

## On the Primality of $k! + 1$ and $2 * 3 * 5 * \dots * p + 1$

By Mark Timpler

**Abstract.** In this paper the results of an investigation of  $k! + 1$  and  $2*3*5*\dots*p + 1$  are reported. Values of  $k = 1(1)230$  and  $2 \leq p \leq 1031$  were investigated. Five new primes were discovered.

In this paper the results of an investigation of  $k! + 1$  and  $2*3*5*\dots*p + 1$  are reported. A PDP-11/70 computer was used for all computations.

Numbers  $N$  of the above forms were first checked for small factors by trial division. Then they were tested for pseudoprimality by computing  $a_1^{(N-1)/2} \pmod{N}$  for a base  $a_1$  such that the Jacobi symbol  $(a_1|N) = -1$ . If  $a_1^{(N-1)/2} \not\equiv -1 \pmod{N}$ , then  $N$  is composite by Euler's criterion. If  $a_1^{(N-1)/2} \equiv -1 \pmod{N}$ , then it was proved prime by the following theorem developed by Brillhart, Lehmer, and Selfridge [1, p. 623].

**THEOREM 1.** *Let  $N - 1 = \prod p_i^{a_i} = F_1 R_1$ , where  $F_1$  is the even factored portion of  $N - 1$ ,  $F_1 > R_1 \geq 1$ , and  $(F_1, R_1) = 1$ . If for each  $p_i$  dividing  $F_1$  there exists an  $a_i$  such that  $a_i^{N-1} \equiv 1 \pmod{N}$ , but  $(a_i^{(N-1)/p_i} - 1, N) = 1$ , then  $N$  is prime.*

The calculations were performed by calculating

$$(1) \quad a_i^{(N-1)/p_i} \equiv b_i \not\equiv 1 \pmod{N}, \quad (N, b_i - 1) = 1, \quad \text{and then } b_i^{p_i} \equiv 1 \pmod{N}.$$

The base  $a_1$  was used as long as (1) was satisfied, otherwise a new  $a_i$  was chosen until (1) was satisfied, at which point the original base  $a_1$  was used for the next  $p_i$  (avoiding the computation of  $b_i^{p_i} \pmod{N}$ ).

Theorem 1 was used in the program in favor of other theorems in [1] with slightly faster running speeds. This was due to the fact that Theorem 1 requires relatively little memory space. The processor time to compute (1) for  $p_i = 2$  was roughly  $(x^3/(\ln x)^2)*C$ , where  $x$  is the number of bits in  $(N-1)/2$  and  $C$  is a constant. The time required to prove  $N$  prime was  $(x^3/(\ln x)^2)*C*D/2$ , where  $D$  is the number of distinct prime factors of  $(N-1)/2$ .

This investigation confirmed all of Borning's [2] results for  $k! + 1$  and  $2*3*5*\dots*p + 1$ . It extended the upper limit on  $k$  from 100 to 230, and on  $p$  from 307 to 1031. The primes  $k = 116$  and 154, and  $p = 379, 1019$ , and 1021 were discovered, with 191, 272, 154, 425, and 428 digits, respectively.

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*Values of  $k$  such that*

*$k! + 1$  is prime*

$1 \leq k \leq 230$

*Values of  $p$  such that*

*$2 \cdot 3 \cdot 5 \cdot \dots \cdot p + 1$  is prime*

$2 \leq p \leq 1031$

<u><math>k</math></u>	<u><math>p</math></u>
1	2
2	3
3	5
11	7
27	11
37	31
41	379
73	1019
77	1021
116	
154	

345 Encanto Drive  
Tempe, Arizona 85281

1. JOHN BRILLHART, D. H. LEHMER & J. L. SELFRIDGE, "New primality criteria and factorizations of  $2^m \pm 1$ ," *Math. Comp.*, v. 29, 1975, pp. 620–647.

2. ALAN BORNING, "Some results for  $k! + 1$  and  $2 \cdot 3 \cdot 5 \cdot \dots \cdot p \pm 1$ ," *Math. Comp.*, v. 26, 1972, pp. 567–570.