The entire book is well presented and easy to read (specially for French reading people). It ends with a good bibliography and a detailed index.

One can object that the book contains too many results for a beginner in the subject, who will certainly be unable to find his way without a guide. A good idea would have been to indicate the sections which could be skipped during a first reading.

C. BREZINSKI

Université de Lille 1
59655 Villeneuve d'Ascq
France

12[9.10].—R. J. EVANS, *Table of Cyclotomic Numbers of Order Twenty-Four*, 98 pp. of computer printout, deposited in the UMT file, 1979.

Let $P = 24F + 1$ be a prime with fixed primitive root g. For integers I, J (mod 24), define the cyclotomic number (I, J) of order 24 to be the number of integers N (mod P) for which N/g^I and $(1 + N)/g^J$ are (nonzero) 24th power residues (mod P). Let $\beta = \exp(2\pi i/24)$ and fix a character χ (mod P) of order 24 such that $\chi(g) = \beta$. For characters λ, ψ (mod P), define the Jacobi sums

$$J(\lambda, \psi) = \sum_{N \,(\mathrm{mod}\, P)} \lambda(N)\psi(1 - N), \qquad K(\lambda) = \lambda(4)J(\lambda, \lambda).$$

It is known [1] that there exist integers X, Y, A, B, C, D, U, V such that

$$K(\chi^6) = -X + 2Yi \quad (P = X^2 + 4Y^2,\ X \equiv 1 \pmod 4),$$
$$K(\chi^4) = -A + Bi\sqrt{3} \quad (P = A^2 + 3B^2,\ A \equiv 1 \pmod 6),$$
$$K(\chi^3) = -C + Di\sqrt{2} \quad (P = C^2 + 2D^2,\ C \equiv 1 \pmod 4),$$

and

$$K(\chi) = U + 2Vi\sqrt{6} \quad (P = U^2 + 24V^2,\ U \equiv -C \pmod 3).$$

Since $J(\chi, \chi^2) \in \mathbf{Z}[\beta]$, there are integers $D_0, D_1, \dots, D_7$, such that

$$J(\chi, \chi^2) = \sum_{i=0}^{7} D_i\beta^i.$$

Each number $576(I, J)$ can be expressed as an integer linear combination of $P, 1, X, Y, A, B, C, D, U, V, D_0, \dots, D_7$. Such formulas are presented in tables of cyclotomic numbers of order twenty-four here deposited in the UMT files. (For references to tables of cyclotomic numbers of other orders, see [2].) The tables were constructed by evaluating the right side of [4, (2.7)] using relations between Jacobi sums of orders dividing 24, particularly the relations in [3, p. 496].

There are 48 tables, corresponding to the 48 choices of the 4-tuples (F', V', Z, T) with $F' = F \pmod 2 \in \{0, 1\}$, $V' = V \pmod 2 \in \{0, 1\}$, $Z = \mathrm{ind}\, 2 \pmod{12} \in \{0, 2, 4, 6\}$, $T = \mathrm{ind}\, 3 \pmod 8 \in \{0, 2, 4\}$, where the indices of 2 and 3 are taken with respect to g. Each of the 48 tables contains 109 formulas. The first formula in Table 1, for example, is:

$$576(0, 0) = P - 71 - 114X - 80A - 24C + 96U + 192D_0 + 96D_4.$$

To obtain a formula for a number (I, J) not listed in a table, one makes use of the facts [3, p. 489] that $(I, J) = (-I, J - I)$ and

$$(I, J) = \begin{cases} (J, I), & \text{if } 2 \mid F, \\ (J + 12, I + 12), & \text{if } 2 \nmid F. \end{cases}$$

For example, to find the formula for 576(20, 4) in Table 1 ($F' = V' = Z = T = 0$), one observes that (20, 4) = (4, 8) = (8, 4). To obtain a formula for (I, J) where $Z = Z_0$, $T = T_0$ with either

(i) $Z_0 \in \{8, 10\}, T_0 = 6$,

(ii) $Z_0 \in \{8, 10\}, T_0 \in \{0, 2, 4\}$,

or

(iii) $Z_0 \in \{0, 2, 4, 6\}, T_0 = 6$,

one uses the fact that the transformation $g \longrightarrow g^M$ yields the transformation $(I, J) \longrightarrow (MI, MJ)$, where $M = -1, 5, 7$ in cases (i), (ii), (iii), respectively. Illustrations are now given for a fixed choice of F' and V'.

Case (i). To find $576(I, J)$, one would find $576(-I, -J)$ in the table for $Z = 12 - Z_0$, $T = 2$ modified so that the heading

$$P\ 1\ X\ Y\ A\ B\ C\ D\ U\ V\ D_0 \cdots D_7$$

is replaced by

$$P\ 1\ X\ {-Y}\ A\ {-B}\ C\ {-D}\ U\ {-V}\ E_0 \cdots E_7,$$

where

$$E_0 = D_0 + D_4, \quad E_1 = D_3, \quad E_2 = D_2, \quad E_3 = D_1, \quad E_4 = -D_4,$$
$$E_5 = -D_3 - D_7, \quad E_6 = -D_2 - D_6, \quad E_7 = -D_1 - D_5.$$

For example, to determine $\alpha = 576(0, 23)$ when $F' = V' = 1, Z_0 = 8, T_0 = 6$, note that the table for $F' = V' = 1, Z = 4, T = 2$ (Table 44) gives

$$576(0, 1) = P + 1 - 26X + 48Y - 4A - 4C - 8U + 96V + 8D_0 - 40D_2 - 8D_4 - 8D_6 - 24D_7.$$

Thus,

$$\alpha = P + 1 - 26X - 48Y - 4A - 4C - 8U - 96V + 8(D_0 + D_4) - 40D_2 + 8D_4 + 8(D_2 + D_6) + 24(D_1 + D_5).$$

Case (ii). To find $576(I, J)$, one would find $576(5I, 5J)$ in the table for $Z = 12 - Z_0$, $T = T_0$ modified so that its heading is replaced by

$$P\ 1\ X\ Y\ A\ {-B}\ C\ {-D}\ U\ V\ E_0 \cdots E_7$$

where

$$E_0 = D_0 + D_4, \quad E_1 = D_5, \quad E_2 = -D_2, \quad E_3 = -D_3 - D_7,$$
$$E_4 = -D_4, \quad E_5 = D_1, \quad E_6 = D_2 + D_6, \quad E_7 = D_7.$$

Case (iii). To find $576(I, J)$, one would find $576(7I, 7J)$ in the table for $Z = Z_0$, $T_0 = 2$ modified so that its heading is replaced by

$$P\ 1\ X\ {-Y}\ A\ B\ C\ {-D}\ U\ V\ E_0 \cdots E_7$$

where

$$E_0 = D_0, \quad E_1 = D_3 + D_7, \quad E_2 = -D_2, \quad E_3 = -D_5, \quad E_4 = D_4,$$
$$E_5 = -D_3, \quad E_6 = -D_6, \quad E_7 = D_1 + D_5.$$

AUTHOR'S SUMMARY

Department of Mathematics
University of California, San Diego
La Jolla, California 92093

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