⁴² | 7 ³⁸ | 2 ³ 5 ² 10 ²⁵ | 1 ² 11 ²⁴ | 3 ² 5 ¹ 15 ¹⁷ | 19 ¹⁴ |

15.1 (2,3,7;10) (W=(AB)²(AB-1)²ABAB-1)6

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A= | 55 | 35 | 186 | 253 | 65 | 80 | 7 | 151 | 71 | 202 | 161 | 170 | 37 | 96 | 162 | 124 | 169 | 110 | 22 | 130 | 125 | 19 | 60 | 241 | 167 | 33 | 45 | 245 | 66 | 237 |
| | 248 | 238 | 26 | 103 | 2 | 261 | 13 | 226 | 61 | 122 | 225 | 236 | 69 | 210 | 27 | 198 | 192 | 164 | 106 | 63 | 243 | 78 | 88 | 183 | 1 | 224 | 250 | 244 | 213 | 23 |
| | 39 | 62 | 50 | 216 | 5 | 29 | 67 | 264 | 43 | 98 | 9 | 233 | 191 | 159 | 75 | 155 | 172 | 52 | 211 | 6 | 260 | 165 | 119 | 112 | 201 | 94 | 144 | 53 | 185 | 160 |
| | 121 | 134 | 196 | 86 | 246 | 14 | 249 | 70 | 180 | 100 | 266 | 207 | 34 | 232 | 147 | 49 | 247 | 115 | 174 | 18 | 227 | 84 | 116 | 133 | 108 | 113 | 209 | 223 | 83 | 141 |
| | 91 | 40 | 175 | 16 | 21 | 184 | 150 | 214 | 221 | 20 | 149 | 178 | 114 | 92 | 166 | 256 | 177 | 142 | 154 | 230 | 120 | 138 | 220 | 87 | 218 | 265 | 105 | 240 | 131 | 127 |
| | 8 | 263 | 157 | 139 | 76 | 228 | 153 | 239 | 74 | 90 | 11 | 15 | 205 | 48 | 82 | 135 | 25 | 229 | 17 | 12 | 176 | 77 | 197 | 109 | 123 | 171 | 137 | 132 | 179 | 99 |
| | 234 | 188 | 54 | 126 | 89 | 3 | 187 | 182 | 203 | 208 | 73 | 47 | 200 | 257 | 258 | 93 | 173 | 46 | 242 | 193 | 85 | 10 | 189 | 235 | 163 | 206 | 102 | 190 | 117 | 44 |
| | 79 | 212 | 59 | 128 | 217 | 64 | 215 | 145 | 219 | 143 | 129 | 231 | 118 | 56 | 41 | 38 | 111 | 156 | 168 | 140 | 222 | 104 | 72 | 181 | 204 | 42 | 30 | 32 | 158 | 148 |
| | 24 | 499 | 51 | 58 | 28 | 95 | 107 | 31 | 97 | 57 | 262 | 254 | 4 | 252 | 259 | 136 | 194 | 195 | 255 | 81 | 36 | 251 | 152 | 68 | 146 | 101 | | | | |
| | 180 | 31 | 145 | 20 | 188 | 196 | 36 | 174 | 197 | 104 | 202 | 50 | 98 | 157 | 183 | 186 | 60 | 177 | 118 | 199 | 71 | 179 | 38 | 132 | 210 | 213 | 155 | 158 | 193 | 154 |
| | 57 | 68 | 240 | 173 | 175 | 66 | 220 | 110 | 90 | 29 | 7 | 13 | 75 | 72 | 114 | 147 | 17 | 82 | 81 | 37 | 257 | 151 | 181 | 96 | 232 | 11 | 12 | 10 | 24 | 8 |
| | 87 | 148 | 141 | 243 | 40 | 225 | 85 | 200 | 255 | 251 | 21 | 15 | 77 | 95 | 41 | 242 | 201 | 129 | 224 | 62 | 128 | 111 | 25 | 86 | 217 | 265 | 172 | 211 | 127 | 229 |
| | 237 | 35 | 152 | 162 | 144 | 16 | 230 | 112 | 130 | 108 | 164 | 189 | 168 | 191 | 219 | 124 | 260 | 113 | 18 | 102 | 28 | 139 | 126 | 27 | 23 | 34 | 26 | 22 | 101 | 185 |
| | 78 | 208 | 131 | 47 | 61 | 39 | 198 | 192 | 258 | 226 | 46 | 136 | 266 | 153 | 121 | 59 | 262 | 190 | 235 | 135 | 109 | 264 | 239 | 53 | 215 | 209 | 212 | 52 | 178 | 32 |
| | 184 | 254 | 194 | 83 | 263 | 161 | 134 | 122 | 54 | 133 | 187 | 58 | 166 | 45 | 249 | 37 | 156 | 44 | 42 | 43 | 245 | 105 | 222 | 163 | 56 | 106 | 55 | 123 | 218 | 207 |
| | 49 | 48 | 205 | 63 | 65 | 116 | 261 | 176 | 170 | 241 | 159 | 236 | 227 | 171 | 84 | 231 | 252 | 195 | 246 | 119 | 160 | 97 | 233 | 234 | 253 | 203 | 165 | 167 | 64 | 214 |
| | 67 | 206 | 256 | 99 | 107 | 120 | 247 | 248 | 228 | 182 | 259 | 223 | 169 | 250 | 244 | 94 | 79 | 221 | 100 | 80 | 115 | 238 | 150 | 117 | 216 | 204 | | | | |

15.2 B --> B⁻¹

15.3 (2,3,7;11)

A = 194 22 160 70 147 6 149 185 80 120 75 67 30 77 197 42 140 242 205 36 122 2 109 259 121 219 55 192 115 13
 230 123 227 179 224 20 98 162 133 116 51 16 84 114 126 142 204 167 220 104 41 163 217 254 27 223 239 81 247 60
 256 164 96 99 129 176 12 243 173 4 148 95 238 166 11 165 14 229 203 9 58 193 105 43 215 158 152 265 145 141
 232 241 170 218 72 63 231 37 64 246 250 236 222 50 83 234 226 187 23 143 244 257 144 44 29 40 168 127 135 10
 25 21 32 134 206 45 118 177 65 228 131 240 39 124 119 237 188 208 191 17 90 46 110 113 89 213 5 71 7 221
 261 87 255 200 253 190 182 86 209 3 161 38 52 62 76 74 48 117 169 93 178 172 69 181 210 66 128 171 34 180
 174 157 263 201 8 207 108 137 266 156 139 28 82 1 195 196 15 211 212 154 184 251 79 47 19 125 186 138 159 175
 198 199 146 262 85 225 53 94 26 49 150 103 56 35 216 107 33 130 78 31 97 91 264 106 258 102 136 73 57 132
 92 18 68 111 249 100 59 260 245 101 202 252 155 54 153 61 112 235 24 248 151 214 183 233 88 189

15.4 B --> B⁻¹

15.5 (2,3,7;15)

A = 122 10 207 4 136 72 153 38 27 2 73 23 235 238 66 47 71 194 201 220 82 130 12 81 43 135 9 99 154 46
 255 181 84 7 170 254 169 8 262 206 158 113 25 190 202 30 16 65 128 260 148 215 53 117 55 157 222 144 131 160
 61 212 83 256 48 15 106 76 196 180 17 6 11 252 101 68 34 213 175 224 24 21 63 33 195 147 126 241 259 204
 91 253 129 134 95 186 140 155 28 140 325 75 139 210 187 205 67 199 127 118 162 191 146 42 171 164 227 54 109 119 249
 123 1 121 125 124 87 108 49 93 22 59 143 159 94 26 5 239 219 102 97 189 229 132 58 231 112 86 51 218 230
 177 228 7 29 98 166 56 41 133 60 264 110 163 115 258 156 247 214 37 35 114 263 242 223 79 240 151 192 251 70
 32 221 188 217 261 96 104 183 141 44 111 178 211 18 85 69 209 232 107 246 19 45 216 90 105 40 3 265 197 103
 193 62 78 168 52 203 184 149 138 20 182 57 174 80 225 226 116 152 142 150 145 198 100 257 13 245 266 14 137 176
 88 173 248 250 236 200 167 243 120 244 179 74 92 36 31 64 234 165 89 50 185 39 172 161 208 237

15.6 B --> B⁻¹

15.7 (2,3,7;19)

A = 129 25 9 142 10 154 261 33 3 5 148 88 207 209 217 179 208 31 49 139 131 61 58 107 2 238 50 215 96 155
 18 196 8 41 151 64 51 91 165 69 34 79 149 150 228 100 110 182 19 27 37 224 53 219 114 120 57 23 247 112
 22 156 230 36 193 225 111 160 40 171 253 239 135 130 186 80 218 257 42 76 143 183 93 163 87 172 85 12 192 188
 38 94 83 92 252 259 255 203 146 46 250 242 122 113 262 249 24 126 117 47 67 60 104 55 236 116 109 199 184 56
 245 103 251 125 152 108 220 213 1 74 21 138 243 200 73 158 244 132 20 153 265 4 81 221 206 99 216 11 43 44
 35 125 140 6 30 62 170 136 198 68 185 164 84 162 39 260 214 204 197 157 70 86 195 181 232 223 191 266 16 226
 174 48 82 119 161 75 248 90 246 210 177 89 65 264 173 32 169 159 118 134 254 202 98 168 258 145 13 17 14 190
 241 235 128 167 28 147 15 77 54 127 144 222 176 52 66 180 227 45 229 63 240 175 263 256 212 115 237 26 72 231
 211 102 133 137 121 189 59 187 106 101 123 95 71 201 97 234 78 205 259 166 7 105 233 194 141 178

15.8 (2,3,10;6)

A = 32 40 228 204 151 225 85 143 27 169 124 105 13 99 128 195 259 107 108 167 75 192 102 257 60 202 9 165 29 120
 227 1 242 234 142 36 206 83 223 2 245 231 233 87 239 251 153 217 149 158 180 255 198 163 154 240 173 86 157 25
 235 69 132 194 266 218 123 201 62 246 103 72 114 121 21 127 81 118 220 134 77 111 38 222 7 58 44 261 210 90
 211 208 94 93 230 265 166 152 14 160 191 23 71 162 12 252 18 19 133 221 82 146 219 73 174 215 117 78 254 30
 74 264 67 11 171 209 76 15 224 188 176 63 109 80 137 189 135 145 205 214 177 35 8 243 138 112 248 200 49 150
 5 98 47 55 207 168 59 50 213 100 181 104 54 262 28 97 20 156 20 216 125 196 57 115 247 131 141 183 249 51
 161 187 178 237 244 253 182 130 136 197 101 22 193 64 16 172 190 53 256 148 68 26 229 4 139 37 155 92 126 89
 91 263 159 140 116 170 48 66 113 79 110 84 39 129 6 226 31 3 203 95 42 232 43 34 61 260 184 258 45 56
 250 33 144 185 41 70 175 147 179 241 46 106 186 119 52 199 24 238 17 236 88 164 212 122 96 65

15.9 B --> B⁻¹

15.10 (2,3,10;10)

A = 18 254 106 176 226 137 132 30 52 108 136 113 97 173 46 60 209 1 130 105 34 220 157 38 247 103 256 259 179 8
 251 99 184 21 118 104 258 24 72 227 148 42 152 207 149 15 138 198 54 83 183 9 217 49 174 163 159 167 110 6
 181 253 168 232 109 78 88 204 189 229 263 39 252 74 156 234 266 66 194 223 211 241 50 134 245 133 124 67 89 175
 123 221 249 94 158 250 13 178 32 248 117 224 26 36 20 3 262 10 65 59 187 155 12 199 122 201 101 35 197 244
 143 115 91 87 125 240 236 246 182 19 218 7 86 84 142 11 6 47 139 239 200 135 121 265 169 202 214 41 45 193
 222 43 153 210 112 75 23 95 57 165 206 192 56 257 160 196 58 63 145 215 235 172 14 55 90 4 260 98 29 255
 61 129 51 33 231 212 111 188 69 237 195 162 150 79 191 166 119 48 114 141 116 146 242 68 264 161 44 243 17 154
 81 186 230 147 170 225 53 131 261 22 92 151 80 102 216 5 40 233 70 213 185 64 228 76 171 127 190 238 140 126
 82 203 208 120 85 128 25 100 93 96 31 73 62 2 180 27 164 37 28 177 219 107 71 205 144 77

15.11 B --> B⁻¹

15.12 (2,3,10;10)

| A= | (w) ⁷ |
|-----|------------------|
| 98 | 42 |
| 222 | 241 |
| 106 | 88 |
| 257 | 86 |
| 68 | 181 |
| 211 | 66 |
| 122 | 180 |
| 151 | 168 |
| 32 | 144 |

15.13 B --> B⁻¹

15.14 (2,3,10;15)

| A= |
|-----|
| 41 |
| 13 |
| 116 |
| 73 |
| 38 |
| 49 |
| 35 |
| 126 |
| 101 |

15.15 (2,3,10;19)

| A= |
|-----|
| 123 |
| 47 |
| 240 |
| 206 |
| 183 |
| 188 |
| 191 |
| 107 |
| 63 |

15.16 (2,3,11;10)

| A= | (w=(AB) ⁴ (AB ⁻¹) ² ABAB ⁻¹) ⁶ |
|-----|---|
| 123 | |
| 171 | |
| 262 | |
| 200 | |
| 211 | |
| 151 | |
| 107 | |
| 188 | |
| 237 | |
| 183 | |
| 189 | |

15.17 B --> B⁻¹

15.18 (2,3,11;19)

| A= |
|-----|
| 206 |
| 171 |
| 262 |
| 200 |
| 211 |
| 151 |
| 107 |
| 188 |
| 237 |
| 183 |
| 189 |

15.19 (2,3,15;5)
 $A = 200 \ 50 \ 46 \ 145 \ 147 \ 6 \ 91 \ 52 \ 94 \ 228 \ 111 \ 120 \ 28 \ 225 \ 155 \ 178 \ 88 \ 125 \ 150 \ 75 \ 137 \ 197 \ 83 \ 45 \ 25 \ 230 \ 81 \ 13 \ 203 \ 36$
 248 99 215 174 260 30 250 127 124 233 258 67 151 264 24 3 234 131 229 2 51 8 119 220 209 123 92 107 65 235
 261 153 241 134 59 231 42 87 207 90 144 188 265 232 20 187 205 136 182 157 27 179 23 112 255 210 68 17 148 70
 7 57 121 9 206 129 195 217 32 100 138 256 166 251 191 106 58 156 263 149 11 84 141 185 218 163 224 252 53 12
 93 242 56 39 18 173 38 227 96 171 48 237 223 64 208 78 21 101 266 160 113 238 222 71 4 213 5 89 110 19
 43 190 62 180 15 108 80 181 201 140 198 162 116 177 243 103 246 219 204 172 130 170 126 34 193 176 164 16 82 154
 158 79 189 226 114 214 76 72 183 152 105 244 175 194 97 196 22 161 211 1 159 253 29 169 77 95 69 135 55 86
 199 212 146 186 33 245 98 115 168 54 249 143 133 117 14 184 128 10 49 26 66 74 40 47 60 257 132 142 247 254
 63 122 165 192 216 167 239 31 221 37 104 118 202 240 85 102 236 41 262 35 61 259 109 44 73 139

15.20 (2,3,15;6)
 $A = 144 \ 28 \ 210 \ 99 \ 36 \ 20 \ 116 \ 140 \ 135 \ 10 \ 123 \ 43 \ 187 \ 53 \ 109 \ 16 \ 249 \ 253 \ 242 \ 6 \ 214 \ 111 \ 166 \ 39 \ 181 \ 200 \ 220 \ 2 \ 100 \ 136$
 182 87 235 233 60 5 203 122 24 245 243 260 12 195 224 103 159 70 238 205 225 112 14 255 265 149 183 101 120 35
 85 125 79 73 169 158 201 256 108 48 175 246 64 251 97 161 191 115 63 105 153 177 131 157 61 241 32 167 142 185
 194 92 162 212 133 152 75 231 4 29 58 227 46 218 80 172 199 69 15 147 22 52 202 248 78 7 213 171 119 59
 264 38 11 259 62 156 215 128 247 154 83 148 95 229 9 30 186 188 262 8 223 89 143 1 145 240 110 132 56 236
 258 96 81 130 226 126 84 66 47 198 76 93 163 221 193 23 88 211 65 217 118 106 266 228 71 196 82 216 219 192
 25 31 57 209 90 137 13 138 239 190 77 180 165 91 44 176 232 160 107 26 67 113 37 230 50 261 254 250 184 3
 168 94 117 21 127 178 170 104 179 27 164 252 141 45 51 155 102 174 134 204 98 197 34 257 33 150 263 49 189 146
 86 19 41 244 40 72 129 114 17 208 74 222 18 207 54 68 234 151 124 42 206 139 237 121 55 173

15.21 B --> B⁻¹

15.22 (2,3,15;7)
 $A = 46 \ 57 \ 147 \ 86 \ 222 \ 30 \ 138 \ 117 \ 110 \ 198 \ 225 \ 187 \ 200 \ 101 \ 36 \ 215 \ 51 \ 18 \ 121 \ 94 \ 207 \ 98 \ 132 \ 24 \ 203 \ 172 \ 109 \ 100 \ 107 \ 6$
 181 178 265 60 65 15 116 217 248 252 235 259 266 210 230 1 63 190 129 50 17 145 66 88 80 214 2 189 164 34
 79 62 47 111 35 53 67 258 232 246 75 72 219 199 71 238 156 139 61 55 91 177 197 176 255 4 162 54 104 123
 81 166 213 20 260 185 209 22 250 28 14 153 155 89 130 173 29 256 27 9 64 205 254 119 161 37 8 146 114 124
 19 193 90 120 160 208 201 128 49 105 151 23 221 144 261 204 163 7 78 216 194 233 143 134 52 118 3 148 263 158
 131 249 102 243 103 77 175 150 167 125 115 87 137 59 171 92 159 211 218 240 165 26 106 227 157 84 82 32 192 231
 31 188 251 196 96 253 12 182 58 48 241 179 122 141 206 184 83 10 74 13 127 237 25 136 112 195 21 126 97 44
 168 224 93 56 16 140 38 169 73 245 133 5 228 212 11 257 174 223 234 45 180 69 142 229 41 239 202 76 236 170
 191 244 154 242 220 70 262 39 152 99 183 40 186 113 85 108 226 68 42 95 135 247 149 264 33 43

15.23 B --> B⁻¹

15.24 (2,3,15;19)
 $A = 186 \ 26 \ 55 \ 226 \ 169 \ 137 \ 62 \ 236 \ 170 \ 202 \ 44 \ 71 \ 134 \ 142 \ 155 \ 237 \ 65 \ 196 \ 243 \ 43 \ 21 \ 51 \ 59 \ 97 \ 167 \ 2 \ 103 \ 216 \ 205 \ 124$
 250 215 35 45 33 61 92 253 261 153 70 151 20 11 34 259 140 199 184 244 22 223 119 56 3 54 248 63 23 213
 36 7 58 245 17 163 75 220 130 41 12 121 204 77 67 162 74 118 211 177 180 115 88 132 86 85 123 83 89 263
 233 37 110 201 95 138 24 225 260 100 239 252 27 189 146 126 221 165 258 93 222 178 214 194 82 128 251 78 53 193
 72 157 87 30 125 106 228 116 247 69 182 84 257 13 139 256 6 96 135 47 200 14 264 175 229 105 265 176 188 156
 42 160 40 166 15 150 122 266 172 152 210 76 66 242 108 154 25 218 5 9 240 159 173 195 144 148 80 112 219 81
 208 131 224 49 185 1 187 149 104 234 235 230 120 24 174 18 197 255 48 141 94 10 232 73 29 212 254 181 262 161
 79 206 60 113 32 28 238 168 179 68 107 111 52 183 98 4 231 127 145 192 227 203 91 190 191 8 16 217 101 171
 249 164 19 50 64 246 129 57 241 31 117 102 38 207 198 136 133 109 46 99 39 209 90 143 147 158

15.25 B --> B⁻¹

15.26 (2,3,15;19)
 $A = 250 \ 77 \ 178 \ 4 \ 159 \ 101 \ 74 \ 62 \ 223 \ 126 \ 31 \ 241 \ 100 \ 70 \ 142 \ 189 \ 198 \ 174 \ 209 \ 94 \ 247 \ 169 \ 48 \ 230 \ 117 \ 84 \ 194 \ 158 \ 76 \ 206$
 11 64 128 87 162 228 124 140 255 231 44 93 236 41 86 145 149 23 135 57 256 132 53 245 55 208 50 221 113 220
 61 8 67 32 88 227 63 187 252 14 92 102 262 7 139 29 2 226 265 244 107 249 109 26 183 45 34 65 171 246
 172 71 42 20 166 110 212 197 190 13 6 72 248 154 192 118 81 214 83 96 239 136 59 161 137 229 25 106 217 167
 188 224 195 37 130 10 251 33 131 125 129 52 146 160 49 112 115 200 75 38 218 115 242 211 46 133 266 181 47 199
 210 165 196 104 201 156 175 28 5 134 114 35 216 191 152 95 120 179 22 186 89 91 215 18 157 238 243 3 168 240
 148 193 85 184 233 170 68 121 16 99 164 105 182 27 123 153 98 17 150 138 155 237 203 219 207 30 205 56 19 151
 144 97 213 108 173 163 119 141 204 60 58 234 9 122 225 78 66 36 116 24 40 253 185 222 261 43 202 176 111 180
 12 143 177 80 54 90 21 103 82 1 127 69 232 258 39 51 260 254 264 257 235 73 263 259 79 147

15.27 (2,3,19;5)
 A=
 39 42 11 178 82 128 33 113 155 200 3 28 69 255 68 209 183 133 167 176 185 184 79 194 127 98 236 12 72 123
 31 89 7 159 35 108 172 150 1 189 262 2 120 76 241 153 244 254 234 81 73 64 220 242 253 115 205 229 179 66
 228 161 257 52 65 60 260 15 13 93 87 29 51 249 130 44 256 163 23 186 50 5 222 84 166 187 71 122 32 140
 217 136 70 264 197 103 263 26 137 112 252 250 96 195 243 192 181 36 232 266 157 100 8 180 56 134 145 149 199 43
 230 88 30 202 154 206 25 6 201 75 138 224 18 116 225 92 99 131 223 90 216 240 211 162 117 165 213 177 118 38
 245 251 46 125 9 175 111 168 34 196 62 144 78 238 146 85 19 158 233 207 171 37 173 221 156 20 148 4 59 114
 107 231 17 22 21 80 86 235 40 258 265 106 261 24 104 160 95 198 119 10 129 124 204 203 57 126 170 259 16 210
 143 248 147 227 215 141 91 237 247 53 174 83 139 132 135 246 214 61 58 121 182 109 169 49 188 27 218 164 239 142
 45 54 105 47 151 226 219 212 74 102 152 101 55 48 14 77 63 190 208 67 193 41 97 94 191 110

15.28 (2,3,19;7)
 A=
 52 7 30 74 75 114 2 105 104 31 25 112 50 43 94 227 182 106 108 223 258 161 159 35 11 150 78 127 147 3
 10 91 123 226 24 231 121 195 215 232 135 116 14 96 54 183 113 62 206 13 148 1 172 45 256 242 241 155 156 184
 70 48 63 64 239 196 68 67 134 61 197 251 238 4 5 194 77 27 80 79 100 261 163 169 205 93 266 253 152 151
 32 222 86 15 245 44 234 115 201 81 199 162 200 9 8 18 136 19 175 178 228 12 47 6 98 42 250 224 218 264
 37 262 33 145 254 132 28 198 154 249 142 126 146 69 41 107 212 246 192 193 157 131 237 164 124 133 29 51 149 26
 90 89 255 129 58 59 141 230 23 203 22 102 83 144 185 229 260 186 84 233 179 53 189 176 109 174 243 110 171 180
 181 17 46 60 165 168 213 207 173 214 259 139 140 76 38 66 71 128 101 103 99 244 160 248 95 49 188 265 221 219
 257 137 187 190 39 217 216 119 210 240 209 92 20 118 225 34 16 111 166 158 36 40 170 97 247 236 143 73 65 220
 57 56 177 202 95 138 235 204 130 117 72 252 88 125 153 55 211 21 191 167 82 122 263 120 208 87

15.29 (2,3,19;10)
 A=
 203 50 49 81 151 121 200 91 38 10 178 135 29 233 189 16 57 265 79 185 76 145 230 45 25 197 83 52 13 181
 243 229 129 217 88 232 67 9 223 158 198 131 99 250 24 147 264 234 3 2 110 28 245 92 188 184 17 61 263 60
 58 209 141 139 113 170 37 109 126 256 73 117 71 225 127 21 204 125 19 175 4 252 27 191 148 235 241 35 104 226
 8 54 97 166 102 107 93 119 43 182 222 95 114 89 137 111 96 220 68 51 106 140 65 103 174 258 72 171 98 161
 6 153 206 183 78 69 75 136 33 154 42 177 201 247 12 128 105 192 64 112 63 261 211 227 22 214 46 85 228 150
 5 254 122 130 159 260 212 40 155 187 120 210 208 253 255 94 244 236 169 66 118 172 259 115 80 231 132 11 249 221
 30 100 124 56 20 262 160 55 15 202 84 138 193 205 195 196 26 41 246 7 133 190 1 77 194 123 213 163 62 162
 143 157 207 146 238 218 34 216 266 108 180 101 39 242 74 90 144 149 32 23 176 36 14 48 86 128 240 215 257 237
 87 224 31 167 53 199 134 251 179 44 248 82 164 152 165 70 239 116 173 156 142 186 59 47 18 219

15.30 B --> B⁻¹

15.31 (2,3,19;11)
 A=
 41 8 51 106 185 234 84 2 26 226 170 137 160 31 44 95 235 34 219 142 193 91 156 133 36 9 151 79 128 230
 14 208 46 18 207 25 97 121 88 216 1 200 74 15 122 33 102 108 63 194 3 77 176 173 136 238 243 240 152 68
 251 94 49 215 254 196 81 60 188 201 206 125 177 43 184 258 52 78 28 103 67 92 205 7 175 195 183 39 107 263
 22 82 114 62 16 210 37 225 239 165 249 47 80 166 212 4 89 48 198 218 115 169 150 93 111 264 182 237 181 127
 38 45 153 241 72 189 120 29 167 252 247 186 24 214 135 55 12 179 221 203 204 20 261 266 240 163 245 232 113
 27 59 123 202 236 23 265 227 256 13 248 229 147 255 100 104 129 262 112 1 180 233 54 231 85 53 73 178 138 171
 119 117 87 75 5 132 213 69 126 146 259 260 21 50 86 66 197 109 199 42 70 154 140 141 83 71 35 32 244 96
 224 105 187 134 64 40 217 110 19 220 139 257 250 211 98 10 158 228 162 30 174 149 172 6 17 155 118 56 99 58
 124 145 57 209 148 246 131 161 101 223 61 130 253 65 164 159 222 76 191 192 143 168 90 116 157 144

15.32 (2,3,19;15)
 A=
 127 158 25 4 245 87 41 150 236 126 160 49 13 240 222 59 182 24 242 66 162 65 244 18 33 37 252 212 68 117
 3 238 226 58 22 20 119 29 89 80 124 139 227 193 77 114 75 137 217 70 104 165 257 200 151 176 6 113 69 97
 234 264 171 233 105 243 90 179 190 103 102 101 100 81 95 260 38 60 220 134 263 181 88 76 265 152 30 214 67 120
 121 223 173 71 43 10 31 148 42 196 194 132 48 110 140 206 78 259 72 135 199 35 143 174 187 180 44 128 230 8
 85 116 224 241 170 156 203 32 219 11 51 21 50 262 82 166 239 36 237 155 93 247 123 144 188 86 251 207 98 146
 112 17 216 209 191 55 145 175 202 99 185 53 74 131 229 130 56 266 141 84 201 189 157 232 205 136 178 39 184 258
 40 28 261 118 249 183 79 45 159 109 246 15 122 153 248 63 73 255 195 149 47 204 94 91 52 9 169 62 167 14
 154 19 96 23 5 221 172 225 215 255 177 27 250 57 228 46 83 210 138 106 213 164 111 92 115 198

16. A(9) ORDER = 181440 DEGREE = 9 |OUT(G)| = 2

| | | | | | | | | | |
|------------|--|--|-------------------------------|-------------------------------|----------------|-------------------------------|--|--|-------------------------------|
| CLASS | 1 | 2A | 2B | 3A | 3B | 3C | 4A | 4B | 5 |
| C(X) | G | 480 | 192 | 1080 | 81 | 54 | 24 | 16 | 60 |
| CYCLE TYPE | 1 ⁹ | 1 ⁵ 2 ² | 1 ¹ 2 ⁴ | 1 ⁶ 3 ¹ | 3 ³ | 1 ³ 3 ² | 1 ³ 2 ¹ 4 ¹ | 1 ¹ 4 ² | 1 ⁴ 5 ¹ |
| CLASS | 6A | 6B | 7 | 9A | 9B | 10 | 12 | 15AB | |
| C(X) | 24 | 6 | 7 | 9 | 9 | 20 | 12 | 15 | |
| CYCLE TYPE | 1 ² 2 ³ 1 ¹ | 1 ² 1 ¹ 6 ¹ | 1 ² 7 ¹ | 9 ¹ | 9 ¹ | 2 ² 5 ¹ | 2 ¹ 3 ¹ 4 ¹ | 1 ¹ 3 ¹ 5 ¹ | |

16.1 (2A,4B,9;4); (AB⁻¹ABAB²)¹²
A= 2 1 3 4 6 5 7 8 9
B= 1 3 4 5 2 7 8 9 6

16.2 B --> B⁻¹

16.3 (2A,4B,9;6)
A= 2 1 3 6 5 4 7 8 9

16.4 (2B,3B,15)
A= 1 9 7 5 4 8 3 6 2
B= 2 3 1 5 6 4 8 9 7

17. PSL(3,5) ORDER = 372000 DEGREE = 31 |OUT(G)| = 2

| | | | | | | | |
|------------|--|--|--|---|--|---|-------------------------------|
| CLASS | 1 | 2 | 3 | 4AB | 4C | 5A | 5B |
| C(X) | G | 480 | 24 | 480 | 16 | 500 | 25 |
| CYCLE TYPE | 1 ³¹ | 1 ⁷ 2 ¹² | 1 ¹ 3 ¹⁰ | 1 ⁷ 4 ⁶ | 1 ² 2 ⁴ 6 ⁶ | 1 ⁶ 5 ⁵ | 1 ¹ 5 ⁶ |
| CLASS | 6 | 8AB | 10 | 12AB | 20AB | 24ABCD | 31(10) |
| C(X) | 24 | 24 | 20 | 24 | 20 | 24 | 31 |
| CYCLE TYPE | 1 ¹ 3 ² 6 ⁴ | 1 ¹ 2 ³ 8 ³ | 1 ² 2 ² 5 ¹ 10 ² | 1 ¹ 3 ² 12 ² | 1 ² 4 ¹ 5 ¹ 20 ¹ | 1 ¹ 6 ¹ 24 ¹ | 31 ¹ |

17.1 (2,3,24;6)
A= 1 29 12 4 26 18 10 14 9 7 11 3 27 8 31 23 17 6 24 20 21 28 16 19 30 5 13 22 2 25
15
B= 23 7 29 17 26 6 9 11 2 5 30 1 16 13 18 14 22 19 15 3 24 4 12 27 28 10 21 31 20 8
25

17.2 (2,3,24;10)
 A= $\begin{matrix} 1 & 16 \\ 31 \end{matrix}$ 3 4 26 14 11 17 27 30 7 13 12 6 28 2 8 18 24 22 29 20 23 19 25 5 9 15 21 10

17.3 (2,3,31;4)
 A= $\begin{matrix} 1 & 26 \\ 31 \end{matrix}$ 14 21 29 13 10 12 17 7 11 6 6 3 15 24 9 27 23 20 4 22 19 16 30 2 18 28 5 25

17.4 (2,3,31;5)
 A= $\begin{matrix} 1 & 29 \\ 31 \end{matrix}$ 13 5 4 8 25 6 12 11 10 9 3 14 28 23 18 17 19 22 21 20 16 26 7 24 27 15 2 30

17.5 (2,3,31;6)
 A= $\begin{matrix} 1 & 22 \\ 23 \end{matrix}$ 3 18 14 26 7 19 27 10 11 13 12 5 29 20 24 4 8 16 28 2 31 17 25 6 9 21 15 30

17.6 (2,3,31;10)
 A= $\begin{matrix} 1 & 16 \\ 22 \end{matrix}$ 13 19 24 18 30 14 12 10 25 9 3 8 15 2 17 6 4 28 29 31 23 5 11 26 27 20 21 7

17.7 (2,3,31;10)
 A= $\begin{matrix} 1 & 16 \\ 20 \end{matrix}$ 12 24 $((AB)^2(AB^{-1})^2)^{10}$ 5 17 25 8 9 11 10 3 27 18 22 2 6 14 26 31 29 15 23 4 7 19 13 28 21 30

18. M(22) ORDER = 443520 DEGREE = 22 |OUT(G)| = 2

| CLASS | 1 | 2 | 3 | 4A | 4B | 5 | 6 | 7AB | 8 | 11AB |
|------------|-----------------|-------------------------------|-------------------------------|--|--|-------------------------------|--|-------------------------------|--|-----------------|
| IC(X) | G | 384 | 36 | 32 | 16 | 5 | 12 | 7 | 8 | 11 |
| CYCLE TYPE | 1 ²² | 1 ⁶ 2 ⁸ | 1 ⁴ 3 ⁶ | 1 ² 2 ² 4 ⁴ | 1 ² 2 ² 4 ⁴ | 1 ² 5 ⁴ | 2 ² 3 ² 6 ² | 1 ¹ 7 ³ | 2 ¹ 4 ¹ 8 ² | 11 ² |

18.1 (2,4A,11;5)
 A= $\begin{matrix} 1 & 13 & 18 & 22 & 19 & 11 & 9 & 8 & 7 & 12 & 6 & 10 & 2 & 15 & 14 & 16 & 17 & 3 & 5 & 20 & 21 & 4 \\ B= & 6 & 12 & 16 & 15 & 20 & 5 & 21 & 13 & 9 & 17 & 11 & 19 & 7 & 2 & 4 & 10 & 3 & 22 & 14 & 1 & 8 & 18 \end{matrix}$

18.2 (2,4A,11;6)
 A= $\begin{matrix} 11 & 18 & 19 & 4 & 14 & 21 & 17 & 8 & 16 & 10 & 1 & 12 & 15 & 5 & 13 & 9 & 7 & 2 & 3 & 20 & 6 & 22 \end{matrix}$

18.3 (2,4B,11;4)
 A= $\begin{matrix} 9 & 14 & 18 & 4 & 13 & 17 & 7 & 10 & 1 & 8 & 11 & 12 & 5 & 2 & 19 & 16 & 6 & 3 & 15 & 22 & 21 & 20 \\ B= & 12 & 5 & 3 & 22 & 4 & 19 & 10 & 8 & 11 & 17 & 21 & 6 & 7 & 18 & 20 & 9 & 13 & 14 & 1 & 15 & 16 & 2 \end{matrix}$

18.4 B --> B⁻¹

18.5 (2,4B,11;5)
 A= $\begin{matrix} 4 & 8 & 11 & 1 & 13 & 6 & 16 & 2 & 12 & 10 & 3 & 9 & 5 & 14 & 22 & 7 & 17 & 21 & 19 & 20 & 18 & 15 \end{matrix}$

18.6 (2,4B,11;6)
 A= $\begin{matrix} 8 & 12 & 6 & 9 & 14 & 3 & 19 & 1 & 4 & 10 & 11 & 2 & 13 & 5 & 22 & 16 & 21 & 18 & 7 & 20 & 17 & 15 \end{matrix}$

| 19. | | J(2) | | ORDER = 604800 | | DEGREE = 100 | | OUT(G) = 2 | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---------------------------------|--------------------------------|--|--------------------------------|---|---|-----------------|----|----|----|----|----|----|----|-----|----|----|----|----|----|-----|----|-----|----|----|-----|----|----|----|
| CLASS | 1 | 2A | 2B | 3A | 3B | 4 | 5AB | 5CD | | | | | | | | | | | | | | | | | | | | | | |
| C(X) | G | 1920 | 240 | 1080 | 36 | 96 | 300 | 50 | | | | | | | | | | | | | | | | | | | | | | |
| CYCLE TYPE | 1 ¹⁰⁰ | 1 ²⁰ 2 ⁴⁰ | 2 ⁵⁰ | 1 ¹⁰ 3 ³⁰ | 1 ⁴ 3 ³² | 1 ⁸ 2 ⁶ 4 ²⁰ | 5 ²⁰ | 5 ²⁰ | | | | | | | | | | | | | | | | | | | | | | |
| CLASS | 6A | 6B | 7 | 8 | 10AB | 10CD | 12 | 15AB | | | | | | | | | | | | | | | | | | | | | | |
| C(X) | 24 | 12 | 7 | 8 | 20 | 10 | 12 | 15 | | | | | | | | | | | | | | | | | | | | | | |
| CYCLE TYPE | 1 ² 2 ⁴ 3 ⁶ 12 | 2 ² 6 ¹⁶ | 1 ² 7 ¹⁴ | 1 ² 2 ³ 4 ³ 8 ¹⁰ | 10 ¹⁰ | 5 ⁴ 10 ⁸ | 1 ² 3 ⁴ 6 ² 12 ⁶ 5 ² 15 ⁶ | | | | | | | | | | | | | | | | | | | | | | | |
| 19.1 (2A,5CD,6) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A= | 1 | 25 | 3 | 1 | 5 | 97 | 35 | 54 | 9 | 10 | 26 | 12 | 13 | 95 | 36 | 23 | 17 | 18 | 19 | 38 | 37 | 91 | 16 | 100 | 2 | 11 | 27 | 28 | 73 | 89 |
| B= | 21 | 55 | 56 | 28 | 14 | 99 | 73 | 95 | 79 | 29 | 35 | 71 | 34 | 77 | 47 | 52 | 24 | 69 | 75 | 45 | 42 | 30 | 1 | 43 | 5 | 19 | 41 | 22 | 44 | 32 |
| 19.2 (2A,5CD,7) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A= | 1 | 25 | 5 | 4 | 3 | 95 | 97 | 100 | 17 | 10 | 11 | 18 | 13 | 91 | 54 | 16 | 9 | 12 | 19 | 20 | 34 | 35 | 23 | 92 | 2 | 26 | 27 | 28 | 29 | 93 |
| B= | 56 | 85 | 65 | 74 | 63 | 49 | 80 | 98 | 44 | 38 | 46 | 32 | 81 | 64 | 52 | 43 | 84 | 60 | 45 | 67 | 73 | 90 | 51 | 77 | 62 | 50 | 87 | 57 | 47 | 82 |
| 19.3 (2A,5CD,10) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 26 | 18 | 31 | 14 | 15 | 16 | 17 | 12 | 27 | 39 | 79 | 22 | 23 | 24 | 25 | 11 | 19 | 28 | 55 | 34 |
| B= | 57 | 80 | 74 | 78 | 50 | 67 | 66 | 76 | 43 | 58 | 47 | 46 | 41 | 63 | 77 | 68 | 75 | 64 | 21 | 62 | 48 | 49 | 44 | 88 | 90 | 60 | 96 | 84 | 42 | 85 |
| 19.4 (2A,5CD,12) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A= | 4 | 2 | 5 | 1 | 3 | 36 | 54 | 95 | 9 | 28 | 11 | 12 | 13 | 92 | 91 | 16 | 17 | 18 | 27 | 81 | 89 | 100 | 23 | 97 | 25 | 26 | 19 | 10 | 38 | 71 |
| B= | 69 | 62 | 63 | 64 | 78 | 85 | 67 | 56 | 61 | 87 | 30 | 53 | 59 | 86 | 43 | 88 | 98 | 65 | 46 | 82 | 20 | 80 | 52 | 44 | 66 | 74 | 70 | 76 | 21 | 90 |
| 19.5 (2A,5CD,15) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A= | 98 | 47 | 70 | 4 | 5 | 80 | 25 | 16 | 24 | 10 | 48 | 53 | 59 | 14 | 15 | 8 | 51 | 52 | 19 | 20 | 21 | 62 | 58 | 9 | 7 | 46 | 100 | 71 | 65 | 64 |
| B= | 86 | 22 | 87 | 30 | 29 | 45 | 74 | 31 | 69 | 3 | 28 | 91 | 89 | 67 | 55 | 54 | 90 | 60 | 84 | 6 | 97 | 49 | 39 | 79 | 50 | 61 | 63 | 88 | 73 | 77 |
| 19.6 (2B,3B,7,10) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A= | 12 | 74 | 72 | 43 | 21 | 45 | 49 | 25 | 30 | 68 | 65 | 51 | 34 | 54 | 36 | 23 | 81 | 7 | 37 | 11 | 42 | 64 | 78 | 44 | 62 | 84 | 13 | 6 | 2 | 67 |
| B= | 38 | 99 | 10 | 13 | 55 | 83 | 60 | 17 | 24 | 68 | 96 | 70 | 34 | 56 | 67 | 49 | 54 | 1 | 19 | 46 | 21 | 36 | 85 | 97 | 16 | 26 | 39 | 69 | 32 | 92 |
| ((AB) ² (AB ⁻¹) ² ABAB ⁻¹) ⁷ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A= | 92 | 59 | 10 | 8 | 75 | 58 | 48 | 4 | 19 | 3 | 50 | 31 | 57 | 71 | 97 | 94 | 83 | 69 | 9 | 61 | 35 | 100 | 46 | 89 | 38 | 82 | 76 | 98 | 93 | 39 |
| B= | 20 | 55 | 73 | 52 | 41 | 87 | 60 | 40 | 18 | 77 | 14 | 33 | 63 | 32 | 5 | 27 | 70 | 53 | 90 | 88 | 47 | 26 | 17 | 56 | 86 | 85 | 66 | 80 | 24 | 79 |
| ((AB) ² (AB ⁻¹) ² ABAB ⁻¹) ⁷ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A= | 92 | 59 | 10 | 8 | 75 | 58 | 48 | 4 | 19 | 3 | 50 | 31 | 57 | 71 | 97 | 94 | 83 | 69 | 9 | 61 | 35 | 100 | 46 | 89 | 38 | 82 | 76 | 98 | 93 | 39 |
| B= | 40 | 29 | 91 | 47 | 22 | 64 | 45 | 3 | 71 | 48 | 28 | 23 | 78 | 2 | 5 | 100 | 76 | 93 | 81 | 7 | 84 | 90 | 51 | 79 | 72 | 87 | 50 | 57 | 27 | 95 |

19.7 B --> B⁻¹

19.8 (2B,3B,7;12)
A=
20 37 48 89 67 19 18 75 49 29 93 60 61 38 21 43 52 7 6 1 15 74 34 31 64 40 58 68 10 87
24 62 92 23 63 79 2 14 41 26 39 94 16 66 98 84 83 3 9 69 56 17 57 95 88 51 53 27 76 12
13 32 35 25 96 44 5 28 50 86 81 78 85 22 8 59 97 72 36 99 71 90 47 46 73 70 30 55 4 82
100 33 11 42 54 65 77 45 80 91

19.9 (2B,3B,7;15) ((AB)²(AB⁻¹)²AB(AB⁻¹)²ABAB⁻¹)³
A=
99 60 49 37 73 64 40 34 61 96 36 50 92 52 70 44 90 97 38 81 55 24 93 22 72 94 95 43 77 56
88 57 91 8 65 11 4 19 98 7 82 67 28 16 54 69 79 89 3 12 80 14 83 45 21 30 32 78 85 2
9 66 76 6 35 62 42 75 46 15 74 25 5 71 68 63 29 58 47 51 20 41 53 100 59 87 86 31 48 17
33 13 23 26 27 10 18 39 1 84

19.10 B --> B⁻¹

19.11 (2B,3B,10;7) ((AB)²(AB⁻¹)²ABAB⁻¹)¹²
A=
62 14 84 65 9 89 43 100 5 98 41 69 19 2 92 21 75 28 13 58 16 90 71 64 35 55 57 18 67 87
56 82 46 60 25 80 61 66 94 93 11 85 7 96 91 33 54 68 52 79 95 49 77 47 26 31 27 20 63 34
37 1 59 24 4 38 29 48 12 99 23 73 72 97 17 83 53 88 50 36 86 32 76 3 42 81 30 78 6 22
45 15 40 39 51 44 74 10 70 8

19.12 B --> B⁻¹

19.13 (2B,3B,10;15) ((AB)²(AB⁻¹)²ABAB⁻¹)⁵
A=
6 29 24 58 61 1 27 30 43 40 48 52 37 92 39 89 34 49 60 23 50 76 20 3 68 67 7 93 2 8
64 51 78 17 88 41 13 62 15 10 36 59 9 69 53 47 46 11 18 21 32 12 45 86 94 72 99 4 42 19
5 38 74 31 95 79 26 25 44 83 97 56 98 63 90 22 82 33 66 100 87 77 70 96 91 54 81 35 16 75
85 14 28 55 65 84 71 73 57 80

19.14 B --> B⁻¹

19.15 (2B,3B,10;15) ((AB)²(AB⁻¹)²ABAB⁻¹)¹⁰
A=
95 49 48 8 99 97 72 4 26 47 63 13 12 38 73 50 60 64 77 30 43 37 25 36 23 9 98 94 92 20
62 65 46 78 86 24 22 14 80 74 82 68 21 52 100 33 10 3 2 16 70 44 54 53 88 93 87 76 91 17
66 31 11 18 32 61 90 42 85 51 81 7 15 40 89 58 19 34 96 39 71 41 84 83 69 35 57 55 75 67
59 29 56 28 1 79 6 27 5 45

19.16 B --> B⁻¹

19.17 (2B,3B,12;7) ((AB)²(AB⁻¹)²ABAB⁻¹)⁸
A=
34 75 22 56 71 74 97 17 90 63 69 59 33 18 68 49 8 14 79 39 89 3 76 25 24 29 41 72 26 40
50 58 13 1 93 81 55 67 20 30 27 78 62 77 85 83 86 66 16 31 88 94 80 96 37 4 73 32 12 82
87 43 10 92 84 48 38 15 11 95 5 28 57 6 2 23 44 42 19 53 36 60 46 65 45 47 61 51 21 9
99 64 35 52 70 54 7 100 91 98

19.18 (2B,3B,12;10) ((AB)²(AB⁻¹)²AB(AB⁻¹)²ABAB⁻¹)⁸
A=
25 82 94 55 35 8 65 6 93 60 16 92 58 81 45 11 59 87 97 49 32 86 24 23 1 30 62 71 63 26
90 21 67 95 5 69 54 42 47 99 56 38 64 83 15 66 39 68 20 76 84 100 74 37 4 41 72 13 17 10
98 27 29 43 7 46 33 48 36 73 28 57 70 53 88 50 89 85 91 96 14 2 44 51 78 22 18 75 77 31
79 12 9 3 34 80 19 61 40 52

19.19 B --> B⁻¹

19.20 (2B,3B,12;15) ((AB)²(AB⁻¹)²ABAB⁻¹)¹⁵
A=
28 48 95 53 19 23 26 50 94 88 84 98 93 59 31 87 20 37 5 17 52 29 6 41 57 7 96 1 22 91
15 70 90 56 55 68 18 51 81 86 24 100 44 43 73 47 46 2 72 8 38 21 4 97 35 34 25 85 14 65
99 82 76 66 60 64 74 36 71 32 69 49 45 67 79 63 92 89 75 83 39 62 80 11 58 40 16 10 78 33
30 77 13 9 3 27 54 12 61 42

19.21 (2B,3B,15;6)
 A= 39 50 73 57 62 23 41 65 14 55 76 90 24 9 35 69 60 47 70 81 51 61 6 13 84 53 30 40 78 27
 42 87 82 96 15 67 91 54 1 28 7 31 77 52 74 72 18 86 85 2 21 44 26 38 10 58 4 56 66 17
 22 5 93 88 8 59 36 99 16 19 98 46 3 45 80 11 41 29 92 75 20 33 97 25 49 48 32 64 95 12
 37 79 63 100 89 34 83 71 68 94

19.22 (2B,3B,15;10) ((AB)²(AB⁻¹)²ABAB⁻¹)¹⁰
 A= 5 93 37 94 1 36 92 38 96 15 12 11 80 99 10 98 20 34 25 17 24 43 88 21 19 71 79 95 56 52
 63 59 83 18 53 6 3 8 51 44 86 60 22 40 69 82 55 97 75 81 39 30 35 84 47 29 91 66 32 42
 78 70 31 90 85 58 76 74 45 62 26 89 77 68 49 67 73 61 27 13 50 46 33 54 65 41 100 23 72 64
 57 7 2 4 28 9 48 16 14 87

19.23 B --> B⁻¹

20. PSP(4,4) ORDER = 979200 DEGREE = 85 |OUT(G)| = 4

| CLASS | 1 | 2A | 2B | 2C | 3A | 3B | 4A | 4B | 5AB |
|------------|-----------------|--|---|--|--|--|---|---|---|
| C(X) | G | 3840 | 3840 | 256 | 180 | 180 | 32 | 32 | 300 |
| CYCLE TYPE | 85 ¹ | 1 ²¹ ₂ ³² | 1 ⁵ ₂ ⁴⁰ | 1 ⁵ ₂ ⁴⁰ | 1 ⁷ ₃ ²⁶ | 1 ¹⁰ ₃ ²⁵ | 1 ¹ ₂ ² ₄ ²⁰ | 1 ¹ ₂ ² ₄ ²⁰ | 1 ⁵ ₅ ¹⁶ |
| CLASS | 5CD | 5E | 6A | 6B | 10AB | 10CD | 15AB | 15CD | 17ABCD |
| C(X) | 300 | 25 | 12 | 12 | 20 | 20 | 15 | 15 | 17 |
| CYCLE TYPE | 5 ¹⁷ | 5 ¹⁷ | 1 ³ ₂ ³ ₆ ¹⁰ | 1 ² ₄ ³ ₁₆ ¹² | 1 ² ₂ ⁵ ₄ ¹⁰ ₆ | 5 ¹ ₁₀ ⁸ | 5 ² ₁₅ ⁵ | 1 ² ₃ ¹ ₅ ¹ ₁₅ ⁵ | 17 ⁵ |

20.1 (2AB,5E,15;5)
 A= 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 24 25 22 23 28 29 26 27 32
 33 30 31 36 37 34 35 40 41 38 39 44 45 42 43 48 49 46 47 52 53 50 51 56 57 54 55 60 61 58
 B= 22 10 42 82 62 26 2 30 34 38 6 54 70 66 14 74 46 78 18 50 58 11 43 85 64 1 23 25 24 12
 83 65 44 13 63 45 84 39 7 56 73 27 3 32 37 79 20 52 61 67 17 76 49 81 60 19 51 29 36 5
 31 69 48 16 75 41 72 9 55 68 77 47 15 28 33 35 4 40 57 71 8 80 53 59 21

20.2 (2AB,5E,17;2)
 A= 1 2 3 4 5 16 17 14 15 20 21 18 19 8 9 6 7 12 13 10 11 32 33 30 31 36 37 34 35 24
 25 22 23 28 29 26 27 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 59 58 61 60 55 54 57
 56 67 66 69 68 62 65 64 85 84 83 82 81 80 79 78 77 76 75 74 73 72 71 70

20.3 (2AB,5E,17;5)
 A= 1 2 3 4 5 14 15 16 17 18 19 20 21 6 7 8 9 10 11 12 13 22 23 24 25 26 27 28 29 30
 31 32 33 34 35 36 37 46 47 48 49 50 51 52 53 38 39 40 41 42 43 44 45 66 67 68 69 62 63 64
 B= 65 58 59 60 61 54 55 56 57 74 75 76 77 70 71 72 73 82 83 84 85 78 79 80 81

20.4 (2C,4A,15;4) (AB⁻¹ABAB²)¹⁵
 A= 46 71 13 60 37 32 63 8 81 45 51 40 3 19 27 57 84 66 14 22 74 20 28 55 85 69 15 23 76 30
 62 6 78 44 53 39 5 73 36 12 59 70 58 34 10 1 49 48 47 72 11 61 35 67 24 77 16 43 41 4
 52 31 7 80 65 18 54 82 26 42 2 50 38 21 83 29 56 33 79 64 9 68 75 17 25
 B= 22 2 26 34 30 38 6 54 70 42 10 62 82 46 14 66 74 50 18 58 78 3 27 37 32 1 23 25 24 4
 35 33 28 5 31 29 36 43 11 64 85 39 7 56 73 51 19 60 81 47 15 68 77 53 80 21 59 41 72 9
 55 49 76 17 67 45 84 13 63 48 69 75 16 40 57 71 8 44 65 83 12 52 61 79 20

20.5 B --> B⁻¹

20.6 (2C,4A,15;10) (AB⁻¹ABAB²)¹⁵
A= 60 80 52 18 24 82 16 31 41 71 37 46 13 77 43 7 26 4 54 65 67 55 62 5 69 17 33 83 38 45
8 74 27 34 47 73 11 29 42 75 9 39 15 85 30 12 35 70 49 66 57 3 63 19 22 81 51 61 59 1
58 23 53 78 20 50 21 79 25 48 10 72 36 32 40 84 14 64 68 2 56 6 28 76 44

20.7 B --> B⁻¹

20.8 (2C,4A,15;17) (ABAB²)¹⁰
A= 84 44 24 64 10 53 34 67 9 5 79 70 77 46 57 19 27 39 16 33 58 74 81 3 71 59 17 41 30 29
54 47 20 7 35 50 69 56 18 48 28 80 72 2 76 14 32 40 60 36 68 52 6 31 61 38 15 21 26 49
55 73 75 4 78 66 8 51 37 12 25 43 62 22 63 45 13 65 11 42 23 83 82 1 85

20.9 (2C,4A,15;17) (ABAB²)¹⁷ (AB⁻¹ABAB²)¹⁷

A= 45 65 10 25 85 68 20 31 70 3 40 46 51 55 34 80 17 58 75 7 28 50 48 39 4 72 30 21 67 27
8 74 60 15 79 36 54 83 24 11 62 44 43 42 1 12 82 23 64 22 13 84 63 37 14 81 57 18 73 33
69 41 53 49 2 77 29 6 61 9 76 26 59 32 19 71 66 78 35 16 56 47 38 52 5

20.10 B --> B⁻¹

20.11 (2C,4A,17;4) (AB⁻¹ABAB²)⁶
A= 1 9 8 7 6 5 4 3 2 10 11 12 13 19 20 18 21 16 14 15 17 83 82 85 84 68 69 66 67 50
51 52 53 37 36 35 34 80 81 78 79 63 62 65 64 49 48 47 46 30 31 32 33 74 75 76 77 61 60 59
58 43 42 45 44 28 29 26 27 73 72 71 70 54 55 56 57 40 41 38 39 23 22 25 24

20.12 B --> B⁻¹

20.13 (2C,4A,17;6) (AB⁻¹(AB)6B)⁶
A= 7 59 74 29 44 8 1 6 9 77 60 43 26 42 61 27 76 28 58 75 45 49 32 79 62 13 16 18 4 72
38 23 57 34 85 51 68 31 46 65 80 14 12 5 21 39 73 56 22 83 36 66 53 78 63 48 33 19 2 11
15 25 55 70 40 52 67 37 82 64 81 30 47 3 20 17 10 54 24 41 71 69 50 84 35

20.14 B --> B⁻¹

20.15 (2C,4A,17;10) (AB⁻¹(AB)6B)¹⁵
A= 63 69 58 4 56 70 52 15 29 21 34 76 41 32 8 46 81 43 83 23 10 38 20 77 36 80 30 49 9 27
71 14 51 11 44 25 82 22 45 84 13 75 18 35 39 16 73 50 28 48 33 7 78 54 68 5 61 3 66 60
57 64 1 62 65 59 67 55 2 6 31 79 47 85 42 12 24 53 72 26 17 37 19 40 74

20.16 B --> B⁻¹

20.17 (2C,4A,17;15) (ABAB²)¹⁰
A= 44 64 10 24 84 54 17 35 81 3 50 41 47 59 29 74 7 69 71 20 30 38 49 4 51 37 78 55 15 -21
31 66 73 75 8 61 26 22 83 65 12 43 42 1 45 85 13 62 23 11 25 63 82 6 28 60 76 80 14 56
36 48 52 2 40 32 72 68 18 70 19 67 33 16 34 57 79 27 77 58 9 53 39 5 46

20.18 (2C,4A,17;17) (AB²)⁶

A= 36 13 70 61 47 51 69 82 9 32 28 24 2 74 39 65 21 57 78 43 17 46 73 12 59 71 58 11 49 60
48 10 72 37 35 1 34 45 15 54 79 63 20 77 38 22 5 31 29 84 6 52 68 40 64 76 18 27 25 30
4 81 42 55 16 66 83 53 7 3 26 33 23 14 80 56 44 19 41 75 62 8 67 50 85

20.19 (2C,4B,5B;17)

A= 44 64 10 24 84 54 17 35 81 3 50 41 47 59 29 74 7 69 71 20 30 38 49 4 51 37 78 55 15 21
31 66 73 75 8 61 26 22 83 65 12 43 42 1 45 85 13 62 23 11 25 63 82 6 28 60 76 80 14 56
36 48 52 2 40 32 72 68 18 70 19 67 33 16 34 57 79 27 77 58 9 53 39 5 46
B= 1 8 7 9 6 3 5 4 2 14 17 15 16 13 11 10 12 19 18 21 20 49 48 47 46 78 79 80 81 64
65 62 63 31 30 33 32 42 43 44 45 77 76 75 74 59 58 61 60 28 29 26 27 52 53 50 51 83 82 85
84 69 68 67 66 34 35 36 37 39 38 41 40 72 73 70 71 54 55 56 57 25 24 23 22

20.20 (2C,4B,15;5) (AB⁻¹ABAB²)¹⁰
 A= 34 21 41 76 63 49 59 72 13 84 53 67 9 55 80 45 17 26 22 30 2 19 77 64 39 18 62 38 74 20
 40 75 65 1 35 37 36 28 25 31 3 57 79 44 16 82 50 66 6 47 60 73 11 54 14 78 42 85 7 51
 68 27 5 24 33 48 12 61 71 83 69 8 52 29 32 4 23 56 43 15 81 46 70 10 58

20.21 B --> B⁻¹

20.22 (2C,4B,15;15) (AB⁻¹(AB)²)¹⁵
 A= 11 67 78 29 40 80 69 38 27 13 1 12 10 26 68 81 39 41 66 28 79 47 34 73 60 14 9 20 4 75
 53 32 54 23 84 42 65 8 17 5 18 36 49 58 71 85 22 64 43 50 76 57 31 33 55 74 52 44 63 25
 82 70 59 48 37 19 2 15 7 62 45 83 24 56 30 51 77 3 21 6 16 61 72 35 46

20.23 (2C,4B,15;15) (AB⁻¹(AB)²)⁴
 A= 8 49 31 78 64 7 6 1 9 79 65 48 30 32 81 46 63 62 33 47 80 59 42 77 28 72 39 25 54 13
 3 14 19 34 52 69 83 74 27 60 45 23 56 70 41 16 20 12 2 68 82 35 53 29 76 43 58 57 22 40
 71 18 17 5 11 85 67 50 36 44 61 26 75 38 73 55 24 4 10 21 15 51 37 84 66

20.24 (2C,4B,15;15) (AB⁻¹ABAB²)¹⁷
 A= 34 2 26 30 22 76 41 63 21 80 45 55 17 72 49 59 13 84 53 67 9 5 33 24 27 3 25 28 31 4
 29 32 23 1 37 36 35 85 51 68 7 73 47 60 11 81 43 56 15 77 39 64 19 71 12 48 61 79 16 44
 57 83 8 52 69 75 20 40 65 78 54 14 42 82 66 6 50 70 58 10 46 74 62 18 38

20.25 B --> B⁻¹

20.26 (2C,4B,15;17) (ABAB²)⁶
 A= 17 54 81 35 44 34 80 45 55 78 43 36 57 14 16 15 1 42 37 79 56 22 67 76 49 38 60 33 83 70
 1 11
 52 58 40 85 31 66 23 48 77 30 84 41 59 74 47 24 69 10 20 7 3 50 29 71 64

20.27 (2C,4B,17;5) (ABAB²)⁶
 A= 4 5 3 1 2 65 67 54 60 56 58 63 69 59 57 68 62 66 64 61 55 79 74 85 72 84 73 78 75 70
 83 76 81 77 80 71 82 48 39 45 50 42 53 47 40 51 44 38 49 41 46 52 43 8 21 10 15 11 14 9
 20 17 12 19 6 18 7 16 13 30 36 25 27 23 29 32 34 28 22 35 33 37 31 26 24

20.28 (2C,4B,17;15) (ABAB²)⁶
 A= 57 30 75 12 52 17 43 36 78 59 68 4 62 41 73 25 6 84 19 27 46 34 44 79 16 26 20 48 83 2
 69 65 61 22 71 8 40 45 81 37 14 72 7 23 38 21 47 28 82 67 64 5 58 55 54 56 1 53 10 77
 33 13 74 51 32 76 50 11 31 80 35 42 15 63 3 66 60 9 24 70 39 49 29 18 85

20.29 (2C,4B,17;15) (ABAB²)¹⁷ (AB⁻¹ABAB²)⁴
 A= 46 71 13 60 37 32 63 8 81 45 51 40 3 19 27 57 84 66 14 22 74 20 28 55 85 69 15 23 76 30
 62 6 78 44 53 39 5 73 36 12 59 70 58 34 10 1 49 48 47 72 11 61 35 67 24 77 16 43 41 4
 52 31 7 80 65 18 54 82 26 42 2 50 38 21 83 29 56 33 79 64 9 68 75 17 25

20.30 B --> B⁻¹

20.31 (2C,4B,17;17) (AB²)¹⁵ (AB⁻¹ABAB²)¹⁷
 A= 36 13 70 61 47 51 69 82 9 32 28 24 2 74 39 65 21 57 78 43 17 46 73 12 59 71 58 11 49 60
 48 10 72 37 35 1 34 45 15 54 79 63 20 77 38 22 5 31 29 84 6 52 68 40 64 76 18 27 25 30
 4 81 42 55 16 66 83 53 7 3 26 33 23 14 80 56 44 19 41 75 62 8 67 50 85

20.32 B --> B⁻¹