REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

6[5.10.3]–PHILIPPE G. CIARLET, *The Finite Element Method for Elliptic Problems*, North-Holland, Amsterdam, 1978, xviii + 530 pp. Price $56.95.

This monograph is very welcome for more than one reason. Firstly, it is a very thorough treatment of an area that has been developing rapidly. But more generally, it is an addition to the small library of mathematical books on numerical solution of partial differential equations. Although this field has seen tremendous growth since the invention of computers, there are very few mathematical texts to which one can turn for reference. This poses greatest difficulty for graduate students in mathematics trying to learn the subject and to begin research in the area. Fortunately, Ciarlet's book has done much to alleviate this problem in one area of discretization techniques for partial differential equations.

The subject of the book is, as the title indicates, finite element discretization methods for approximating the solution of elliptic boundary value problems. The basic idea of this discretization technique is to use piecewise-polynomial approximating functions in a Ritz-Galerkin variational formulation, rather than the more classical eigenfunction expansion. Although the idea was anticipated very early by Courant, it was not developed by mathematicians. Rather, finite element discretizations grew out of so-called "matrix methods" in structural engineering, with the major developments occurring in the 1950's and 1960's. Although remnants of this era of finite elements can still be seen from time to time, the mathematical point of view seems well established even in the engineering literature: methods are described in terms of bilinear forms and piecewise-polynomial subspaces, rather than the original terminology of matrix methods.

The power of finite element methodology has many facets. It allows discretization of problems on domains having irregular shape (e.g., an airplane wing). It automatically (and accurately) discretizes complex boundary conditions, something that is often not clear how to do in a finite difference context. There are simple ways to generate schemes of any desired accuracy automatically, and often several different approaches to choose from: nonconforming elements, mixed methods, hybrid methods, etc., as well as the more standard displacement methods. Virtually all of these advantages were discovered by engineers, but there is one more that was discovered by mathematicians: there is a natural framework in which to analyze the convergence properties of finite element methods. Indeed, the techniques of the variational approach to partial differential equations are in many cases directly applicable to the analysis of finite element methods. While this may seem to be only of academic interest, it has yielded detailed information about

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finite element discretizations that can be quite useful by indicating what (and what not) to expect from a fully debugged finite element computer code.

The basic goal of Ciarlet's book is to report on the mathematical investigations into various finite element methods for discretizing elliptic problems, with a focus on many of the more successful ideas that have been put forward in engineering practice. In this regard, it shares a common goal with the earlier book by Strang and Fix, *An Analysis of the Finite Element Method*, Prentice-Hall, Englewood Cliffs, N.J., 1973. Thus, to describe Ciarlet's work in more detail, perhaps it is fruitful to contrast it with that of Strang and Fix.

One obvious difference between the two books is that Ciarlet has written with much more mathematical rigor and detail. His book is aimed toward a mathematically sophisticated audience, whereas Strang and Fix sought to appeal to a broad readership. Moreover, Ciarlet's treatment is more like a treatise (contrary to his view expressed in the first sentence of the preface) in that the subjects included are treated completely, without many unsettled issues and conjectures. In contrast, Strang and Fix gave, in many cases, only preliminary arguments and sought thereby to give direction for future research. Thus, someone wanting a leisurely introduction to finite elements would be best advised to peruse first the book by Strang and Fix, coming later to Ciarlet's book for a more complete statement of results and proofs.

In addition to presenting material at a higher level of rigor and completeness, Ciarlet's book also reflects mathematical advances made in several areas since the writing of Strang and Fix: nonlinear problems, mixed methods, maximum-norm estimates, shell problems, etc. On the other hand, Strang and Fix provided a chapter each on the subjects of eigenvalue approximation and problems with singular solutions, while Ciarlet only gives these subjects brief mention. And, as the title suggests, Ciarlet focuses only on time-independent problems; in the book by Strang and Fix, one finds a chapter on parabolic and hyperbolic problems.

Perhaps the only drawback to Ciarlet's excellent monograph is its price, a shortcoming shared by all advanced research texts today. The recent appearance of a paper-back version of the book softens the blow; further, for the purposes of a graduate course on finite elements, one could consider the lecture notes by Ciarlet, *Numerical Analysis of the Finite Element Method*, Presses de l'Université de Montréal, 1976. These were a precursor to the larger monograph and have been used successfully as the text in graduate mathematics courses at the University of Michigan, for about one-fifth of the price of the monograph. With the more complete monograph on reserve in the library, this can offer students a low-budget introduction to finite element methods without a great sacrifice of mathematical content.

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7[2.05].–WALTER SCHEMPP & KARL ZELLER, Editors, *Multivariate Approximation Theory*, International Series of Numerical Mathematics, Birkhäuser Verlag, Basel, Switzerland, 1979, 455 pp., 24 cm. Price $38.00.

This volume contains papers presented at a meeting organized by the editors.