REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

13[7.30].—SALVADOR CONDE, *Raices de Ecuaciones Trascendentes con Productos Cruzados de Funciones de Bessel de Diferentes Ordenes,* Facultad de Ingenieria Universidad del Zulia, Maracaibo, Venezuela, 1977; a report of 78 pages with 67 pages of computer printed tables, deposited in the UMT file.

Let

$$B_\nu(x) \equiv J_\nu(x)Y_{\nu-1}(\beta x) - J_{\nu-1}(\beta x)Y_\nu(x) = 0,$$

$$C_\nu(x) = J_\nu(\beta x)Y_{\nu-1}(x) - J_{\nu-1}(x)Y_\nu(\beta x) = 0.$$

This report gives the first 28 positive zeros of these functions for $\nu = 0(0.1)1.0$ and $\beta = 0.1(0.1)0.9$, $1.1(0.1)2.0$, to 12S. If $\beta = 1$, there are no zeros.

If $\nu = 1$, some scattered and low accuracy tables are reported in the FMRC index [1]. Again, if $\nu = 1$, the first 10 positive zeros for a large number of β values are given to 10D in reports by Fettis and Caslin [2], [3]. If $\nu = 1/2$, the positive zeros are $(2m - 1)\pi/2(1 - \beta)$, m a positive integer. Thus, the tables under review are the most extensive known to me.

The method of computation is described in the introduction. The zeros were determined to a precision of order 10^{-13}. I am not sure what this means, but I should think the data were intended to be correct to within a half unit of the last place recorded.

I have compared the common entries of the report under review with those of Fettis and Caslin. According to the method of computation used by the latter authors, their entries should be correct to within a half unit of the last place recorded. In the comparison, we found numerous discrepancies of as many as 6 units in the last place, where, if necessary, the Conde tables are rounded to 10D. We also found entries which differed by 10, 11, 14 and 31 units in the last place. We have not attempted to determine which are correct. Neither do we record the errata more precisely since the data is believed to be more than of sufficient accuracy for most applications.

The question of interpolation is not considered. For interpolation in the β direction, the technique discussed by Fettis and Caslin should prove helpful.

Y. L. L.

1. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables,* 2nd ed., published for Scientific Computing Service Ltd. by Addison-Wesley, Reading, Mass., 1962, pp. 414, 415.

2. HENRY E. FETTIS & JAMES C. CASLIN, *An Extended Table of Zeros of Cross Products of Bessel Functions,* Report ARL 66-0023, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1966, v + 129 pp., 28 cm. (Copies obtainable from the Defense Documentation Center, Cameron Station, Alexandria, Virginia.) See also *Math. Comp.,* v. 21, 1967, pp. 507, 508.

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3. HENRY E. FETTIS & JAMES C. CASLIN, *More Zeros of Bessel Function Cross Products*, Report ARL 68-0209, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, December 1968, v + 56 pp., 28 cm. (Released to the Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151.) See also *Math. Comp.*, v. 23, 1969, p. 884.

14[3.05].—PHILIP J. DAVIS, *Circulant Matrices*, Wiley, New York, 1979, xv + 250 pp. Price $18.95.

A circulant matrix of order n is a square matrix of the form

$$C = \begin{pmatrix} c_1 & c_2 \cdots\cdots\cdots c_n \\ c_n & c_1 \cdots\cdots\cdots c_{n-1} \\ \cdot & \\ \cdot & \\ \cdot & \\ c_2 & c_3 \cdots\cdots\cdots c_1 \end{pmatrix} = \operatorname{circ}(c_1, c_2, \ldots, c_n).$$

This special structure affords an intimate relationship between the study of circulants and Fourier analysis; this relationship provides the backbone for the elegant analysis of circulants presented in this compact, well-organized book by the well-known numerical analyst Philip J. Davis.

Circulant matrices and their generalizations have important applications in physics, image processing, probability and statistics, number theory, geometry, and numerical analysis, e.g. in the numerical solution of periodic boundary value problems. Although the book does not actively pursue any of these applications, it does fulfill the author's avowed purpose of serving as a general reference on circulants, so that the basic facts need not be "rediscovered over and over again" by someone researching a specific application.

Chapter 1 motivates the study of circulants with a geometric example of nested polygons, the vertices of successive polygons being related by a circulant transformation. In fact, those with a taste for geometry will find throughout the book a host of connections between circulant matrix properties and geometrical results (cf. Chapter 4).

Chapter 2 is packed full of useful information on general matrix theory and is intended to provide background for the more specific study of circulants to follow in Chapter 3. In particular, the reader will find an interesting self-contained presentation of material on least squares, the singular value decomposition, and generalized inverses, although computational aspects are not addressed.

In Chapter 3, a central result is the representation of all circulants as polynomials in the special circulant $\pi = \operatorname{circ}(0, 1, 0, \ldots, 0)$, from which many interesting properties of circulants may be obtained. Generalizations of circulants (e.g. skew-circulants, block circulants) appear here and in the fifth chapter. Finally, the study of centralizers in Chapter 6 is used by Davis "to encompass and unify a number of results previously obtained as well as to point us in several new directions."

In order to appreciate the material in the book, the reader should have a reasonable background in matrix theory and some knowledge of abstract algebra. As such,