

domain of applicability of the techniques. The programs of this type might well be implemented on standard computers as well as on pocket calculators.

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1. P. HENRICI, *Elements of Numerical Analysis*, Wiley, New York, 1964.
2. P. HENRICI, *Applied and Computational Complex Analysis*, Vol. 1, Wiley, New York, 1974.
3. P. HENRICI, *Applied and Computational Complex Analysis*, Vol. 2, Wiley, New York, 1977.

**16[5.10.3].**—R. GLOWINSKI, E. Y. RODIN & O. C. ZIENKIEWICZ (Editors), *Energy Methods in Finite Element Analysis*, Wiley, New York, 1979, xviii + 361 pp., 23½ cm. Price \$43.95.

This volume is dedicated to Professor Fraeijs de Veubeke. A short biography of Professor Fraeijs de Veubeke is given at the beginning and a list of his main publications at the end.

The book contains nineteen chapters with thirty-three authors. The following description of its contents is taken from the editors' preface.

"Chapter 1, by J. T. Oden, gives the mathematical foundations of variational mechanics. It describes the various formulations existing for a given problem with a detailed discussion of the classical variational principles, the dual principles and their applications to elastostatics.

In Chapter 2, by P. G. Ciarlet and P. Destuynder, it is proved *without a priori assumptions*, that the classical two-dimensional linear models in elastic plate theory are indeed limits of the standard three-dimensional models of linear elasticity. This result is proved using variational formulations of the elastic problems and singular perturbation methods.

In Chapter 3, A. Samuelsson introduces the concept of 'global constant strain condition' to study non-conforming finite elements and shows its relationship to the well-known 'patch test'.

In Chapter 4, G. B. Warburton gives a survey of the recent developments in structural dynamics computational methods via finite elements. Modal methods and numerical integration methods are described with their main properties, and their use is discussed with many details.

In Chapter 5, by O. C. Zienkiewicz, D. W. Kelly, and P. Bettess, one studies how standard finite element methods and boundary integral methods can be coupled in order to solve, for example, boundary value problems on unbounded domains; various examples from fluid mechanics, electrotechnics, etc., illustrate the possibilities of this new class of methods.

The chapters from 6 to 11 are all related to complementary energy methods and dual variational principles applied to finite element approximations.

In Chapter 6, D. J. Allman discusses the use of compatible and equilibrium models and finite elements, applied to the stretchings of elastic plates. A new triangular equilibrium element is introduced and the properties of the associated matrix is studied in detail.

In Chapter 7, L. S. D. Morley, starting from Koiter's first approximation shell theory, develops an approximation of elastic shell problems, based on a new finite element stiffness formulation.

In Chapter 8, R. L. Taylor and O. C. Zienkiewicz show that the computational difficulties associated with complementary energy methods can be overcome by an appropriate penalty technique. Numerical examples illustrate the feasibility of the method. The two following chapters are more mathematical in nature.

In Chapter 9, P. A. Raviart and J. M. Thomas give the theoretical foundations of the dual finite element models for second order linear elliptic problems.

In Chapter 10, F. Brezzi does the same for fourth order linear elliptic problems taking the biharmonic plate bending problem to illustrate the various possible approaches.

Chapter 11, by C. Johnson and B. Mercier, is dedicated to a class of mixed equilibrium finite element methods. Applications to problems in linear elasticity, plasticity, and fluid dynamics (Navier-Stokes equations) are given. The next two chapters deal with incompressible media.

Chapter 12, by J. H. Argyris and P. C. Dunne describes a new finite element approximation for incompressible or near-incompressible materials. The method is based on displacement models and allows the use of low order elements.

Chapter 13, by R. Glowinski and O. Pironneau, discusses the approximation of the Stokes problem for incompressible fluids, by means of low order conforming elements to approximate velocity and pressure. This method is based on a new variational formulation of the Stokes problem and leads to approximate problems which can be easily solved by standard finite element Poisson solvers. Chapters 14 to 18 are related to nonlinear problems.

In Chapter 14, P. G. Bergan, I. Holand, and T. H. Soreide present a new incremental method for solving problems in nonlinear finite element analysis. The method is based on the use of a 'current stiffness parameter' which is a normalized measure of the incremental stiffness in the direction of motion. Several numerical tests illustrate the feasibility of the method.

In Chapter 15, S. Cescotto, F. Frey, and G. Fonder give a unified approach for Lagrangian description in nonlinear, large displacement, and large strain structural problems. Total and updated Lagrangian descriptions appear as particular cases of this more general theory.

In Chapter 16, B. M. Irons analyzes some of the difficulties arising from curve fitting and shows how nonlinear effects can make difficult the numerical solution of problems apparently easy to solve.

In Chapter 17, H. Matthies, G. Strang, and E. Christiansen analyze from a mathematical point of view a fairly difficult infinite dimensional saddle-point problem, which describes the duality between the static and kinematic theories of limit analysis. The theoretical difficulty lies in the fact that the natural functional spaces to study this class of problems, of great interest in perfect plasticity, are  $L_1$  and spaces of functions of bounded variation. They introduce the space of functions of bounded deformation, required because Korn's inequality fails in  $L_1$ .

In Chapter 18, A. R. S. Ponter and P. Brown discuss a new finite element method for computing the deformation of creeping structures. The finite element

formulation is discussed in detail for a strain-hardening creep relationship and computed solutions are presented for a thermally loaded plate.

The last chapter by M. Geradin concerns modal analysis. The biorthogonal Lanczos algorithm is proposed as a very efficient tool to compute the upper eigenvalue spectrum of an arbitrary square matrix."

**17[4.00, 5.00].**—M. K. JAIN, *Numerical Solution of Differential Equations*, Wiley, New York, 1979, xiii + 443pp., 25cm. Price \$16.95.

This book attempts the impossible: It is concerned with the numerical solution of ordinary as well as partial differential equations, with the associated initial value as well as boundary value problems (even eigenvalue problems are not omitted), and it assumes virtually no knowledge of the theory of differential equations or of numerical analysis. Consequently, it has to cover many pages with introductory material but cannot afford to do it in a rigorous or even seriously intuitive form for sake of brevity. There are a good number of farther reaching results; but they are often unmotivated or their context is not clear. A good deal of important material is missing and the present state of the art is not well represented. Computational aspects (e.g. step size control) are virtually unreflected.

Even as a gateway to the more specialized literature on the subject the volume is not suitable since the references are unsystematic and largely outdated. Under these circumstances, I cannot imagine for whom the book could be of any value.

H. J. S.

**18[3.35].**—F. J. PETERS, *Sparse Matrices and Substructures with a Novel Implementation of Finite Element Algorithms*, Mathematical Centre Tracts 119, Mathematisch Centrum, Amsterdam, 1980, ii + 98pp., 24cm. Price Dfl. 12,—.

This short book, a revision of the author's Ph.D. thesis, takes a fresh look at the problem of solving sparse linear systems. The main result is that the finite element technique of factoring a sparse matrix as it is generated, building up in the process larger and larger factored substructures, is applicable to any sparse matrix problem. Moreover, the resulting code needs only to manipulate dense submatrices (or submatrices with a dense profile).

Unfortunately, the book is sprinkled with the claim that this new viewpoint will make all previous work on sparse equation solvers obsolete, a thesis not proved by the contents. There is no serious attempt, either analytical or experimental, to gauge the efficiency or simplicity of the methods in comparison with standard software. Only a very sketchy outline of a program is provided.

The reader wanting a thorough treatment of sparse matrix methods should look elsewhere—to George and Liu's new book, for example.

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