REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

12 [4.05.3, 12.05.1].–L. F. SHAMPINE & M. K. GORDON, *Computer Solution of Ordinary Differential Equations*, Freeman, San Francisco, 1975, vi + 318 pp., 24 cm. Price $13.95.

This book concerns the computer solution of the initial value problem in ordinary differential equations. It differs from the numerous other texts available in this area in that the authors discuss only the mathematical and computational considerations relevant to one particular state-of-the-art nonstiff solver, rather than survey many particular methods or several classes of methods. The exposition is quite elementary and self-contained and avoids almost all mathematics not directly related to the given code. In particular, only the Adams-Bashforth-Moulton methods in PECE form are studied.

The centerpiece of the book is the suite of FORTRAN programs DE/STEP, INTRP. These form an exceptionally well written code which, while judged among the best available general purpose codes for the solution of nonstiff initial value problems, is also very readable. In addition to the mathematical theory of the Adams algorithms employed in the code, there is a full discussion of the variable-step implementation used (Krogh's modified divided differences), and of the step design and order selection procedures. Such discussions enable the reader to understand a significant code in full detail and appreciate the many decisions that are made in its development, and thereby furnish a perspective rather different from and complementary to that furnished by the asymptotic analysis of algorithms traditionally taught in numerical analysis courses. This makes *Computer Solution of Ordinary Differential Equations* a distinct and valuable addition to the numerical analysis literature.

Of course, one can question some of the choices made by the authors in the code and their heuristic justifications, and when the book is used as a text it is useful to supplement these by a discussion of their limitations and of other possibilities which can be pursued. For example, in STEP the user supplied error tolerance controls the local error *per step* in the *predictor* formula. The chief justification of this is that the error *per unit step* in the accepted *corrector* value is then roughly a constant multiple of the tolerance. This constant, however, depends on a ratio of higher derivatives of the solution and so is unavailable to the user even a posteriori, and, in fact, the "constant" changes with the order. It is possible, though more expensive, to directly control the local error-per-unit step in the accepted solution. This is preferable because this quantity relates directly to the solution furnished the user and under certain hypotheses may be viewed as an asymptotically correct estimate of its residual.

0025-5718/81/0000-0122/$02.50

One of the main uses of this work is as a textbook. Because of the authors' novel approach, however, there is some difficulty in incorporating it into a more or less standard curriculum. Most teachers of courses on the numerical solution of ordinary differential equations will not wish to use it as the primary textbook because it leaves vast areas untouched: single-step methods; Dahlquist's theory of multistep methods; stiffness, A-stability, and related concepts; to list a few. Fortunately, the elementary and lucid style of this exposition makes it quite accessible to the student who has already encountered the basic mathematical theory of multistep methods, and therefore it can be used as a supplementary text relatively easily. Worked exercises to test the reader's understanding are included. Through lectures and reading assignments the reviewer very successfully covered virtually the entire text in an introductory graduate level course in two and one half weeks. The authors also suggest that their book be used as text for a topics course or as a source of supplementary readings in a survey course on numerical analysis.

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13 [2.05.5].—CLAUDE BREZINSKI, *Padé-Type Approximation and General Orthogonal Polynomials*, Birkhäuser Verlag, Basel, Boston, 1980, 250 pp. Price $34.00.

In recent years, we have witnessed a surge of interest in the field of rational approximation with emphasis on computation and evaluation of functions. In particular, considerable emphasis has been placed on approximations of analytical type with remainder as opposed to those approximations which require tabular values of the functions being approximated, that is, approximations of the curve fitting variety.

Let $f(x)$ be a function which possesses the at least formal representation $f(x) = \sum_{k=0}^{\infty} c_k x^k$. Let $P_m(x)$ and $Q_n(x)$ be polynomials in x of degree m and n, respectively, such that $Q_n(x)f(x) - P_m(x) = O(x^{r+1})$ where r is some positive integer. Then $T_{mn}(x) = P_m(x)/Q_n(x)$ is a rational approximation to $f(x)$ with the property that its expansion in powers of x agrees with that of $f(x)$ through the first r powers. The fraction $T_{mn}(x)$ fills out a matrix $m, n = 0, 1, 2, \ldots$. If $m = n$, we have the main diagonal while if $m = n - 1$, we have the first subdiagonal etc. If $r = m + n$, the approximations go by the name Padé,for which there is considerable literature.

In previous studies the reviewer [1], [2], [3] has developed rational approximations principally for generalized hypergeometric series with $m = n - a$, $a = 0$ or $a = 1$ such that $r = n - a$ at least. Originally, they were developed on the basis of Lanczos' τ-method and later it was shown how they could be obtained by appropriate weighting of the partial sum of the series for $f(x)$. The latter weighting or summability procedure can be extended at least formally for any series. For the series ${}_2F_1(1, b; c; -z)$ and its confluent forms, a certain set of the above rational approximations are those of the Padé class. There are other methods for getting rational approximations. For example, appropriate quadrature of the Stieltjes