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Misstatements in Milne-Thomson, Calculus of Finite Differences, Macmillan, London, 1933

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Abstract. Milne-Thomson's "proofs" of the interpolating property of the continued fraction employing reciprocal differences, as well as the symmetry property of reciprocal differences, are shown to be faulty. He also makes a similar misleading, though correct, statement in connection with divided differences.

In Chapter V, "Reciprocal Differences", p. 107, in his "proof" of the interpolating property of the continued fraction employing reciprocal differences, Milne-Thomson's argument for the vanishing of the remainder partial quotient $(x - x_n)/(\rho_n(xx_1 \cdots x_n) - \rho_{n-2}(x_1x_2 \cdots x_{n-1}))$ when $x = x_n$, is given in his statement (lines 8–9): ". . . if a numerator of one of the constituent partial fractions vanish, this and all the following constituents do not affect the value, and can be ignored." Now that argument does not apply here because $x - x_n$ is a factor in the denominator which is, from the definition of reciprocal differences, $(x - x_n)/(\rho_{n-1}(xx_1 \cdots x_{n-1}) - \rho_{n-1}(x_1x_2 \cdots x_n))$. Thus the remainder partial quotient, i.e., $\rho_{n-1}(xx_1 \cdots x_{n-1}) - \rho_{n-1}(x_1x_2 \cdots x_n)$, becomes, for $x = x_n$, $\rho_{n-1}(x_nx_1 \cdots x_{n-1}) - \rho_{n-1}(x_1x_2 \cdots x_n)$, which vanishes from the symmetry property of reciprocal differences. But Milne-Thomson's proof of the symmetry, which is given later on pp. 110–111, is based on the interpolation property (Eq. (6), p. 110), so that even if page 107 were rectified, his presentation would still be circular. However, the results in that chapter happen to be correct because the symmetry property was proven directly from the definition of reciprocal differences, and independently of the interpolation property, in the original article of T. N. Thiele, "Différences Réciproques," *Kongelige Danske Videnskabernes Selskab., Oversigt over Forhandlinger,* **1906**, no. 3, pp. 153–171.

Oddly enough, Milne-Thomson makes a similar misleading, though correct, statement in connection with divided differences (p. 3, last two lines, p. 4, first three lines), where he states *without showing why,* that the interpolation property holds because the remainder term

$$R_n(x) = (x - x_1)(x - x_2) \cdots (x - x_n)[xx_1 \cdots x_n]$$

vanishes when $x = x_i$. The truth of that statement is not apparent just from the

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vanishing of the factor $(x - x_1)(x - x_2) \cdots (x - x_n)$, since $x - x_i$ occurs also in the denominator of $[xx_1 \cdots x_n]$. But from the explicit expression for $[xx_1 \cdots x_n]$ which is derived, on p. 7, independently of the preceding text, it becomes apparent, even after the partial cancellation of $x - x_i$, that $R_n(x_i) = 0$.

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