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Conjectures on the Taylor Series Expansion Coefficients of the Jacobian Elliptic Function sn(x, k)

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Abstract. Two conjectures are posed for the coefficients introduced by Alois Schett in the Taylor series expansion of the Jacobian elliptic function sn(x, k). The first conjecture is furnished with a proof revealing a procedure which might be useful when calculating further coefficients. Some of the coefficients are tabulated.

The Taylor series expansion coefficients of the Jacobian elliptic function sn(x, k) were given by Schett [2], [3] for powers of x up to and including 49. In accordance with Wrigge [7] we write

$$\text{(1)} \qquad \operatorname{sn}(x, k) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} s_{2n+1}(1, k),$$

where we have the polynomials

$$\text{(2)} \qquad s_{2n+1}(1, k) = \sum_{j=0}^{n} c_j^{(2n+1)} k^{2j}.$$

The coefficients $c_j^{(2n+1)}$ may be obtained using some rather complicated recurrence formulae or in a combinatorial way as in Schett [4] and in Dumont [1], where they are denoted by $a_{2n+1,0,j}$.

CONJECTURE I. *The coefficient* $c_j^{(m)}$ *of* k^{2j} *in the polynomials* $s_m(1, k)$, *where* $m = 2n + 1$, *is obtained as a solution to the difference equation over* m, *the characteristic equation of which is*

$$\text{(3)} \qquad \prod_{p=0}^{j} \left(u - [2(j-p) + 1]\right)^{p+1} = 0. \quad \square$$

Proof. Jacobi in "Fundamenta Nova Theoriae Functionum Ellipticarum," 1829, and afterwards Glaisher [*Messenger of Math.*, Vol. XI, 1882], [5, p. 516] gives the formula

$$\text{(4)} \qquad \frac{d^2 \operatorname{sn}^n(x, k)}{dx^2} = n(n-1)\operatorname{sn}^{n-2}(x, k) - n^2(1 + k^2)\operatorname{sn}^n(x, k) + n(n+1)k^2 \operatorname{sn}^{n+2}(x, k).$$

Putting $n = 1$, we get

$$\text{(5)} \qquad \frac{d^2 \operatorname{sn}(x, k)}{dx^2} = -(1 + k^2)\operatorname{sn}(x, k) + 2k^2 \operatorname{sn}^3(x, k).$$

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Now, by writing

$$\operatorname{sn}(x, k) = \sum_{n=0}^{\infty} (-1)^n r_{2n}(x) k^{2n} \tag{6}$$

and differentiating twice with respect to x, we get for $n \geqslant 1$ the differential equations

$$r_0'' + r_0 = 0, \tag{7}$$

$$r_{2n}'' + r_{2n} = r_{2n-2} - 2\sum_{j=0}^{n-1} r_{2j} \sum_{i=0}^{n-1-j} r_{2i} r_{2n-2j-2i-2}, \tag{8}$$

with the obvious initial conditions

$$r_0(0) = r_{2n}(0) = 0, \tag{9}$$

$$r_0'(0) = 1, \quad \text{and} \quad r_{2n}'(0) = 0. \tag{10}$$

These differential equations may be easily solved recursively, using de Moivre's theorem to linearize in the right member. The calculations are quite lengthy and therefore will be omitted, but there is no difficulty in proving by induction that

$$r_{2n} = \sum_{j=0}^{n} \sum_{i=0}^{n-j} b_{ji}^{(n)} x^i \left\{ \begin{matrix} \sin \\ \cos \end{matrix} \right\} (2j + 1)x, \tag{11}$$

where the sine is chosen for even values of i and the cosine for odd values of i and where the coefficients $b_{ji}^{(n)}$ are real. From (6) we see that r_{2n} are odd functions of x.

The result follows by introducing the Taylor series expansion for the sines and cosines and identifying coefficients of like powers of k in (1) and (2) as was done by Wrigge [7] for $n = 1, 2$. □

Example 1. The comprehensive Tables I–X show calculated values of the coefficients $c_j^{(m)}$ for $j = 1$ to 10 and $m < 100$. Using the procedure described in the proof, we may easily extend these tables to values of $j > 10$. Note the periodic behavior of the last digit in the Tables I–X. □

Formulae for the coefficients $c_j^{(m)}$ for $j \leqslant 10$ are presented in Table XI. These formulae may be used to extend the Tables I–X to values of $m > 100$.

The values and expressions in Tables I–XI have been verified directly and found to be in agreement with those published by Schett [2], [3]. Conjecture I has also been proved by Wrigge [7] for $j \leqslant 2$.

The special structure of the coefficients given in Table XI leads to

CONJECTURE II. *The coefficient $c_j^{(m)}$ of k^{2j} in the polynomials $s_m(1, k)$, where $m = 2n + 1$, is*

$$4^{-2j} \sum_{p=0}^{j} \frac{(-1)^p}{p!} P_p(n; j)[2(j - p) + 1]^m, \tag{12}$$

where P_p are polynomials of degree p with integral coefficients defined by

$$P_p(n; j) = \left(\sum_{i=0}^{p} (-1)^{[i/2]} A_i(j, p) D^i \right) \beta^p, \tag{13}$$

where D is the differentiation operator with respect to β and

$$\beta = 4[2(n - j) + 3] = 4[m - 2j + 2]. \tag{14}$$

We have further for $i \geqslant 1$

$$(15) \qquad A_0(j, p) \equiv 1,$$

$$(16) \qquad A_i(j, p) = \sum_{n=0}^{[i/2]} Q_{i-2n}^{(i)}(p) j^n + \sum_{n=0}^{[(i-1)/2]} Q_{i-2n-1}^{(i)}(p)\delta_{j,p+n},$$

where $Q_r^{(i)}$ are polynomials of degree r and $\delta_{j,p+n}$ is the Kronecker symbol. (The brackets indicate, as usual, the integer part.)

Some of the polynomials $Q_r^{(i)}$ have been empirically determined. They are all numbered below to make it easier to refer to a possible wrong formula.

We have the following expressions

(17) $Q_0^{(1)}(p) = -1,$
(18) $Q_1^{(1)}(p) = p - 1,$
(19) $Q_0^{(2)}(p) = 4,$
(20) $Q_1^{(2)}(p) = p - 3,$
(21) $Q_2^{(2)}(p) = -(3p^2 - 7p - 106)/6,$
(22) $Q_0^{(3)}(p) = 1,$
(23) $Q_1^{(3)}(p) = 4(p - 3),$
(24) $Q_2^{(3)}(p) = (3p^2 - 43p - 72)/6,$
(25) $Q_3^{(3)}(p) = -(p - 3)(p^2 - p - 114)/6,$
(26) $Q_0^{(4)}(p) = 8,$
(27) $Q_1^{(4)}(p) = -(p - 4),$
(28) $Q_2^{(4)}(p) = -(6p^2 - 38p - 166)/3,$
(29) $Q_3^{(4)}(p) = -(p^3 - 34p^2 + 45p + 498)/6,$
(30) $Q_4^{(4)}(p) = (15p^4 - 90p^3 - 3475p^2 + 23022p + 18352)/360,$
(31) $Q_0^{(5)}(p) = -1,$
(32) $Q_1^{(5)}(p) = 8(p - 5),$
(33) $Q_2^{(5)}(p) = -(3p^2 - 49p - 82)/6,$
(34) $Q_3^{(5)}(p) = -2(p - 5)(p^2 - 5p - 111)/3,$
(35) $Q_4^{(5)}(p) = -(15p^4 - 930p^3 + 7985p^2 + 38762p - 41760)/360,$
(36) $Q_5^{(5)}(p) = (p - 5)(3p^4 - 10p^3 - 1255p^2 + 6542p + 51120)/360.$

In particular, we have

(37) $P_0(n; j) \equiv 1,$
(38) $P_1(n; j) = \beta - \delta_{j1},$
(39) $P_2(n; j) = \beta^2 + 2(1 - \delta_{j2})\beta - 2(4j + 18 - \delta_{j2}),$
(40) $P_3(n; j) = \beta^3 + 3(2 - \delta_{j3})\beta^2 - 4(6j + 25)\beta - 6(\delta_{j4} - 29\delta_{j3}),$

$$(41) \qquad \begin{aligned} P_4(n; j) = {} & \beta^4 + 4(3 - \delta_{j4})\beta^3 - 4(12j + 43 + 3\delta_{j4})\beta^2 \\ & -8(12j + 51 + 3\delta_{j5} - 98\delta_{j4})\beta + 24(8j^2 + 74j + 147 - 33\delta_{j4}), \end{aligned}$$

$$(42) \qquad \begin{aligned} P_5(n; j) = {} & \beta^5 + 5(4 - \delta_{j5})\beta^4 - 20(4j + 11 + 2\delta_{j5})\beta^3 \\ & -20(24j + 94 + 3\delta_{j6} - 106\delta_{j5})\beta^2 \\ & +8(120j^2 + 1030j + 1863 - 15\delta_{j6} + 5\delta_{j5})\beta \\ & -120(\delta_{j7} - 42\delta_{j6} + 680\delta_{j5}). \quad \square \end{aligned}$$

Example 2. The polynomials $P_p(n; j)$ corresponding to the expressions given in Example 1 and in Table XI are listed in Table XII. □

The conjectured expressions in Eqs. (17)–(42) seem to be generally valid. Eqs. (37)–(42) are only consequences of Eqs. (17)–(36). The expressions in Conjecture II have been verified directly and found to be in agreement with the results from Conjecture I.

Perhaps Conjecture II may be proved in a similar way as Conjecture I, or, alternatively, by use of the technique in [7], Theorem VIII in [6], the recurrence formulae (2) in [1], or Corollary 1 in [1]. However, there will be a great amount of labor doing so. I look forward to more elegant proofs which hopefully might lead to an explicit expression for the polynomials $P_p(n; j)$. Thus we need only a recurrence formula for the polynomials $Q_r^{(i)}(p)$, which is probably an impossible task to perform.

Similar conjectures may be posed for the corresponding coefficients in the Taylor series expansions of the Jacobian elliptic functions $\mathrm{cn}(x, k)$ and $\mathrm{dn}(x, k)$.

The necessary computer calculations were carried out using REDUCE and the FORTRAN subroutine package NUMBIG for handling large numbers.

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TABLE I. *Coefficients* $c_1^{(m)}$ *for* $m < 100$

m	coefficient
1	0
3	1
5	14
7	135
9	1228
11	11069
13	99642
15	896803
17	8071256
19	72641337
21	653772070
23	5883948671
25	52955538084
27	476599842805
29	4289398585298
31	38604587267739
33	347441285409712
35	3126971568687473
37	28142744118187326
39	253284697063686007
41	2279562273573174140
43	20516060462158567341
45	184644544159427106154
47	1661800897434843955475
49	14956208076913595599368
51	134605872692222360394409
53	1211452854230001243549782
55	10903075688070011191948143
57	98127681192630100727533396
59	883149130733670906547800677
61	7948342176603038158930206210
63	71535079589427343430371856011
65	643815716304846090873346704224
67	5794341446743614817860120338145
69	52149073020692533360741083043438
71	469341657186232800246669747391079
73	4224074914676095202220027726519852
75	38016674232084856819980249538678813
77	342150068088763711379822245848109466
79	3079350612798873402418400212632985347
81	27714155515189860621765601913696868280
83	249427399636708745595890417223271814681
85	2244846596730378710363013755009446332294
87	20203619370573408393267123795085016990815
89	181832574335160675539404114155765152917508
91	1636493169016446079854637027401886376257749
93	14728438521148014718691733246616977386319922
95	132555946690332132468225599219552796476879483
97	1193003520212989192214030392975975168291915536
99	10737031681916902729926273536783776514627240017

TABLE II. *Coefficients* $c_2^{(m)}$ *for* $m < 100$

m	coefficient
1	0
3	0
5	1
7	135
9	5478
11	165826
13	4494351
15	116294673
17	2949965020
19	74197080276
21	1859539731885
23	46535238000235
25	1163848723925346
27	29100851707716150
29	727566807977891803
31	18189614152200873621
33	454744658216502193656
35	11368657974646161302248
37	284216848055029040209305
39	7105425014717554019615631
41	177635661714292879129333150
43	4440891888211006424569211370
45	111022300477586804328521775591
47	2775557542867631254917084671065
49	69388938863336838872742230963028
51	1734723474327885153932200566560316
53	43368086883974153468865596179537861
55	1084202172341038681140525152576628723
57	27105054310788355233787656136744958490
59	677626357790855396141703190125407409886
61	16940658944968768734261555200748618699315
63	423516473626059259570422483352507810221405
65	10587911840668614140822229932564269315050736
67	264697796016874697910341197721597886185564880
69	6617444900423347901998172936625910415385519153
71	165436122510597438830695275898188315660019925911
73	4135903062765063397027307959659974862336914826966
75	103397576569127765554621350960108392056172432974098
77	2584939414228205068659375498399045545726053194512575
79	64623485355705227821829507689652851380624977979163041
81	1615587133892631630128648676699398918500088983590130380
83	40389678347315799386175659899126556642487220706327730340
85	1009741958682895064346445681406608133949168820493538050781
87	25243548967072377343848402464364230992616225277932125097595
89	631088724176809440374524360436484290756525665199326327860178
91	15777218104420236071822598295039799316698454842488488223670406
93	394430452610505902370792306285275779986366286193975266846360971
95	9860761315262647564564681305484605791028835257008185041541038373
97	246519032881566189162831343045812203457793789138390278743779171560
99	6162975822039154729518756397985282528121213141022235644288093575096

TABLE III. *Coefficients* $c_3^{(m)}$ *for* $m < 100$

m	coefficient
1	0
3	0
5	0
7	1
9	1228
11	165826
13	13180268
15	834687179
17	47152124264
19	2504055894564
21	128453495887560
23	6460701405171285
25	321298267540551700
27	15875718186751193446
29	781562415106660985428
31	38396599486084770569951
33	1884152729554433297404688
35	92396925087242863212482504
37	4529421792220618780953132624
39	221994390052130259394532925609
41	10879128434075642651641785959580
43	533114507941647087221696108146570
45	26123594546702044085526699031503100
47	1280082056495642083638458387900885491
49	62724702167664030806105463320712967928
51	3073528273867302640364620157246357015276
53	150603352927474237430404905305182067706008
55	7379576501562696222059774285180327454181629
57	361599566789909704272324537843571128856976484
59	17718387053299667526031545607505598124208043054
61	868201180758052308058092498645580622348221305700
63	42541863439091685468680525191321502602258565710855
65	2084551453146368331328407816966461069963353999847200
67	102143024946998883912060618070412257387835508479989136
69	5005008319149989755864557792931267995261245675448141728
71	245245410136434947947233510301682741729736833122062014801
73	12017025161122682167292682866887544425611761756718064677228
75	588834234555576505897519550727047727706594157791796053897554
77	28852877535978146700312837959228814345064795180987018772648844
79	1413791000362820909069430781859101554523460598462523339446959899
81	69275759046050999387523149899608772486006904428452001460999318920
83	3394512193982705386673372304698880348082747847203332386248895177140
85	166331097523792400504281484743484618065720631566238058487324251377000
87	8150223779143940442146143569148734403748289709497311935946014637417381
89	399360965190308824688674674124147749478467527758766757429904973600699828
91	19568687294639099050023021729003768183122137142701256787941737537946081526
93	958865677445354346075330175002928043890960370572479662512477698603070911348
95	46984418195028058438727557403259497589083457219075019327136964660397663743279
97	2302236491561635579659342935235689566050015238241056954724432903440109251142704
99	112809588086654619535744698182739734225534785161397399261557122602565598173389144

TABLE IV. *Coefficients* $c_4^{(m)}$ *for* $m < 100$

m	coefficient
1	0
3	0
5	0
7	0
9	1
11	11069
13	4494351
15	834687179
17	109645021894
19	11966116940238
21	1171517154238290
23	107266611330420090
25	9412382749388124015
27	803475280086029066515
29	67362921649153881472361
31	5581153512072331417781229
33	458814920174904775826257436
35	37524907781760654616571819884
37	3058692313447287528959880082164
39	248766472286660081843970414904068
41	20203253868959518771392559825506285
43	1639243235717722648852313037046453305
45	132923444218451509189072119405846654355
47	10774309557783598613466933578712446737055
49	873107580262755356033649662814680367103746
51	70741754576010871308624691715257326195146714
53	5731113290279152541326385472093837037802652646
55	464273113615330874205458871170525051939811187614
57	37608834197862575635540194937171017460468024168059
59	3046454243478505849181714021377418216044082870191023
61	246769872217866505286387899911965804995809092599313805
63	19988720387250704386816630085561606423919839750356146145
65	1619104708181727658393199917087006933289769137469787558520
67	131148414199504505950511524085219752842782306573102992452120
69	10623068893452175929537188378523502804346389351687829600463656
71	860470980271121894987621790622004366806344536467680361737790024
73	69698270921061772033660484086640334795499619278622518952808851481
75	5645566091314378121385665969345392830216205191730285628055448714869
77	457291164006523479765911360005829862217088143745179587927784380053079
79	37040599966067345943430184518726212084974014280130860729196966092967603
81	3000289388267519811041906205768838832567190131414282613433153417262036350
83	243023480317868437982345960763515177049403956655712430259455436590047907110
85	19684903913601306277560007277543950934946489175178004339548723280609496814010
87	1594477318047847555770749910058986247584147261867723635803045251560220136862210
89	129152667843540178355316053553179066844239567141953324310863614078628407143131591
91	10461366350718091687075961902785517739932518942731607165257860318695580126342603659
93	847370687235439948317733276455279065871339577809280686414100465206698752766218035441
95	68637026309948938219042131784988507624527449867680275757198120676999223732320791536149
97	5559599163407651504843830388458504030347226984482342866570357123093211917852775628520724
99	450327533855643143706473828529435122695590797150942787207082105644630092709645293824610308

TABLE V. *Coefficients* $c_5^{(m)}$ *for* $m < 100$

m	coefficient
1	0
3	0
5	0
7	0
9	0
11	1
13	99642
15	116294673
17	47152124264
19	11966116940238
21	2347836365864484
23	393938089395885894
25	59752013018382750024
27	8470841585571575617239
29	1146456994425541774291534
31	150221961163114696686151695
33	19239380962379456298762250416
35	2424371762015227695363084225932
37	301977301501927982712251650296648
39	37303324488483426954302995423715292
41	4580796878616173620118408244176914608
43	560128160549135541529462577248201125213
45	68283013125971106451240903324125753716898
47	8306051633303508006760284701965046518394653
49	1008798346891106069674845062004217099062126168
51	122387239143761942370134589735761526561609251514
53	14836411248162087093958930724630959446874364584940
55	1797552727028703742409838111858884788669326444960066
57	217703289290719893119610564691182352540852373464053432
59	26359003389295094137454436781749489920440891125650500371
61	3190869731746670456494579893507932023000521229249637555382
63	386216031996085211951466631639698533360911551112660503019707
65	46742324156362328453362456356889930862379745555497650995931872
67	5656678540537174782458944367931781198896985172716772125630795992
69	684530169546542728974860447157249316167333115274473396060302763152
71	82834200357474696225136733741844478645992684053740898922870062953208
73	10023445491138254021182251750947543065340812214333320206188809840618464
75	1212879385536427560757054321273270313613301699778251975024971065577768825
77	146761959534416982520417689677633682655914041238410388100972995955762393866
79	17758494114309515402592109198965559053947441378368379195237623638531140613225
81	2148802586514895133475760425426106930300995075825118454047542594577823629498056
83	260007181667927050037249417014472081102294141321935465167822720380491496091361126
85	31461041406983193820596042698912468917715834478657102613130971418104631603437046196
87	3806800370382616050708887360179398042144644597948480420894664127065652370494315515006
89	460624039877059039364345942395441035601240515541927686344786090463447209790789079021736
91	55735608208102699614264644675214285857816097210045729226935083779143331530993953793280527
93	6744016852429194007103161314193977682602690568114953471146495086918078488727901123433566238
95	816026725090505511883465047593210372947771351318924327404354903174522013771013100970746433959
97	98739290670364579346289257945746437123211955216213857203865378272839638771432232746250370157200
99	11947458893993607944703746710663034978283956573688437242242559689242698633062536818609092374021604

TABLE VI. *Coefficients* $c_6^{(m)}$ *for* $m < 100$

m	coefficient
1	0
3	0
5	0
7	0
9	0
11	0
13	1
15	896803
17	2949965020
19	2504055894564
21	1171517154238290
23	393938089395885894
25	107947764316226205276
27	25835579116799316507780
29	5632500127524872577252027
31	1149330973559307337432235521
33	223559382769795167319093086664
35	41982964485265754951017173213880
37	7681155057059283727400087851836804
39	1378203273696399945207173716059020652
41	243689884438907962985939130480387999720
43	42615064610639130927440580694098016933848
45	7390289780812377609071588455917782806706037
47	1273545261977469819494923690008367374545329695
49	218418024200718459604948339142805127847887061764
51	37324047896877960910487643121149207814565764374652
53	6360602606896893278605945623434118096367465200757462
55	1081702364611734372643785460312066828756978894878990642
57	183669851891278309656233280810239226653112605198447877348
59	31149838563756912781904165066659528225222951040497179968700
61	5278228854605774550123352041401016719352133008835241607105135
63	893780596049351319241784522715259814906894276337940189968882237
65	151271201635475662440469612337299415669962712906992565051619507408
67	25592856285671142291647651280294492609480471006230099860202835907120
69	4328719754656534647501429198211450657613220368862910063923412536979336
71	731996880797898127285428282811042222662851497763639221623287613775591768
73	123763096798438881659878649682921202794957901852515237666867938949267461520
75	20922934732566763275487153351748449256434222584686062230701467271421146836464
77	3536848642329376308011644052623327898601200704924177641517615242275568754380777
79	597836538570840686595552416713877318073658269206485137263443032246658120866637083
81	101048004705717048217731371440657166536144782259683704957991628133153823537248631660
83	17078813576205170044222924934136890198577657408166641029617322498429159439484133324436
85	2886531530551672154178089647431424714548706489734247397865577694391054459033193854999514
87	487850240234559884235982684251132764026368160362486568251919172922303911999161896248886430
89	82449977774072786731056686992049978367005256294370333489760246369097792146183262782604429868
91	13934455047850968511717613860711038124028193567211341013786388753685261751869834349371300296244
93	2354973706110503837565603249686337152881887406205964978562269449759748504067920582702519698172963
95	397996865361870931299727260335792973157928488403482416699536274433298544294234439526267359939385721
97	67262253223920482917658922105017745926465509151636274380933733317077570675046600176409618503084776216
99	11367417904945257138758662685765408094951580626074308228125792395169316136429699980523283246666186499176

TABLE VII. *Coefficients $c_7^{(m)}$ for $m < 100$*

m	coefficient
1	0
3	0
5	0
7	0
9	0
11	0
13	0
15	1
17	8071256
19	74197080276
21	128453495887560
23	107266611330420090
25	59752013018382750024
27	2583557911679931650780
29	94249795206380533005166 32
31	3051808875538951440990525939
33	906467723949073501017465886864
35	25258329864405746940357841626 9848
37	67077985737611839850488056248053296
39	1717307839195662401174213071700216 3700
41	4274452522959262726615911031357902157680
43	1040957982950195520590134594765664197681560
45	249230724309924443773931882861214431260678608
47	58883620568814372676383312320742625862669958405
49	13767561449229500950631899040013363589097919031464
51	3192702852818512869565024425110557295955449028990988
53	735626181666515796117330415214751485584281448738278968
55	168635192212400894193307900135719498693093032415655178670
57	38503473516730302272210907192978703446459191024388801732216
59	876357652610145440846582031719982193 2[illegible]75862261989065269840764
61	1989682810953764446408601448264291805771908703359490804640967720
63	450855289900571670976434320387818147058190169625675338872105769335
65	102005312144214878176315969299484950403375028478589072555741428795552
67	23050687106630409945112066224613534619112594228518858159076178524034800
69	5203952559077120915562077215897791679403082207686352885969372894469187296
71	1173978395775419021516573114563874562565977514545510929998536712510555556072
73	264687983383028415761629956595224826111314504749173468167932547377342121110112
75	59550040859679089778481316157348108115791055527611804495573003691354131170977392
77	13437946383953430167810578913737779147286660628033351674458479233652288435103414816
79	3026457753315643384723719397594941662693792859803550202426244392499022771906720960393
81	681463267084845722470307375279311919328562008684459536562621584490305778405608462991480
83	153418311365179097187656005267584539119553807900766012145951479705863350986866314565102340
85	34534653553038507472258882447456568035772737193734882155103118770602798700652546592010735656
87	7773003213075635012355961194840929382429051158913581890201716691405762760070900301960120529890
89	1749396747506108758200898576082698740052699853721355310084514755156179859094027357793940550661928
91	393696182900240731358672395901835177841473096414668552941964017751017499278364286471123581497192116
93	88595875282335566617040573834587895798073815199208236810323518771550681940781475280174915823674759096
95	19936543494119121778220366273615733721445974023943846233343178494410197345253371456544097246937171586939
97	4486151129158500862694468221656418567148034545197632361046754817772016940762897867130262547743445642702896
99	1009458362654244304897308185990562848826484147610289686243443167875482369814521517027000081375172948540901512

TABLE VIII. *Coefficients* $c_8^{(m)}$ *for* $m < 100$

m	coefficient
1	0
3	0
5	0
7	0
9	0
11	0
13	0
15	0
17	1
19	72641337
21	1859539731885
23	6460701405171285
25	9412382749388124015
27	8470841585571575617239
29	5632500127524872577252027
31	3051808875538951440990525939
33	1429953329302734392093044646982
35	602297594518030428986818986545686
37	234170438234669757816987374536542702
39	85635607007228962104291998560813839198
41	29863765165573633115609534911253825271570
43	10033617862597411302371670253845686246467650
45	3273247158340961478628421988032806047303042330
47	1043080227289567521787610710182510189123900602538
49	326218687075211486962131649800062757253999650634479
51	100497064047140010952638526349514774119194500606021303
53	30585889728496253194861269764541626286998738286104401699
55	9217757157269812697438495974693276793093928509041610145147
57	2755994085084054971511560288177634236560431577789619149271433
59	818717879094402292194138289644779511181850993577257259386003169
61	241947686684018436605939607517979943086644387725610158433744839165
63	71197948074053128108997896279872153849836130984817094096818204675845
65	20879448824133832736843393718552829008341723966538448086195782789864220
67	6106012099464862394577144219869889102156438868865210587199566775400102972
69	1781607899873454007945801076934331788461017608571776392927390884116330358956
71	518881754326771290602459758556339397309432422867555402555907512973388727956492
73	150895976111580246356909001162888264027760218873479742837062672337239599059305076
75	43829125679335574630014061598741241705681559040846683402506806117988329717912897108
77	12718143344655483038960398902652372104963548878706419075147444211203629858658568709444
79	3687580399440309646452396939916563651788852920538027068872359679905044099193394675075300
81	1068518368642176972410252049481446305467258643681877587235088493102869014413610795924907245
83	309455779372809954525810807422583651413455463671352077961638314215420908240253881628745922245
85	89584853682051973750781575641139388249982496950338145267136724075822095367379121657459157412585
87	25925372371968293109315257995161364187544758306258873084996217708226887115091155776423760830224241
89	7500637636043273733423617159224562849183066230261071976477174317849387389981368577434798322407085683
91	2169586855785888483164412771637468998478739494070751150310047101376068987892205438596119320641602505531
93	627451363825143887208259403623391293745650383661166822121788605115905531076817728621855381324459972211663
95	181435465792330460742305067561772920368669590169914589288166794997115711144003686619871388697884801145178919
97	52458445135265793453017642441049708151847889168227518761926510780204460499582031992548139076063604494164148610
99	1516594367131594496331164251528862269101150984305054849839348998217990222683195677566391419680356908289344488 1266

TABLE IX. *Coefficients* $c_9^{(m)}$ *for* $m < 100$

m	coefficient
1	0
3	0
5	0
7	0
9	0
11	0
13	0
15	0
17	0
19	1
21	653772070
23	46535238000235
25	321298267540551700
27	8034752800860290665 15
29	1146456994425541774291534
31	11493309735593073374322355 21
33	906467723949073501017465886864
35	6022975945180304289868189865456 86
37	352513571679334580855533139395470836
39	1873772214728107703459201092074172750 58
41	924466950582857163919586524293410869902 80
43	4301728954342187295209774847272630018478809 0
45	19106900875568186798772740152583987267862674420
47	8175785866750908617220315276737911020940067654750
49	3394443742229202076972991369333038170312239508202448
51	1375165137228358124023201671353179440007859890273083647
53	546053909698878831582976562716106830864331257673556106058
55	213291493516574196216927646501097703637962881450506799942037
57	82192535700849366227925196143395891439771286878392549122793532
59	31321288108127815961032046749042011706989536048087953826158015605
61	11825924781466959535228923103018912892372814651330160149342919376850
63	4431065508349411985491242913005746262319438047710276794892122404745815
65	1649781485841549739501476489229396267954694080274853477764931082117486560
67	611024480335889602721775982599002406681118207314000704018184452116542688380
69	225316314254968054865417088522063651171213116533303651168272630477760941174792
71	82784702233053781275160842650875043585123483747817692207855120473185823466339028
73	30324797478238765209922002575502352328558469864786594193386015369467877008921778832
75	1108046251203334704727736984619486244895301585694472013614461,18423341036593782013082 28
77	4040307643891266321437478076665653666401989370218025084888111516325049790196032893030088
79	1470689275444244177748380192123538888205944643433834822603935868962988529116195114882190892
81	534570452448582442611587373969888321954743514008914846230801486202742676616731488407779770080
83	194076363182402156073518375844777936815475806822459658818503818291336485088410450462178409726285
85	70390213743516854317613298679314489812205941326996712203304525603526587187828028650287910787840270
87	25509224672988378902709584206366072185494123026751342353426250197671347519960719166480715622226993455
89	9238225626401614391815021149194951238598761165568108829001582046259687849999655360290241617651442950116
91	3343773619132305859652507097485008626438348774626779154936743378177575312221402661294039001600333722214439
93	1209718230573904970223005105193880833427230497305116557363767606405973247937942311054275079033424049130741686
95	437487385008093980646813139359026907713666574848456999506472423677406591652582919474255236342253843407544577309
97	158164758962165862304010383745304090426147964863950455996281834162876534266738139382364754949635592036993088843504
99	57166389064951599460003801395731460205260328195142527763724790081330994336136063189414296492075926155965556754239794

TABLE X. *Coefficients* $c_{10}^{(m)}$ *for* $m < 100$

m	coefficient
1	0
3	0
5	0
7	0
9	0
11	0
13	0
15	0
17	0
19	0
21	1
23	5883948671
25	1163848723925346
27	15875718186751193446
29	67362921649153881472361
31	150221961163114696686151695
33	223559382769795167319093086664
35	252583298644057469403578416269848
37	234170438234669757816987374536542702
39	187377221472810770345920109207417275058
41	133969576487544409027917496382111749031668
43	87728918732198931944006930116683301862240028
45	53584249785424102912573513119405869087185148918
47	30949875721911960369001953290125975409197457599738
49	17084515829874885039358166642716434050375758695548328
51	9087875953293371893352181289662329870352667885606878776
53	4689074024466085396655627199412197301179538368431695410679
55	2359177828374264864084824823488500441802841116030800861505577
57	1162335047039741530533510629264494262807427254934441601999711110
59	562738416082891966791964988845459259248514962583484118577368116882
61	268487657994393236547167649973684555787738305032866427629975964445911
63	126533947224431192515987708414774764615334531615284970658658273740739921
65	59020556364450350858181375348753058153264670745913963111345919072285074416
67	27291046642529386411271348993591196065265821085545701783246397903489368784976
69	12527057617385541872341205493096509966115351953660771952999340629372827905414476
71	5714600834892969456349199299674441560869687975107160155056442841258748718271466868
73	2593269353455404859238644702786310316617311564371261953264362999487060340551499730920
75	1171616579873846099029933960534824740949005200202911177093501674284480814806715501376312
77	527346325223948287098669907706243617651561148384585574373439367569257326387543835401150524
79	236607920413936122873665792585474025141845935384099531163913550385478247823546550367153732772
81	105875956865232704136900542371917797103129270379770261821381844895922746789688097540367864023280
83	47269322235299636195939325564321147822624879141641097722030644779005710185131453453249055707397200
85	2106333554107190118538873805947195718497235292775814595537391750088214976715890560303161289560731165
87	9370639764819802274706778260699723806937047993396636344359889647199963631289884526161953308626625689635
89	4163078575268038739792813428265922499044565094636378087461596631309524564398438036214545073448668689809130
91	18473770733940130428856901766291576410947582496852652828421222196256962331792154634222837671665296365252702846
93	818974297380411779552589794921503464260888977905293844966166378669164297125468053169628680507434882641917239637
95	362764699341676668168545091459233686492843699086368676758364547897482257729361406111207809313619433349116482684803
97	160574244306631000607415942696727895509178928498946929016228878184507679034091199679372632505131612037842780921706744
99	71034668912082503286817495795978819257473949847700059344239638542311898617948727912243913685114430680197272550292351528

TABLE XI. *Some expressions for the coefficients $c_j^{(m)}$*

$$4^0\, c_0^{(m)} = 1$$

$$4^2\, c_1^{(m)} = 3^m - (4m - 1)$$

$$4^4\, c_2^{(m)} = 5^m - (4m - 8)3^m + (8m^2 - 32m + 7)$$

$$4^6\, c_3^{(m)} = 7^m - (4m - 16)5^m + (8m^2 - 60m + 82)3^m - (\tfrac{32}{3}m^3 - 120m^2 + \tfrac{1000}{3}m - 67)$$

$$4^8\, c_4^{(m)} = 9^m - (4m - 24)7^m + (8m^2 - 92m + 230)5^m - (\tfrac{32}{3}m^3 - 176m^2 + \tfrac{2488}{3}m - 945)3^m + (\tfrac{32}{3}m^4 - \tfrac{704}{3}m^3 + \tfrac{5008}{3}m^2 - \tfrac{11716}{3}m + 738)$$

$$4^{10} c_5^{(m)} = 11^m - (4m - 32)9^m + (8m^2 - 124m + 442)7^m - (\tfrac{32}{3}m^3 - 240m^2 + \tfrac{4936}{3}m - 3264)5^m + (\tfrac{32}{3}m^4 - \tfrac{928}{3}m^3 + \tfrac{9160}{3}m^2 - \tfrac{34376}{3}m + 11661)3^m - (\tfrac{128}{15}m^5 - \tfrac{928}{3}m^4 + \tfrac{12256}{3}m^3 - \tfrac{69728}{3}m^2 + \tfrac{733832}{15}m - 8808)$$

$$4^{12} c_6^{(m)} = 13^m - (4m - 40)11^m + (8m^2 - 156m + 718)9^m - (\tfrac{32}{3}m^3 - 304m^2 + \tfrac{8152}{3}m - 7440)7^m + (\tfrac{32}{3}m^4 - \tfrac{1184}{3}m^3 + \tfrac{15400}{3}m^2 - \tfrac{81292}{3}m + 46519)5^m - (\tfrac{128}{15}m^5 - 384m^4 + \tfrac{19360}{3}m^3 - 49176m^2 + \tfrac{2402192}{15}m - 150486)3^m + (\tfrac{256}{45}m^6 - \tfrac{1536}{5}m^5 + \tfrac{58208}{9}m^4 - \tfrac{198304}{3}m^3 + \tfrac{14730424}{45}m^2 - \tfrac{9599332}{15}m + 110728)$$

$$4^{14} c_7^{(m)} = 15^m - (4m - 48)13^m + (8m^2 - 188m + 1058)11^m - (\tfrac{32}{3}m^3 - 368m^2 + \tfrac{12136}{3}m - 13984)9^m + (\tfrac{32}{3}m^4 - 480m^3 + \tfrac{23176}{3}m^2 - 51956m + 120337)7^m - (\tfrac{128}{15}m^5 - \tfrac{1408}{3}m^4 + 9824m^3 - \tfrac{288944}{3}m^2 + \tfrac{6463012}{15}m - 668241)5^m + (\tfrac{256}{45}m^6 - \tfrac{5504}{15}m^5 + \tfrac{84224}{9}m^4 - \tfrac{356320}{3}m^3 + \tfrac{34587544}{45}m^2 - \tfrac{33993476}{15}m + 2004451)3^m - (\tfrac{1024}{315}m^7 - \tfrac{11008}{45}m^6 + \tfrac{337408}{45}m^5 - \tfrac{1074656}{9}m^4 + \tfrac{46720768}{45}m^3 - \tfrac{209978872}{45}m^2 + \tfrac{906028148}{105}m - 1443574)$$

TABLE XI. *Some expressions for the coefficients* $c_j^{(m)}$ (*continued*)

$$
\begin{aligned}
4^{16}c_8^{(m)} = {} & 17^m - (4m - 56)15^m + (8m^2 - 220m + 1462)13^m - \\
& (\tfrac{32}{3}m^3 - 432m^2 + \tfrac{16888}{3}m - 23408)11^m + \\
& (\tfrac{32}{3}m^4 - \tfrac{1696}{3}m^3 + \tfrac{32488}{3}m^2 - \tfrac{264236}{3}m + 253307)9^m - \\
& (\tfrac{128}{15}m^5 - \tfrac{1664}{3}m^4 + \tfrac{41632}{3}m^3 - \tfrac{496432}{3}m^2 + \tfrac{13907612}{15}m - 1907640)7^m + \\
& (\tfrac{256}{45}m^6 - \tfrac{2176}{5}m^5 + \tfrac{119936}{9}m^4 - \tfrac{620864}{3}m^3 + \tfrac{76174744}{45}m^2 - \\
& \tfrac{101048912}{15}m + 9677802)5^m - \\
& (\tfrac{1024}{315}m^7 - \tfrac{2560}{9}m^6 + \tfrac{460288}{45}m^5 - \tfrac{1744480}{9}m^4 + \tfrac{92504128}{45}m^3 - \\
& \tfrac{106684168}{9}m^2 + \tfrac{3405419728}{105}m - 27334691)3^m + \\
& (\tfrac{512}{315}m^8 - \tfrac{10240}{63}m^7 + \tfrac{20480}{3}m^6 - \tfrac{7004288}{45}m^5 + \tfrac{31106848}{15}m^4 - \\
& \tfrac{144814528}{9}m^3 + \tfrac{4235963624}{63}m^2 - \tfrac{12493128428}{105}m + 19333223)
\end{aligned}
$$

$$
\begin{aligned}
4^{18}c_9^{(m)} = {} & 19^m - (4m - 64)17^m + (8m^2 - 252m + 1930)15^m - \\
& (\tfrac{32}{3}m^3 - 496m^2 + \tfrac{22408}{3}m - 36224)13^m + \\
& (\tfrac{32}{3}m^4 - \tfrac{1952}{3}m^3 + \tfrac{43336}{3}m^2 - \tfrac{412540}{3}m + 469749)11^m - \\
& (\tfrac{128}{15}m^5 - 640m^4 + \tfrac{55840}{3}m^3 - 260752m^2 + \tfrac{26162132}{15}m - 4398784)9^m + \\
& (\tfrac{256}{45}m^6 - \tfrac{7552}{15}m^5 + \tfrac{161792}{9}m^4 - 329280m^3 + \tfrac{145470664}{45}m^2 - \\
& \tfrac{238528748}{15}m + 29914578)7^m - \\
& (\tfrac{1024}{315}m^7 - \tfrac{14848}{45}m^6 + \tfrac{124928}{9}m^5 - \tfrac{2803712}{9}m^4 + \tfrac{179442208}{45}m^3 - \\
& \tfrac{1290056152}{45}m^2 + \tfrac{2193088532}{21}m - 141218826)5^m + \\
& (\tfrac{512}{315}m^8 \ \tfrac{19456}{105}m^7 + \tfrac{133888}{15}m^6 - 235520m^5 + \tfrac{55247168}{15}m^4 - \\
& \tfrac{171886624}{5}m^3 + \tfrac{57240313192}{315}m^2 - \tfrac{9842673160}{21}m + 379604650)3^m - \\
& (\tfrac{2048}{2835}m^9 - \tfrac{9728}{105}m^8 + \tfrac{689152}{135}m^7 - \tfrac{2361856}{15}m^6 + \tfrac{400323968}{135}m^5 - \\
& \tfrac{522074336}{15}m^4 + \tfrac{702603488128}{2835}m^3 - \tfrac{102679699768}{105}m^2 + \tfrac{75111559808}{45}m - \\
& 264337010)
\end{aligned}
$$

TABLE XI. *Some expressions for the coefficients* $c_j^{(m)}$ (*continued*)

$$
\begin{aligned}
4^{20}c_{10}^{(m)} = {} & 21^m - (4m - 72)19^m + (8m^2 - 284m + 2462)17^m - \\
& (\tfrac{32}{3}m^3 - 560m^2 + \tfrac{28696}{3}m - 52944)15^m + \\
& (\tfrac{32}{3}m^4 - 736m^3 + \tfrac{55720}{3}m^2 - 202308m + 798079)13^m - \\
& (\tfrac{128}{15}m^5 - \tfrac{2176}{3}m^4 + 24032m^3 - \tfrac{1158704}{3}m^2 + \tfrac{44900812}{15}m - \\
& 8877288)11^m + \\
& (\tfrac{256}{45}m^6 - \tfrac{8576}{15}m^5 + \tfrac{209792}{9}m^4 - 491200m^3 + \tfrac{252319864}{45}m^2 - \\
& \tfrac{488845444}{15}m + 74380386)9^m - \\
& (\tfrac{1024}{315}m^7 - \tfrac{5632}{15}m^6 + \tfrac{813568}{45}m^5 - \tfrac{1404416}{3}m^4 + \tfrac{314705248}{45}m^3 - \\
& \tfrac{895704688}{15}m^2 + \tfrac{27937189928}{105}m - 466277137)7^m + \\
& (\tfrac{512}{315}m^8 - \tfrac{13312}{63}m^7 + \tfrac{525568}{45}m^6 - \tfrac{16048256}{45}m^5 + \tfrac{294010624}{45}m^4 - \\
& \tfrac{655243360}{9}m^3 + \tfrac{149485794464}{315}m^2 - \tfrac{169259066656}{105}m + 2074575409)5^m - \\
& (\tfrac{2048}{2835}m^9 - \tfrac{32768}{315}m^8 + \tfrac{1221632}{189}m^7 - \tfrac{10179328}{45}m^6 + \tfrac{658423424}{135}m^5 - \\
& \tfrac{2990989696}{45}m^4 + \tfrac{319276747232}{567}m^3 - \tfrac{875326272344}{315}m^2 + \\
& \tfrac{2151552753452}{315}m - 5348556339)3^m + \\
& (\tfrac{4096}{14175}m^{10} - \tfrac{131072}{2835}m^9 + \tfrac{3056128}{945}m^8 - \tfrac{122420224}{945}m^7 + \tfrac{2205426688}{675}m^6 - \\
& \tfrac{7244229248}{135}m^5 + \tfrac{1621995503776}{2835}m^4 - \tfrac{10792995524608}{2835}m^3 + \\
& \tfrac{22579172944784}{1575}m^2 - \tfrac{1495484636944}{63}m + 3674007443)
\end{aligned}
$$

TABLE XII. *The polynomials* $P_p(n; j)$

j	$P_1(n;j)$	$P_2(n;j)$	$P_3(n;j)$
1	$8n + 3$		
2	$8n - 4$	$64n^2 - 64n - 34$	
3	$8n - 12$	$64n^2 - 176n + 60$	$512n^3 - 2112n^2 + 1504n + 942$
4	$8n - 20$	$64n^2 - 304n + 292$	$512n^3 - 3456n^2 + 6112n - 1686$
5	$8n - 28$	$64n^2 - 432n + 652$	$512n^3 - 4992n^2 + 14368n - 11088$
6	$8n - 36$	$64n^2 - 560n + 1140$	$512n^3 - 6528n^2 + 25696n - 30096$
7	$8n - 44$	$64n^2 - 688n + 1756$	$512n^3 - 8064n^2 + 40096n - 61776$
8	$8n - 52$	$64n^2 - 816n + 2500$	$512n^3 - 9600n^2 + 57568n - 109200$
9	$8n - 60$	$64n^2 - 944n + 3372$	$512n^3 - 11136n^2 + 78112n - 175440$
10	$8n - 68$	$64n^2 - 1072n + 4372$	$512n^3 - 12672n^2 + 101728n - 263568$

j	$P_4(n;j)$
4	$4096n^4 - 36864n^3 + 98816n^2 - 58944n - 41328$
5	$4096n^4 - 51200n^3 + 210176n^2 - 299392n + 70968$
6	$4096n^4 - 67584n^3 + 385280n^2 - 862656n + 580104$
7	$4096n^4 - 83968n^3 + 609536n^2 - 1819328n + 1815288$
8	$4096n^4 - 100352n^3 + 882944n^2 - 3267520n + 4212072$
9	$4096n^4 - 116736n^3 + 1205504n^2 - 5305536n + 8304984$
10	$4096n^4 - 133120n^3 + 1577216n^2 - 8031680n + 14726856$

j	$P_5(n;j)$
5	$32768n^5 - 512000n^4 + 2816000n^3 - 6123520n^2 + 3239552n + 2478720$
6	$32768n^5 - 655360n^4 + 4802560n^3 - 15376640n^2 + 19118592n - 4012560$
7	$32768n^5 - 819200n^4 + 7710720n^3 - 33395200n^2 + 63810112n - 38919000$
8	$32768n^5 - 983040n^4 + 11274240n^3 - 61002240n^2 + 152562112n - 135913440$
9	$32768n^5 - 1146880n^4 + 15493120n^3 - 100160000n^2 + 306230592n - 347689440$
10	$32768n^5 - 1310720n^4 + 20367360n^3 - 152834560n^2 + 549637312n - 749618400$

TABLE XII. *The polynomials* $P_p(n; j)$ (*continued*)

j	$P_6(n;j)$
6	$262144n^6 - 6291456n^5 + 57794560n^4 - 248770560n^3 + 474789376n^2 - 229256064n - 188510400$
7	$262144n^6 - 7667712n^5 + 87654400n^4 - 489000960n^3 + 1338789376n^2 - 1511561088n + 285879600$
8	$262144n^6 - 9240576n^5 + 129433600n^4 - 909434880n^3 + 3305084416n^2 - 5645881344n + 3196743840$
9	$262144n^6 - 10813440n^5 + 179077120n^4 - 1510809600n^3 + 6761529856n^2 - 14911155840n + 12192150240$
10	$262144n^6 - 12386304n^5 + 236584960n^4 - 2324520960n^3 + 12291083776n^2 - 32772475776n + 33789126240$

j	$P_7(n;j)$
7	$2097152n^7 - 71565312n^6 + 983564288n^5 - 6892462080n^4 + 25434472448n^3 - 44282208768n^2 + 19853396352n + 17363596320$
8	$2097152n^7 - 84410368n^6 + 1385431040n^5 - 11841249280n^4 + 55522009088n^3 - 136115348992n^2 + 142793809920n - 24616141200$
9	$2097152n^7 - 99090432n^6 + 1930428416n^5 - 19914424320n^4 + 115872985088n^3 - 371756732928n^2 + 583441743744n - 311292162720$
10	$2097152n^7 - 113770496n^6 + 2563506176n^5 - 30906122240n^4 + 213465878528n^3 - 833956680704n^2 + 1671638425344n - 1277031616560$

j	$P_8(n;j)$
8	$16777216n^8 - 771751936n^7 + 14797504512n^6 - 152265293824n^5 + 898239627264n^4 - 2974298865664n^3 + 4830541058048n^2 - 2033894716416n - 1877990002560$
9	$16777216n^8 - 889192448n^7 + 19803406336n^6 - 239681404928n^5 + 1698640297984n^4 - 7040782794752n^3 + 15879147393024n^2 - 15708098414592n + 2487754080000$
10	$16777216n^8 - 1023410176n^7 + 26438795264n^6 - 375328866304n^5 + 3172917837824n^4 - 16131399122944n^3 + 47085624918016n^2 - 69097969483776n + 35099607070080$

TABLE XII. *The polynomials $P_p(n; j)$ (continued)*

j	$P_9(n;j)$
9	$134217728n^9 - 8002732032n^8 + 203893506048n^7 - 2885777620992n^6 + 24649416572928n^5 - 128707575349248n^4 + 392752637673472n^3 - 603599318765568n^2 + 240614820163584n + 233239937736960$
10	$134217728n^9 - 9059696640n^8 + 262781534208n^7 - 4268947931136n^6 + 42384027549696n^5 - 262750997053440n^4 + 991971647782912n^3 - 2094260870836224n^2 + 1973487689726976n - 288763376630400$

j	$P_{10}(n;j)$
10	$1073741824n^{10} - 80530636800n^9 + 2629828608000n^8 - 48911792209920n^7 + 568348190441472n^6 - 4250313314795520n^5 + 20235563712512000n^4 - 57890789535252480n^3 + 85021138163810304n^2 - 32282512864174080n - 32707469473862400$