19[7.15].—H. J. J. TE RIELE, *Tables of the First* 15,000 *Zeros of the Riemann Zeta Function to* 28 *Significiant Figures, and Related Quantities,* Report NW 67/79, Stichting Mathematisch Centrum, Amsterdam, June 1979, 5 pp. text + 154 pp. tables.

Table 1 of this report is a table of the imaginary parts of the first 15,000 zeros ρ_n of the Riemann zeta function $\zeta(s)$ in the upper half of the critical strip. Table 2 gives $|\rho_n \zeta'(\rho_n)|^{-1}$, and Table 3 gives $\arg(\rho_n \zeta'(\rho_n))$.

The computation of Table 1 was performed using (mainly) double-precision arithmetic on a Cyber 73/173 and required about 21 hours of CPU time. The author claims an accuracy of "about 28 digits" and the table gives each ρ_n to 28 significant digits.

The reviewer checked the accuracy of a few entries in Table 1, using 40S computation and his multiple-precision arithmetic package [1], and both the Euler-Maclaurin formula for $\zeta(s)$ (as used by te Riele) and the Riemann-Siegel formula with a sufficient number of terms [2]. The largest error found was 66 units in the last place (ρ_{10142} = 9998.850397089674049057631757 is correct, te Riele gives 9998.850. . . 631691). Thus, the final two digits of entries in the table should be regarded with suspicion. Despite this, the table is a significant advance over the 9S table of $\rho_1, \ldots, \rho_{1600}$ given in [3] and other tables known to the reviewer.

Tables 2 and 3 are given to 10S, although computed to "about 14 significant digits". The reviewer has not found any errors exceeding 0.5 units in the last place in these tables.

The tabulated quantities were used by the author in computations concerning Mertens' conjecture [4]. However, the reviewer wonders what point there is in publishing *all* of them in this manner. Anyone wishing to continue the Mertens conjecture computation would hardly use the values of ρ_n given in Table 1; a horrifying amount of key-punching and verifying would be required. Instead, he would either recompute them, using Table 1 for checking purposes, or obtain them in machine-readable form, e.g. on magnetic tape. Unfortunately, te Riele does not say if his tables are available on magnetic tape, nor does he reproduce the program that generated them, although he does clearly describe the computational method.

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1. R. P. BRENT, "A Fortran multiple-precision arithmetic package," *ACM Trans. Math. Software,* v. 4, 1978, pp. 57–70.

2. R. P. BRENT, *Numerical Investigation of the Riemann Siegel Approximation,* Tech. Report, Dept. of Computer Science, Australian National University. (To appear.)

3. C. B. HASELGROVE in collaboration with J. P. C. MILLER, *Tables of the Riemann Zeta Function,* Royal Soc. Math. Tables, Vol. 6, Cambridge, 1960.

4. H. J. J. TE RIELE, "Computations concerning the conjecture of Mertens," *J. Reine Angew. Math.,* v. 311/312, 1979, pp. 356–360.

20[9.10].—SIRPA MÄKI, *The Determination of Units in Real Cyclic Sextic Fields,* Computer Table, 122 pages, University of Turku, Finland, 1979. Reference [3] subsequently appeared as Lecture Notes No. 797, 1980 and contains a photographic copy of this table.

There are 1337 real cyclic sextic fields having conductor less than 2022. Fundamental units and class numbers have been computed for all but 12 of these fields. In six of these cases the necessary information on the cubic subfield was unavailable and for the remaining six fields the numbers became too large to be handled by the program.

For any real sextic field there exist two units ϵ_1 and ϵ_2 which together with the units of the proper subfields generate the whole unit group. Although the table does not list values for ϵ_1 and ϵ_2, they can be expressed as products and quotients of units which are listed.

The class number h_6 of the sextic field can be expressed in the form

$$h_6 = h_2 h_3 h_R,$$

where h_2 and h_3 are the class numbers of the quadratic and cubic subfields and h_R is the so-called relative class number. In fact $h_R = (U_6 : U_6')$ where U_6 is the unit group of the sextic field and U_6' is a subgroup of U_6 which is generated by the units of the proper subfields together with the so-called cyclotomic units of the sextic fields. For approximately 90 percent of the fields listed $h_R = 1$. Moreover h_R assumes the values 3, 4, and 7 for approximately 3, 4.5, and 1.7 percent of the fields respectively. In addition h_R assumes the values 9, 12, and 16 each one time.

The relatively large number of occurrences of 3 and 4 as relative class numbers can be partially explained by the fact that there exists a subgroup U_6^* of U_6 containing U_6' such that $(U_6 : U_6^*) = 1, 3, 4$ or 12. In 70 percent of the cases where 3 divides h_R, this index is also divisible by 3. When 3 is replaced with 4, the percentage becomes 85.

The table tempts one to conjecture that h_R never assumes the values 2, 5, 6 or 8. This, in fact, is true. Let H_2', H_3' and H_6' denote the 3-complements of the class groups of the quadratic, cubic, and sextic fields and denote the orders of these groups by h_2', h_3', and h_6'. An automorphism of the sextic field (and hence of its cubic subfield) of order 3 will be denoted by σ. By decomposing H_3' into orbits under σ it is seen that

$$h_3' \equiv 1 \pmod 3.$$

A similar decomposition of H_6' leads to

$$h_6'/h_2' \equiv 1 \pmod 3.$$

But

$$h_6' = h_2' h_3' h_R',$$

where h_R' is the largest factor of h_R which is prime to 3. Thus $h_R' \equiv 1 \pmod 3$.

The method of computation which is described in an accompanying manuscript [3] is essentially a refinement of the method of Leopoldt [1] and [2].

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1. H. W. LEOPOLDT, "Über Einheitengruppe und Klassenzahl reeler abelscher Zahlkörper," *Abh. Deutsch. Akad. Wiss. Berlin Math.-Nat. Kl.*, **1953**, No. 2, Berlin, 1954.

2. H. W. LEOPOLDT' "Über ein Fundamental-problem der Theorie der Einheiten algebraischer Zahlkörper," *Sitz. Ber. Bayr. Akad. Wiss. Math.-Nat. Kl.*, **1956**, No. 5.

3. S. MÄKI, *The Determination of Units in Real Cyclic Sextic Fields*, Lecture Notes in Math., Springer-Verlag, Berlin, Heidelberg and New York, 1980.