

NUMERICAL ANALYSIS

edited by

Gene H. Golub and Joseph Oliger

*Lecture Notes from the Short Course
sponsored by the AMS, Atlanta, January 3–4, 1978*

This is the collection of texts prepared by the lecturers of the Numerical Analysis Short Course given at the A. M. S. meeting in Atlanta, Georgia in January 1978. Computational linear algebra, optimization and the solution of nonlinear equations, the approximation of functions and functionals, and approximations for initial and boundary value problems for ordinary and partial differential equations are discussed. Methods such as the QR factorization, singular value decomposition, quasi-Newton and secant methods, finite difference, finite element and collocation methods are included in these discussions.

The subject matter was chosen to emphasize prominent research areas and attitudes in numerical analysis. These are introductory lectures on the subject matter for presentation to an audience of scientists from other areas or disciplines. Typically, there is an introduction to a given problem area and to techniques used, an application to applied problems, and a discussion of current research questions or directions.

Several trends in modern numerical analysis are discussed in these lectures. There has always been the quest to find the best way to do things. More realistic notions of "best" are evolving which in-

corporate the classical notions and realistic costs of producing the desired result. The discussion of good vs. best approximation is an example. More attention is being given to providing not only an answer, but a computed guarantee that it is a good answer—or a poor one. Easily computed and sharp a posteriori estimates are needed. The discussion of estimates of condition numbers is an example. There is progress being made in algorithm design based on operator splittings which allow one to take advantage of being able to solve simpler sub-problems very efficiently. Updating strategies for optimization and splitting methods for differential equations are examples.

These texts should be useful to the practicing users of numerical methods, programmers, scientists, and engineers who would like to know what progress is being made on the theoretical and developmental side of the subject. They should be useful to numerical analysts to review progress in areas other than their own, and to mathematicians in general who would like to understand what the concerns of numerical analysts are. The texts should be useful for the development of seminars and reading courses in the academic environment. Many will probably find the bibliographies of current work most useful.

THE LECTURERS AND TITLES

CLEVE MOLER, Three Research Problems in Numerical Linear Algebra

J. E. DENNIS, JR., A Brief Introduction to Quasi-Newton Methods

CARL De BOOR, The Approximation of Functions and Linear Functionals:
Best vs. Good Approximation

JAMES M. VARAH, Numerical Methods for the Solution of Ordinary Differential Equations

JOSEPH E. OLIGER, Methods for Time Dependent Partial Differential Equations

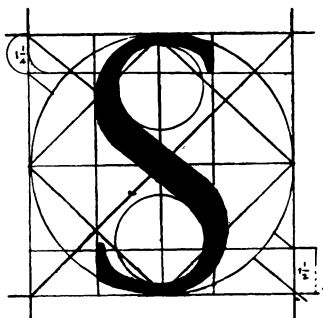
GEORGE J. FIX, Variational Methods for Elliptic Boundary Value Problems

Publication: November 1978, viii + 135 pages.

Soft Cover: List \$10.00, Individual \$5.00. CODE: PSAPM/22 E

ORDERS MUST BE PREPAID, PLEASE SPECIFY CODE

**American Mathematical Society, P. O. Box 1571, Annex Station
Providence, RI 02901**



TEX and METAFONT
New Directions in Typesetting
Donald E. Knuth

The TEX Manual describes in Knuth's inimitable style a new system for technical typesetting which makes possible, and even easy, composition and typesetting of the most complex scientific displays of difficult mathematical notation by authors and technical typists.

METAFONT, a system for alphabet design, is Knuth's solution to the problem of limited character sets on raster-based typesetting machines. Like TEX, it is a tool for simplifying technical typesetting and is designed to prepare alphabets and special characters to be used by TEX.

The TEX and METAFONT systems, being implemented in PASCAL, are in the public domain and available to all who typeset.

This volume, containing manuals for both systems, introduces TEX and METAFONT to anyone concerned with typesetting*. Only a knowledge of high school mathematics is required to master the systems.

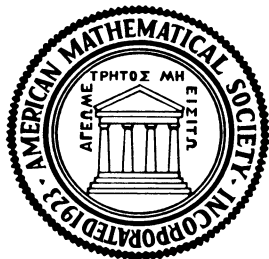
Foreword by Gordon Bell.

December 1979, 360 pages, illustrated, indexed, paperback—\$12.00**
Co-published by Digital Press and the American Mathematical Society

Send orders to **DIGITAL PRESS**
Department AMS, Educational Services
Digital Equipment Corporation
12A Esquire Road
North Billerica, MA 01862

*Part I is a reprint of the Gibbs Lecture given in January 1979, and Part II is a reprint of *TEX, A Manual for Technical Text*. Part III, METAFONT, has not been previously published.

**Price applies to U.S. only. Contact the nearest office of Digital Equipment Corporation, Educational Services, for prices outside the U.S. On prepaid single orders, individual members of the AMS are entitled to a 50% discount; institutional members to a 25% discount on orders for 1 to 5 copies. A 10% discount is extended to all other customers ordering two or more copies. Add \$1 for postage and handling unless the order is prepaid.



REVIEWS IN GRAPH THEORY

Compiled and Edited by William G. Brown
MCGILL UNIVERSITY
DEPARTMENT OF MATHEMATICS

This publication is a four-volume compendium of about 9,600 reviews in graph theory published by **Mathematical Reviews** in Volumes 1 through 56, i.e. between 1940 and 1978 inclusive. Reviews were selected from the several sections of **Mathematical Reviews** which were the usual repositories of such items; from the subject lists in **Mathematical Reviews** indexes, where available; and through a systematic perusal of about half of all reviews published by **Mathematical Reviews** during the 39 years under investigation. Every review cited in a selected review was also read, and the process iterated until stable.

A classification scheme containing over 500 categories was developed for the purpose. Every review has been assigned one primary classification and, on the average, one secondary classification. Reviews are reprinted in strict chronological order of **Mathematical Review** numbers in their primary subject area, with a brief citation at each secondary location.

The final volume provides a detailed author index, which can serve as an effective bibliography of the subject.

These volumes are a research tool. They are directed to anyone who has occasion to consult the literature of graph theory: mathematicians, computer scientists, engineers, and management scientists, as well as students, teachers, and practicing researchers.

The potential reader requires no more background than would be required to read papers which are reviewed in the compendium. These vary from highly erudite papers in other areas of mathematics where graph theory is used as a tool to solve specific problems, to elementary descriptive papers which would be understandable to high school students.

A few of the reviews are themselves gems of the mathematical literature. But, for the most part, the reader will use this book as a research tool—to determine what has been done in a particular area of the subject, or to locate known papers when the values of not all parameters are available.

There has been nothing of this scope or magnitude in the subject before. This is the first major bibliography in graph theory which incorporates reviews.

The editor's previous work includes research papers in graph theory and related fields, and many reviews.

It is estimated that *Reviews in Graph Theory* will be 2,156 pages. Publication is scheduled for December 1980. For prices call or write the Sales Department of the Society after December 1.

American Mathematical Society, P.O. Box 6248, Providence, RI 02940 (401) 272-9500

Mathematical Surveys

Recent volumes in this distinguished series

APPROXIMATION BY POLYNOMIALS WITH INTEGRAL COEFFICIENTS

Le Baron O. Ferguson

Results in the approximation of functions by polynomials with coefficients which are integers have been appearing since that of Pál in 1914. The body of results has grown to an extent which seems to justify the present book. The intention here is to make these results as accessible as possible.

The book addresses essentially two questions. The first is the question of what functions can be approximated by polynomials whose coefficients are integers and the second question is how well are they approximated (Jackson type theorems). For example, a continuous function f on the interval $-1, 1$ can be uniformly approximated by polynomials with integral coefficients if and only if it takes on integral values at $-1, 0$ and $+1$ and the quantity $f(1) + f(0)$ is divisible by 2. The results regarding the second question are very similar to the corresponding results regarding approximation by polynomials with arbitrary coefficients. In particular, nonuniform estimates in terms of the modulus of continuity of the approximated function are obtained.

Aside from the intrinsic interest to the pure mathematician, there is the likelihood of im-

portant applications to other areas of mathematics; for example, in the simulation of transcendental functions on computers. In most computers, fixed point arithmetic is faster than floating point arithmetic and it may be possible to take advantage of this fact in the evaluation of integral polynomials to create more efficient simulations. Another promising area for applications of this research is in the design of digital filters. A central step in the design procedure is the approximation of a desired system function by a polynomial or rational function. Since only finitely many binary digits of accuracy actually can be realized for the coefficients of these functions in any real filter, the problem amounts (to within a scale factor) to approximation by polynomials or rational functions with integral coefficients.

This book should make the task of finding out what is known in this field significantly easier as it presents an introduction to most of the known results in the area of approximation by polynomials with integral coefficients and pointers to the literature for the rest. It is accessible to students at the graduate level and above.

Volume 17, vi + 160 pages, 1980, hard cover. List price \$25.60; institutional member \$19.20; individual member \$12.80.
To order, please specify SURV/17

SYMPLECTIC GROUPS

O. T. O'Meara

This volume, the sequel to the author's *Lectures on Linear groups*, is the definitive work on the isomorphism theory of symplectic groups over integral domains. Recently discovered geometric methods which are both conceptually simple and powerful in their generality are applied to the symplectic groups for the first time. There is a complete description of the isomorphisms of the symplectic groups and their congruence subgroups over integral domains. The

new geometric approach used in the book is instrumental in extending the theory from subgroups of $\text{PSp}_n(n \geq 6)$ where it was known to subgroups of $\text{PFSp}_n(n \geq 4)$ where it is new. There are extensive investigations and several new results on the exceptional behavior of PFSp_4 in characteristic 2.

The author starts essentially from scratch and the reader need be familiar with no more than a first course in algebra.

Volume 16, 128 pages, 1978, hard cover. List \$22.80; institutional member \$17.10; individual member \$11.40
To order, please specify SURV/16

Prepayment is required for all American Mathematical Society publications.
Send for the book(s) above to: AMS, P. O. Box 1571, Annex Station, Providence, RI 02901

M. K. Agrawal, J. H. Coates, D. C. Hunt and A. J. van der Poorten, Elliptic Curves of Conductor 11.....	991
Carl Pomerance, J. L. Selfridge and Samuel S. Wagstaff, Jr., The Pseudoprimes to $25 \cdot 10^9$	1003
Peter Hagsis, Jr., Outline of a Proof That Every Odd Perfect Number Has at Least Eight Prime Factors.....	1027
Reviews and Descriptions of Tables and Books.....	1033
Dongarra, Bunch, Moler & Stewart 10, Gilewicz 11, Evans 12	

No microfiche supplement in this issue

MATHEMATICS OF COMPUTATION

TABLE OF CONTENTS

July 1980

James H. Bramble and Peter H. Sammon, Efficient Higher Order Single Step Methods for Parabolic Problems: Part I	655
Carl de Boor and Blair Swartz, Collocation Approximation to Eigenvalues of an Ordinary Differential Equation: The Principle of the Thing.....	679
Alan E. Berger, Jay M. Solomon, Melvyn Ciment, Stephen H. Leventhal and Bernard C. Weinberg, Generalized OCI Schemes for Boundary Layer Problems.....	695
Blair Swartz, Compact, Implicit Difference Schemes for a Differential Equation's Side Conditions.....	733
L. R. Lundin, A Cardinal Function Method of Solution of the Equation $\Delta u = u - u^3$	747
David P. Maher, Existence Theorems for Transforms Over Finite Rings With Applications to 2-D Convolution.....	757
J. R. Cash, A Note on Olver's Algorithm for the Solution of Second-Order Linear Difference Equations.....	767
Jorge Nocedal, Updating Quasi-Newton Matrices With Limited Storage.....	773
Athena Makroglou, Convergence of a Block-By-Block Method for Nonlinear Volterra Integro-Differential Equations.....	783
D. Meek, A Mean Value Theorem for Linear Functionals	797
F. M. Larkin, Root-Finding by Fitting Rational Functions.....	803
Robert D. Skeel, Iterative Refinement Implies Numerical Stability for Gaussian Elimination.....	817
Avram Sidi, Analysis of Convergence of the T -Transformation for Power Series.....	833
Avram Sidi, Numerical Quadrature and Nonlinear Sequence Transformations; Unified Rules for Efficient Computation of Integrals with Algebraic and Logarithmic Endpoint Singularities.....	851
Adhemar Bultheel, Recursive Algorithms for the Matrix Padé Problem.....	875
T. S. Horner, Recurrence Relations for the Coefficients in Chebyshev Series Solutions of Ordinary Differential Equations.....	893
J. R. Philip, The Convergence and Partial Convergence of Alternating Series.....	907
G. Allasia and F. Bonardo, On the Numerical Evaluation of Two Infinite Products...	917
Godfrey L. Isaacs, Exponential Laws for Fractional Differences.....	933
Marietta J. Tretter and G. W. Walster, Further Comments on the Computation of Modified Bessel Function Ratios.....	937
Jeffrey S. Leon, On an Algorithm for Finding a Base and a Strong Generating Set for a Group Given by Generating Permutations.....	941
Gary B. Gostin, A Factor of F_{17}	975
F. T. Howard, A Special Class of Bell Polynomials.....	977