One of the main uses of this work is as a textbook. Because of the authors' novel approach, however, there is some difficulty in incorporating it into a more or less standard curriculum. Most teachers of courses on the numerical solution of ordinary differential equations will not wish to use it as the primary textbook because it leaves vast areas untouched: single-step methods; Dahlquist's theory of multistep methods; stiffness, A-stability, and related concepts; to list a few. Fortunately, the elementary and lucid style of this exposition makes it quite accessible to the student who has already encountered the basic mathematical theory of multistep methods, and therefore it can be used as a supplementary text relatively easily. Worked exercises to test the reader's understanding are included. Through lectures and reading assignments the reviewer very successfully covered virtually the entire text in an introductory graduate level course in two and one half weeks. The authors also suggest that their book be used as text for a topics course or as a source of supplementary readings in a survey course on numerical analysis.

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13 [2.05.5].—CLAUDE BREZINSKI, *Padé-Type Approximation and General Orthogonal Polynomials*, Birkhäuser Verlag, Basel, Boston, 1980, 250 pp. Price $34.00.

In recent years, we have witnessed a surge of interest in the field of rational approximation with emphasis on computation and evaluation of functions. In particular, considerable emphasis has been placed on approximations of analytical type with remainder as opposed to those approximations which require tabular values of the functions being approximated, that is, approximations of the curve fitting variety.

Let $f(x)$ be a function which possesses the at least formal representation $f(x) = \sum_{k=0}^{\infty} c_k x^k$. Let $P_m(x)$ and $Q_n(x)$ be polynomials in x of degree m and n, respectively, such that $Q_n(x)f(x) - P_m(x) = O(x^{r+1})$ where r is some positive integer. Then $T_{mn}(x) = P_m(x)/Q_n(x)$ is a rational approximation to $f(x)$ with the property that its expansion in powers of x agrees with that of $f(x)$ through the first r powers. The fraction $T_{mn}(x)$ fills out a matrix $m, n = 0, 1, 2, \ldots$. If $m = n$, we have the main diagonal while if $m = n - 1$, we have the first subdiagonal etc. If $r = m + n$, the approximations go by the name Padé,for which there is considerable literature.

In previous studies the reviewer [1], [2], [3] has developed rational approximations principally for generalized hypergeometric series with $m = n - a$, $a = 0$ or $a = 1$ such that $r = n - a$ at least. Originally, they were developed on the basis of Lanczos' τ-method and later it was shown how they could be obtained by appropriate weighting of the partial sum of the series for $f(x)$. The latter weighting or summability procedure can be extended at least formally for any series. For the series ${}_2F_1(1, b; c; -z)$ and its confluent forms, a certain set of the above rational approximations are those of the Padé class. There are other methods for getting rational approximations. For example, appropriate quadrature of the Stieltjes integral $f(z) = \int dg(t)/(1 - zt)$ leads to rational approximations of the above kind and if the quadrature is Gaussian, then Padé approximations emerge.

There are many voids in our understanding of rational approximations. Claude Brezinski's research on rational approximations and summability is brought to focus in the present volume which contains a number of novel and ingenious ideas on the subject. We present an illustration which is at the basis of his work.

Let c be a linear functional which acts on some normed linear space of functions whose domain includes polynomials in t and $(1 - xt)^{-1}$ where x lies in some region of the complex plane. Let $c(x^i) = c_i$, $i = 0, 1, 2, \ldots$. c_i is called the moment of order i of the functional c. Then $f(t) = c((1 - xt)^{-1})$. Let $v(x)$ be a polynomial in x of degree k. Then $w(t) = c((v(x) - v(t))/(x - t))$ is a polynomial of degree $k - 1$. Let $\bar{v}(t) = t^k v(1/t)$, $\bar{w}(t) = t^{k-1}w(1/t)$. Then $f(t) - T_{k-1,k}(t) = R(t)$, where $T_{k-1,k}(t) - \bar{w}(t)/\bar{v}(t)$ is a rational approximation to $f(t)$ and $R(t)$ is the remainder, $R(t) = \{t^k/\bar{v}(t)\}c[v(x)/(1 - xt)]$. The approximation $T_{k-1,k}(t)$ is called a Padé-type approximation for $f(t)$. It is readily shown how the analysis can be extended to give generally rational approximants of the form $T_{mn}(t)$ as discussed above.

The volume under review is devoted to the study of these Padé-type approximants. Basic properties are taken up in Chapter 1. They are divided into three classes. The first deals with algebraic properties; the second concerns the error; and the third explains the connection with polynomial interpolation and summation methods.

Suppose one desires to construct a Padé-type approximation whose order exceeds the degree of the numerator polynomial. This can be accomplished say for the first subdiagonal approximant by putting additional constraints on the polynomial $v(x)$. For instance, we can write $v(x) = u(x)S_m(x)$, $m < k - 1$, and determine the polynomial $S_m(x)$ by the conditions $c(x^i v(x)) = 0$, $i = 0, 1, \ldots, m - 1$. Define the functional c^* by $c^*(x^i) = c(x^i u(x))$. Then the added conditions are $c^*(x^i S_m(x)) = 0$. So $S_m(x)$ is a polynomial of degree m belonging to a family of orthogonal polynomials with respect to c^*. If $m = k - 1$, then we get the Padé approximation which occupies the $(k - 1, k)$ position in the matrix T. Many results on Padé-type approximation can be gotten by means of the theory of general orthogonal polynomials. For this reason the latter theory is extensively studied in Chapter 2.

In Chapter 3, the theory of general orthogonal polynomials is used to derive old and new results for Padé approximations. Some related items such as continued fractions and the ε-algorithm are treated. Two generalizations of Padé-type approximations are taken up in Chapter 4. The first is the case of series with coefficients in a topological vector space. Its application to sequences produces an ε-algorithm which generalizes the scalar ε-algorithm due to Wynn. The second generalization is concerned with Padé-type approximants for double power series.

An appendix gives a 'conversational program' in FORTRAN which computes recursively sequences of Padé approximants. There is a list of 149 references which is admittedly far from complete. The author has prepared a more extensive bibliography of some 2,000 items which has been published in the form of internal reports and is available on request. The shortened reference list aside, the reviewer noticed very few blemishes. One such is on page 30 and has to do with the Laplace transform $f(t) = \int_0^\infty e^{-xt} g(x)\, dx$. Suppose one approximates $f(t)$ by a Padé-type approximation where the degree of the denominator polynomial exceeds that of the numerator polynomial. Then an approximation for $g(x)$ as a series of exponentials follows by inversion of the approximation for $f(z)$. It is stated that this idea is due to Longman in a paper published in 1972. Actually an application of this idea was used by Longman in 1966. I recognize, there is some danger in saying who expressed an idea first. However, the reviewer gave several applications of this idea in papers published in 1957, 1962, 1963, and 1964. See [2, Volume 2, Section 16.4] for details.

The volume is well written and organized. It is valuable both for the information it provides and for pointing out directions for future research. For anyone interested in rational approximations, the book is a must on one's bookshelf.

Y. L. L.

1. Y. L. LUKE, *The Special Functions and Their Approximations*, Vols. 1, 2, Academic Press, New York, 1969.

2. Y. L. LUKE, *Mathematical Functions and Their Approximations*, Academic Press, New York, 1975. Also in Russian, Izdat. "Mir", Moscow, 1980.

3. Y. L. LUKE, *Algorithms for the Computation of Mathematical Functions*, Academic Press, New York, 1977.

14 [8.00, 2.00, 3.00].—W. J. KENNEDY & J. E. GENTLE, *Statistical Computing*, Marcel Dekker, New York, 1980, xi + 591 pp., $23\frac{1}{2}$ cm. Price $26.50.

Historically, statistical practice was limited by what was computationally feasible. Modern computing capability has expanded the horizons on computational feasibility to where what might have been the toys of mathematical statisticians in the past have become part of today's common statistical practice. This success and aim have been the fruits and the goal of the field of statistical computing.

The technique of nonlinear regression epitomizes the toy that is now common practice. It also illustrates the fragmentation that statistical computing suffered from. That is, contributors have come from many disciplines, from those in applications like chemical engineers, to those in operations research and numerical analysis, mathematicians, economists, and, finally, statisticians themselves. These advances have appeared in a great variety of publications. Many wheels have been reinvented many times.

The publication of this book, I believe, is a milestone, although it is not the first in its field (Hemmerle, [1]). Kennedy and Gentle have done an outstanding job of assembling the best techniques from a great variety of sources, establishing a benchmark for the field of statistical computing. Its imperfections will be discussed later.

Disregarding four preparatory chapters on the fundamentals of computing, the authors treat four main topics in depth:

1. Computing probabilities and percentile points.
2. Random numbers and simulation.
3. Linear models computations.
4. Nonlinear regression and optimization.