

Evaluation of the Integral $\int_0^p u^n e^{-u^2} (u + x)^{-1} du$

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Abstract. The representations of the above integral, in a power series form for small values of x and in an asymptotic form for large values of x , are given for integer values of n . In view of the usefulness of this integral, tabulated values are also presented for a wide range of values of x and p , and for a few values of n .

1. Introduction. Integrals involving exponential functions occur frequently in several problems in kinetic theory and generally in mathematical physics (cf. [1]–[5], [7]). An integral of the form $\int_0^\infty e^{-u^2} (u + x)^{-1} du$ has been discussed in [2], [3], [4], [7], and a particular integral of the form $\int_0^p e^{-u^2} (u + x)^{-1} du$ has been evaluated in [5]. This last integral is generalized and its representation, both in a power series form for small values of x and in an asymptotic form for large values of x , are given for integral values of n . In view of the usefulness of this generalized integral, tabulated values are also presented for a wide range of values of x and p , and for a few values of n .

The integral

$$(1) \quad f_n(x, p) = \int_0^p u^n e^{-u^2} (u + x)^{-1} du$$

is considered, and its analytic representations are given for small as well as for large values of x . For small values of x , we can write

$$(2) \quad \begin{aligned} f_n(x, p) &= \int_0^p [u^{n-1} - xu^{n-2} + x^2 u^{n-3} \\ &\quad + \cdots + (-1)^r x^r u^{n-r-1} + \cdots + (-1)^{n-1} x^{n-1}] e^{-u^2} du \\ &\quad + (-1)^n x^n \int_0^p e^{-u^2} (u + x)^{-1} du. \end{aligned}$$

All the integrals except the last one in (2) can be expressed in terms of the incomplete gamma functions, and the last one can be evaluated as follows.

If we write

$$f_0(x, p) = \int_0^p e^{-u^2} (u + x)^{-1} du$$

and set $y = f_0 + e^{-x^2} \ln x$, then it can be seen that y satisfies the differential equation

$$(3) \quad \frac{dy}{dx} + 2xy = \sqrt{\pi} \operatorname{erf} p + \frac{e^{-p^2}}{p+x} - \frac{1}{x}(1 - e^{-x^2}),$$

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where

$$\operatorname{erf} p = \frac{2}{\sqrt{\pi}} \int_0^p e^{-u^2} du.$$

Assuming a series solution of the form

$$(4) \quad y = \sum_{k=0}^{\infty} a_k x^k$$

for the differential equation (3), we obtain the coefficients a_k . They are given below.

$$(5) \quad a_0 = \ln p - \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1} p^{2n}}{n \cdot n!},$$

$$(6) \quad a_1 = 2 \int_0^p e^{-u^2} du + e^{-p^2}/p,$$

$$(7) \quad a_{2k+2} = \frac{a_{2k}}{k+1} - \frac{e^{-p^2}}{2(k+1)p^{2k+2}} - \frac{(-1)^k}{2(k+1)[(k+1)!]}, \quad k = 0, 1, 2, \dots,$$

$$(8) \quad a_{2k+1} = \frac{-2a_{2k-1}}{2k+1} + \frac{e^{-p^2}}{(2k+1)p^{2k+1}}, \quad k = 1, 2, \dots.$$

The coefficients a_{2k+1} and a_{2k} may also be expressed as follows:

$$(9) \quad a_{2k+1} = \frac{(-1)^k a_1}{(\frac{3}{2})_k} - \frac{(-1)^k e^{-p^2}}{(\frac{3}{2})_k} \sum_{r=1}^k \frac{(-1)^{r+1} (\frac{1}{2})_r}{p^{2r+1}}, \quad k = 1, 2, \dots,$$

$$(10) \quad a_{2k} = \frac{(-1)^k a_0}{k!} + \frac{(-1)^k e^{-p^2}}{2k!} \sum_{r=0}^{k-1} \frac{(-1)^r r!}{p^{2r+0}} + \frac{(-1)^k}{2k!} \sum_{r=0}^{k-1} \frac{1}{r+1}, \quad k = 1, 2, \dots,$$

where $(a)_k = \Gamma(k+a)/\Gamma(a)$. But for machine computations, it is more economical to use (7) and (8), rather than (9) and (10).

Thus $f_n(x, p)$ is given by

$$(11) \quad \begin{aligned} f_n(x, p) &= \int_0^p u^n e^{-u^2} (u+x)^{-1} du \\ &= \frac{1}{2} \gamma\left(\frac{n}{2}, p^2\right) - \frac{x}{2} \gamma\left(\frac{n}{2} - \frac{1}{2}, p^2\right) + \frac{x^2}{2} \gamma\left(\frac{n}{2} - 1, p^2\right) \\ &\quad + \cdots + (-1)^r \frac{x^r}{2} \gamma\left(\frac{n}{2} - \frac{r}{2}, p^2\right) + \cdots + (-1)^{n-1} \frac{x^{n-1}}{2} \gamma\left(\frac{1}{2}, p^2\right) \\ &\quad + (-1)^n x^n \left[\sum a_k x^k - e^{-x^2} \ln x \right], \end{aligned}$$

where a_k 's are given by (5), (6), (7), and (8); and $\gamma(a, x)$ is the incomplete gamma function given by

$$(12) \quad \gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt.$$

2. An Alternative Approach. Since $1/(u + x) = (1 - x/(x + u))/u$, it follows that

$$(13) \quad f_n(x, p) = C_{n-1}(p) - xf_{n-1}(x, p),$$

where

$$(14) \quad C_n(p) = \int_0^p u^n e^{-u^2} du.$$

Clearly,

$$(15) \quad C_0(p) = \int_0^p e^{-u^2} du = \frac{\sqrt{\pi}}{2} \operatorname{erf}(p),$$

$$(16) \quad C_1(p) = \frac{1}{2}(1 - e^{-p^2}).$$

Further,

$$(17) \quad C_{n+2}(p) = -\frac{1}{2}p^{n+1}e^{-p^2} + \frac{1}{2}(n+1)C_n(p).$$

So

$$(18) \quad C_{2n} = -\left(\frac{1}{2}\right)_n p e^{-p^2} \sum_{k=0}^{n-1} \frac{p^{2k}}{\left(\frac{3}{2}\right)_k} + \left(\frac{1}{2}\right)_n C_0,$$

and

$$(19) \quad C_{2n+1} = -\frac{1}{2}n!p^2 e^{-p^2} \sum_{k=0}^{n-1} \frac{p^{2k}}{(k+1)!} + n!C_1.$$

In most cases it is easier to compute C_n by the recursion relation (17) than it is to use (18) and (19), and the same is true for f_n using (13). Of course, if n is large, stability problems can arise. If the C_n are known, use of (13) in the forward (backward) directions is stable as n becomes large if $x < 1$ ($x > 1$). On the other hand, use of (17) is not stable in the forward direction as n becomes large. In (14), we use the exponential series to get (in hypergeometric notation)

$$(20) \quad C_n(p) = \frac{p^{n+1}}{n+1} {}_1F_1\left(\begin{array}{c} \frac{n+1}{2} \\ \frac{n+1}{2} + 1 \end{array} \middle| -p^2\right) = \frac{p^{n+1}e^{-p^2}}{n+1} {}_1F_1\left(\begin{array}{c} 1 \\ \frac{n+1}{2} + 1 \end{array} \middle| p^2\right),$$

which is convenient for p fixed and n large.

Combining (13) and (17), we have

$$(21) \quad xf_n + f_{n+1} - \frac{2xf_{n+2}}{n+1} - \frac{2f_{n+3}}{n+1} = \frac{p^{n+1}e^{-p^2}}{n+1}.$$

This is advantageous since the need of C_n is bypassed. On the other hand, there are obvious stability and round-off difficulties. Still the expression is quite useful for moderate n and x provided sufficient round-off controls can be initiated.

Repeated use of (13) gives

$$(22) \quad f_n = \sum_{k=0}^{n-1} (-1)^k x^k C_{n-k-1} + (-1)^n x^n f_0,$$

which is the same as Eq. (11) given above.

For large values of x and for $p < x$

$$(23) \quad f_n(x, p) = \frac{1}{x} \sum_{k=0}^{\infty} x^{-2k} C_{n+2k} - \frac{1}{x^2} \sum_{k=0}^{\infty} x^{-2k} C_{n+2k+1},$$

which is the same as Eq. (24) below.

3. Asymptotic Representation for Large x . When $p < x$, the function $f_n(x, p)$ can be written as

$$(24) \quad \begin{aligned} f_n(x, p) &= \int_0^p \frac{u^n e^{-u^2}}{x(1+u/x)} du = \sum_{k=0}^{\infty} \frac{1}{x^{2k+1}} \int_0^p u^{2k+n} e^{-u^2} du \\ &\quad - \sum_{k=0}^{\infty} \frac{1}{x^{2k+2}} \int_0^p u^{2k+n+1} e^{-u^2} du. \end{aligned}$$

Now we consider the odd and even values of n separately. Let $n = 2m$,

$$(25) \quad \begin{aligned} f_{2m}(x, p) &= \sum_{k=0}^{\infty} \frac{1}{x^{2k+1}} \int_0^p u^{2k+2m} e^{-u^2} du \\ &\quad - \sum_{k=0}^{\infty} \frac{1}{x^{2k+2}} \int_0^p u^{2k+2m+1} e^{-u^2} du \\ &= \sum_{k=0}^{\infty} \left[\left\{ \frac{(2k+2m-1)(2k+2m-3) \cdots 3 \cdot 1 \sqrt{\pi}}{2^{k+m+1}} \frac{\operatorname{erf} p}{x^{2k+1}} \right. \right. \\ &\quad \left. \left. - \frac{(2k+2m)(2k+2m-2) \cdots 4 \cdot 2}{2^{k+m+1}} \frac{(1-e^{-p^2})}{x^{2k+2}} \right\} \right. \\ &\quad \left. + \frac{e^{-p^2}}{2} \left\{ \frac{p^{2k+2m}}{x^{2k+2m}} - \frac{p^{2k+2m-1}}{x^{2k+1}} \right\} \right] \\ &\quad + \frac{e^{-p^2}}{2} \sum_{k=0}^{\infty} \sum_{r=1}^{k+m-1} \left\{ \frac{(2k+2m)(2k+2m-2) \cdots (2k+2m-2r+2)}{2^r} \frac{p^{2k+2m-2r}}{x^{2k+2}} \right. \\ &\quad \left. - \frac{(2k+2m-1)(2k+2m-3) \cdots (2k+2m-2r+1)}{2^r} \frac{p^{2k+2m-2r-1}}{x^{2k+1}} \right\}. \end{aligned}$$

If $n = 2m+1$,

$$(26) \quad \begin{aligned} f_{2m+1}(x, p) &= \sum_{k=0}^{\infty} \frac{1}{x^{2k+1}} \int_0^p u^{2k+2m+1} e^{-u^2} du \\ &\quad - \sum_{k=0}^{\infty} \frac{1}{x^{2k+2}} \int_0^p u^{2k+2m+2} e^{-u^2} du \\ &= \sum_{k=0}^{\infty} \left[\left\{ \frac{(2k+2m)(2k+2m-2) \cdots 4 \cdot 2}{2^{k+m+1}} \frac{1-e^{-p^2}}{x^{2k+1}} \right. \right. \\ &\quad \left. \left. - \frac{(2k+2m+1)(2k+2m-1) \cdots 3 \cdot 1 \sqrt{\pi}}{2^{k+m+2}} \frac{\operatorname{erf} p}{x^{2k+2}} \right\} \right. \\ &\quad \left. + \frac{e^{-p^2}}{2} \left\{ \frac{p^{2k+2m+1}}{x^{2k+2}} - \frac{p^{2k+2m}}{x^{2k+1}} \right\} \right] \\ &\quad + \frac{e^{-p^2}}{2} \sum_{k=0}^{\infty} \sum_{r=1}^{k+m} \frac{(2k+2m+1)(2k+2m-1) \cdots (2k+2m-2r+3)}{2^r} \frac{p^{2k+2m-2r+1}}{x^{2k+2}} \\ &\quad - \frac{e^{-p^2}}{2} \sum_{k=0}^{\infty} \sum_{r=1}^{k+m-1} \frac{(2k+2m+1)(2k+2m-1) \cdots (2k+2m-2r+3)}{2^r} \frac{p^{2k+2m-2r+1}}{x^{2k+2}}. \end{aligned}$$

Asymptotic expansions for the case $p > x$ have not been shown, although they can be easily derived. However, mention may be made about the difficulties associated with asymptotic expansions of functions of several independent variables such as p , x , and n that characterize the present problem. General methods for deriving the appropriate expansions are discussed in [6], [8], and [9].

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N = 0	P	X				
		0.2	0.4	0.6	0.8	1.0
0.10	0.40425003	0.22244239	0.15365773	0.11740285	0.09500092	
0.25	0.79747987	0.47673512	0.34178519	0.26674104	0.21882963	
0.50	1.18251896	0.75982618	0.56584454	0.45221686	0.37708789	
0.75	1.39053917	0.92654371	0.70498991	0.57163489	0.48168701	
1.00	1.50040054	1.01802471	0.78470379	0.64174098	0.54425645	
1.25	1.55467796	1.06599808	0.86252818	0.67893842	0.57792646	
1.50	1.57925510	1.08777523	0.84581006	0.69657739	0.59415781	
1.75	1.58935165	1.09686184	0.85407192	0.70425236	0.60115069	
2.00	1.59308624	1.10026073	0.85719633	0.70714056	0.60385701	
2.25	1.59432411	1.10140324	0.85825127	0.70812249	0.60475606	
2.50	1.59469318	1.10174561	0.85856795	0.70841950	0.60503554	
2.75	1.59478283	1.10183907	0.85865402	0.70849955	0.60511231	
3.00	1.59480858	1.10185432	0.85867333	0.70851868	0.60512335	
3.25	1.59481430	1.10185814	0.85867757	0.70852298	0.60513109	
3.50	1.59481239	1.10186005	0.85867823	0.70852375	0.60513276	
3.75	1.59481239	1.10185909	0.85867858	0.70852339	0.60513288	
4.00	1.59481430	1.10185623	0.85867852	0.70852345	0.60513312	
4.25	1.59481335	1.10185814	0.85867882	0.70852345	0.60513276	
4.50	1.59481239	1.10186005	0.85867882	0.70852351	0.60513365	
4.75	1.59481430	1.10186195	0.85867888	0.70867401	0.60513377	
5.00	1.59481335	1.10186291	0.85867780	0.70852345	0.60513377	

N = 0	P	X				
		1.5	2.0	3.0	4.0	5.0
0.10	0.06432742	0.04862998	0.03268173	0.02461105	0.01973713	
0.25	0.15111786	0.11544436	0.07843828	0.05940349	0.04780449	
0.50	0.26693827	0.20679057	0.14265782	0.10892212	0.08809996	
0.75	0.34678549	0.27136558	0.18937039	0.14551497	0.11817753	
1.00	0.39610386	0.31206781	0.21953613	0.16947830	0.13805443	
1.25	0.42332035	0.33450866	0.23682004	0.18338042	0.14968193	
1.50	0.43671268	0.34630644	0.24560277	0.19052434	0.15570223	
1.75	0.44257915	0.35135949	0.24955833	0.19377470	0.15846068	
2.00	0.44485487	0.35334849	0.25113757	0.19508415	0.15957916	
2.25	0.44565600	0.35404283	0.25169617	0.19555110	0.15998018	
2.50	0.44589829	0.35425764	0.25187111	0.19569808	0.16010743	
2.75	0.44596553	0.35431713	0.25191957	0.19573951	0.16014320	
3.00	0.44598073	0.35433161	0.25193466	0.19574970	0.16015208	
3.25	0.44598460	0.35433441	0.25193423	0.19575191	0.16015387	
3.50	0.44598502	0.35433513	0.25193423	0.19575226	0.16015428	
3.75	0.44598514	0.35433531	0.25193465	0.19575244	0.16015458	
4.00	0.44598514	0.35433507	0.25193483	0.19575238	0.16015458	
4.25	0.44598496	0.35433489	0.25193447	0.19575244	0.16015452	
4.50	0.44598496	0.35433471	0.25193441	0.19575244	0.16015446	
4.75	0.44598484	0.35433483	0.25193393	0.19575197	0.16015428	
5.00	0.44598556	0.35433549	0.25193465	0.19575208	0.16015440	

N = 0	P	X				
		6.0	7.0	8.0	9.0	10.0
0.10	0.01647458	0.01413765	0.01238136	0.01101321	0.00991734	
0.25	0.03959564	0.03437996	0.03014719	0.02684249	0.02419074	
0.50	0.07396466	0.06373960	0.05599504	0.04935534	0.04505673	
0.75	0.09949666	0.08591968	0.07560515	0.06750280	0.06096961	
1.00	0.11647779	0.10074162	0.08875507	0.07931966	0.07169884	
1.25	0.12647051	0.10950249	0.09655458	0.08634782	0.07809472	
1.50	0.13167268	0.11408240	0.10064507	0.09004366	0.08146501	
1.75	0.13406843	0.11616973	0.10254198	0.09176177	0.08303505	
2.00	0.13504446	0.11706561	0.10332000	0.09246796	0.08368176	
2.25	0.13539600	0.11737859	0.10360159	0.09272456	0.08391726	
2.50	0.13550818	0.11747867	0.10369253	0.09280705	0.08399314	
2.75	0.13553965	0.11750686	0.10371792	0.09283036	0.08401459	
3.00	0.13554758	0.11751401	0.10372442	0.09283644	0.08401996	
3.25	0.13554925	0.11751550	0.10372591	0.09283763	0.08402097	
3.50	0.13554949	0.11751568	0.10372591	0.09283787	0.08402133	
3.75	0.13554966	0.11751580	0.10372606	0.09283793	0.08402139	
4.00	0.13554966	0.11751592	0.10372615	0.09283787	0.08402133	
4.25	0.13554972	0.11751580	0.10372615	0.09283787	0.08402139	
4.50	0.13554955	0.11751586	0.10372615	0.09283787	0.08402145	
4.75	0.13554960	0.11751574	0.10372621	0.09283793	0.08402139	
5.00	0.13554960	0.11751580	0.10372615	0.09283793	0.08402157	

P	0.2	0.4	X	0.8	1.0
N = 1	0.10	0.01881764	0.01069060	0.00747251	0.00574524
0.25	0.08539164	0.05419369	0.03981648	0.03149450	0.02605814
0.50	0.22477663	0.15734977	0.12177354	0.09950727	0.08419275
0.75	0.35213637	0.25962687	0.20725089	0.17293733	0.14855766
1.00	0.44674289	0.33925259	0.27600139	0.23343104	0.20256764
1.25	0.50696224	0.39150029	0.32214355	0.27474767	0.23997553
1.50	0.54033589	0.42107749	0.34870112	0.29884565	0.26202553
1.75	0.55654377	0.43566942	0.36197056	0.31101239	0.27326298
2.00	0.56346309	0.44197500	0.36776292	0.31636798	0.27824306
2.25	0.56606424	0.444436800	0.36997920	0.31843185	0.28017426
2.50	0.56692636	0.44516832	0.37072438	0.31912953	0.28082907
2.75	0.56717771	0.44540328	0.37054486	0.31933182	0.28102368
3.00	0.56724611	0.44546437	0.37100202	0.31939101	0.28107780
3.25	0.56725907	0.44547838	0.37101555	0.31940424	0.28108966
3.50	0.56726176	0.44548118	0.37101841	0.31940651	0.28109199
3.75	0.56726235	0.44548112	0.37101889	0.31940711	0.28109235
4.00	0.56726259	0.44548172	0.37101947	0.31940699	0.28109229
4.25	0.56726235	0.44548124	0.37101883	0.31940693	0.28109205
4.50	0.56724241	0.44548112	0.37101941	0.31940734	0.28109294
4.75	0.56726205	0.44548088	0.37101889	0.31940681	0.28109181
5.00	0.56726182	0.44548190	0.37101895	0.31940693	0.28109211

P	1.5	2.0	X	4.0	5.0
0.10	0.00317645	0.00240769	0.00162244	0.00122344	0.00098196
0.25	0.01821086	0.01399892	0.00957268	0.00727372	0.00586525
0.50	0.06087315	0.04769936	0.03330659	0.02559210	0.02078055
0.75	0.11006665	0.08751369	0.06213342	0.04818486	0.039355676
1.00	0.15266752	0.12268740	0.08821547	0.06891060	0.05655171
1.25	0.18291724	0.14398065	0.10743785	0.08437556	0.06948811
1.50	0.20111829	0.16357434	0.11937940	0.09409028	0.07767606
1.75	0.21054476	0.17169440	0.12573791	0.09931523	0.08211040
2.00	0.21478248	0.17538303	0.12866668	0.10174394	0.08418500
2.25	0.21644557	0.17684317	0.12984075	0.10272563	0.08502847
2.50	0.21701616	0.17734855	0.13025177	0.10307187	0.08532768
2.75	0.21718717	0.17750198	0.13037783	0.10317880	0.08542049
3.00	0.21722413	0.17754275	0.13041103	0.10320771	0.08544576
3.25	0.21724430	0.17755228	0.13041196	0.10321468	0.08545172
3.50	0.21724713	0.17755413	0.13042116	0.10321605	0.08545297
3.75	0.21724731	0.17755467	0.13042140	0.10321617	0.08545321
4.00	0.21724713	0.17755467	0.13042164	0.10321623	0.08545321
4.25	0.21724749	0.17755479	0.13042164	0.10321635	0.08545321
4.50	0.21724755	0.17755473	0.13042158	0.10321623	0.08545315
4.75	0.21724755	0.17755473	0.13042158	0.10321635	0.08545327
5.00	0.21724749	0.17755473	0.13042158	0.10321623	0.08545327

P	6.0	7.0	X	9.0	10.0
0.10	0.00082009	0.00070404	0.00061676	0.00054873	0.00049422
0.25	0.01491373	0.00422790	0.00371007	0.00330525	0.00298009
0.50	0.01749247	0.01510302	0.01328804	0.01186254	0.01071329
0.75	0.03326480	0.02880696	0.02540322	0.02271910	0.02054814
1.00	0.04795693	0.04163236	0.03678290	0.03294581	0.02983411
1.25	0.05907555	0.05138110	0.04546222	0.04076734	0.03695206
1.50	0.06615138	0.05761093	0.05102665	0.04579488	0.04153718
1.75	0.07000309	0.06101525	0.05407679	0.04855751	0.04406188
2.00	0.07181364	0.06262141	0.05552004	0.04986782	0.04526170
2.25	0.07255310	0.06327963	0.05611311	0.05040750	0.04575678
2.50	0.07281643	0.06351477	0.05632557	0.05060127	0.04593484
2.75	0.07289845	0.06358826	0.05639211	0.05066206	0.04599082
3.00	0.07292080	0.06360841	0.05641037	0.05067877	0.04600624
3.25	0.07292616	0.06361324	0.05641475	0.05068280	0.04600996
3.50	0.07292736	0.06361425	0.05641569	0.05068365	0.04601074
3.75	0.07292753	0.06361443	0.05641589	0.05068380	0.04601088
4.00	0.07292759	0.06361443	0.05641591	0.05068384	0.04601092
4.25	0.07292753	0.06361443	0.05641592	0.05068386	0.04601094
4.50	0.07292747	0.06361437	0.05641592	0.05068386	0.04601094
4.75	0.07292753	0.06361443	0.05641592	0.05068384	0.04601093
5.00	0.07292753	0.06361437	0.05641592	0.05068388	0.04601093

P	0.2		0.4		X		0.8	1.0
	0.10	0.25	0.50	0.75	1.00	1.25		
N = 2	0.00121155	0.00069884	0.00049133	0.00037889	0.00030834			
0.25	0.01321511	0.00861596	0.00640354	0.00509784	0.00423529			
0.50	0.06564420	0.04765955	0.03753531	0.03099367	0.02649670			
0.75	0.14468104	0.11257555	0.09075773	0.07675856	0.06655061			
1.00	0.22671139	0.18035877	0.15045923	0.12931526	0.11349213			
1.25	0.29380119	0.23859382	0.20190752	0.17539585	0.15522027			
1.50	0.33232374	0.27886897	0.23807825	0.20822304	0.18527013			
1.75	0.36530548	0.30234617	0.25943154	0.22780383	0.20335078			
2.00	0.37814897	0.31405151	0.27018398	0.23774701	0.21259785			
2.25	0.38362169	0.31908703	0.27484651	0.24208903	0.21666014			
2.50	0.38564949	0.32096690	0.27659541	0.24373031	0.21871203			
2.75	0.38630367	0.32157838	0.27717263	0.24426967	0.21871203			
3.00	0.38648891	0.32175201	0.27733594	0.24442416	0.21885955			
3.25	0.38652491	0.32179493	0.27737773	0.24446315	0.21889639			
3.50	0.38654464	0.32180434	0.27738589	0.24447137	0.21890450			
3.75	0.38654649	0.32180625	0.27738744	0.24447310	0.21890599			
4.00	0.38654673	0.32180643	0.27738786	0.24447334	0.21890610			
4.25	0.38654697	0.32180643	0.27738804	0.24447322	0.21890628			
4.50	0.38654691	0.32180655	0.27738786	0.24447340	0.21890628			
4.75	0.38654697	0.32180661	0.27738798	0.24447322	0.21890634			
5.00	0.38654661	0.32180673	0.27738792	0.24447352	0.21890640			

P	1.5		2.0		X		4.0	5.0
	0.10	0.25	0.50	0.75	1.00	1.25		
0.10	0.00021041	0.00015970	0.00010776	0.00008131	0.00006529			
0.25	0.00297715	0.00295611	0.00157483	0.00119855	0.00096742			
0.50	0.01928976	0.01520072	0.01067846	0.00823114	0.00696977			
0.75	0.05000827	0.04008095	0.02870792	0.02236875	0.01832464			
1.00	0.08705854	0.07068509	0.05141361	0.04041724	0.03330154			
1.25	0.12081778	0.09903234	0.07287957	0.05769125	0.04775330			
1.50	0.14562213	0.12015124	0.08916116	0.07093841	0.05891573			
1.75	0.16079682	0.13322502	0.09940034	0.07935327	0.06606203			
2.00	0.16866714	0.14007533	0.10484082	0.08386523	0.06591625			
2.25	0.17216605	0.14314812	0.10731184	0.08593154	0.07169181			
2.50	0.17350906	0.14433682	0.10827845	0.08674604	0.07239556			
2.75	0.17395633	0.14473540	0.10860598	0.08702391	0.07263690			
3.00	0.17408597	0.14485180	0.10870242	0.08710629	0.07270879			
3.25	0.17411888	0.14488143	0.10872716	0.08712751	0.07272738			
3.50	0.17412603	0.14488804	0.10873270	0.08713222	0.07273149			
3.75	0.17412746	0.14488918	0.10873371	0.08713329	0.07273245			
4.00	0.17412770	0.14488947	0.10873383	0.08713335	0.07273245			
4.25	0.17412770	0.14488953	0.10873389	0.08713347	0.07273251			
4.50	0.17412788	0.14488959	0.10873401	0.08713341	0.07273257			
4.75	0.17412782	0.14488947	0.10873401	0.08713347	0.07273251			
5.00	0.17412788	0.14488959	0.10873395	0.08713347	0.07273257			

P	6.0		7.0		X		9.0	10.0
	0.10	0.25	0.50	0.75	1.00	1.25		
0.10	0.00005454	0.00004683	0.00004103	0.00003651	0.00003289			
0.25	0.000081103	0.00069887	0.00061288	0.00054616	0.00049255			
0.50	0.00564468	0.00487834	0.00429523	0.00383666	0.00346656			
0.75	0.01551963	0.01345968	0.01188266	0.01063651	0.00962697			
1.00	0.04074075	0.02463354	0.02179771	0.01954768	0.01771887			
1.25	0.05039145	0.04402349	0.03149621	0.02828798	0.02567327			
1.50	0.05659563	0.04950739	0.04399978	0.03514597	0.03192795			
1.75	0.05995946	0.05249156	0.04668131	0.04203111	0.03822461			
2.00	0.06151553	0.05387712	0.04792980	0.04316718	0.03926687			
2.25	0.06213548	0.05443044	0.04842966	0.04362302	0.03968578			
2.50	0.06234874	0.05462147	0.04860265	0.04378113	0.03983136			
2.75	0.06241249	0.05467878	0.04865465	0.04382876	0.03987527			
3.00	0.06242909	0.05469373	0.04866829	0.04384123	0.03988678			
3.25	0.06242388	0.05469713	0.04867139	0.04384407	0.03988644			
3.50	0.06243361	0.05469775	0.04867201	0.04384465	0.03988696			
4.00	0.06243372	0.05469789	0.04867211	0.04384474	0.03989003			
4.25	0.06243372	0.05469790	0.04867212	0.04384475	0.03989004			
4.50	0.06243370	0.05469788	0.04867211	0.04384473	0.03989005			
4.75	0.06243374	0.05469793	0.04867205	0.04384469	0.03989002			
5.00	0.06243377	0.05469794	0.04867214	0.04384476	0.03989005			

P	0.2	0.4	X	0.8	1.0
N = 3					
0.10	0.00008903	0.00005180	0.00003654	0.00002823	0.00002300
0.25	0.00237428	0.00157092	0.00117518	0.00093903	0.00078202
0.50	0.02281143	0.01687645	0.01341908	0.01114537	0.00953358
0.75	0.07251757	0.05695090	0.04699928	0.04004719	0.03490334
1.00	0.14412981	0.11732858	0.09916667	0.08601553	0.07597989
1.25	0.21918035	0.18250448	0.15679747	0.13762534	0.12272161
1.50	0.28119743	0.23749679	0.20619708	0.18246603	0.16377413
1.75	0.32322109	0.27534401	0.24062377	0.21403962	0.19293189
2.00	0.34705400	0.29710376	0.26061457	0.23252732	0.21012682
2.25	0.35861939	0.30770892	0.27043553	0.24167275	0.21868342
2.50	0.36338806	0.31213278	0.274555980	0.24553531	0.22231579
2.75	0.36509275	0.31372249	0.27605039	0.24693835	0.22364086
3.00	0.36561722	0.31421727	0.27651644	0.24737883	0.22405833
3.25	0.36576134	0.31435050	0.27664214	0.24749869	0.224117235
3.50	0.36579472	0.31438220	0.27667254	0.24752700	0.22419953
3.75	0.36580151	0.31438857	0.27667868	0.24753261	0.22420496
4.00	0.36580294	0.31438977	0.27668007	0.24753422	0.22420639
4.25	0.35589324	0.31438845	0.27668005	0.24753392	0.22420627
4.50	0.36580318	0.31438994	0.27667987	0.24753404	0.22420609
4.75	0.36580330	0.31439024	0.27668017	0.24753410	0.22420633
5.00	0.36580205	0.31439018	0.27667993	0.24753404	0.22420627

P	1.5	2.0	X	4.0	5.0
0.10	0.00001572	0.00001124	0.00000806	0.00000609	0.00000489
0.25	0.00055159	0.00042609	0.00029286	0.00022310	0.00018019
0.50	0.00700563	0.00553884	0.00390492	0.00301569	0.00245642
0.75	0.02644150	0.02129211	0.01533018	0.01197895	0.00983077
1.00	0.05088421	0.04810187	0.03523127	0.02780307	0.02296451
1.25	0.09671515	0.07987714	0.05930206	0.04717706	0.03917562
1.50	0.13961094	0.10374212	0.08156091	0.06529081	0.05444588
1.75	0.15509172	0.12983257	0.09808459	0.0786564	0.06597233
2.00	0.16972369	0.14257365	0.10820186	0.08726358	0.07314301
2.25	0.17709464	0.14904737	0.11340833	0.09161764	0.07688463
2.50	0.18025577	0.15184563	0.11568397	0.09353524	0.07854158
2.75	0.18141925	0.15288264	0.11653566	0.09425807	0.07916533
3.00	0.18178550	0.15321404	0.11681062	0.09445279	0.07937407
3.25	0.18182037	0.15330571	0.11688751	0.09455872	0.07943189
3.50	0.18191475	0.15332776	0.11690587	0.09457463	0.07944596
3.75	0.18191981	0.15333241	0.11690980	0.09457809	0.07944484
4.00	0.18192071	0.15333319	0.11691040	0.09457874	0.079444942
4.25	0.18192059	0.15333325	0.11691058	0.09457874	0.079444947
4.50	0.18192077	0.15333325	0.11691052	0.09457868	0.079444542
4.75	0.18192083	0.15333331	0.11691058	0.09457880	0.079444953
5.00	0.18192083	0.15333337	0.11691058	0.09457874	0.079444559

P	6.0	7.0	X	9.0	10.0
0.10	0.00000408	0.00000351	0.00000307	0.00000274	0.00000246
0.25	0.00015112	0.00013013	0.00011426	0.00010184	0.00009185
0.50	0.00207216	0.00175188	0.00157839	0.00141036	0.00127467
0.75	0.00833616	0.00723616	0.00639268	0.00572535	0.00518420
1.00	0.01951151	0.01703738	0.01505047	0.01354302	0.01228351
1.25	0.03349756	0.02925841	0.02597234	0.02335026	0.02120928
1.50	0.04669604	0.04088014	0.03635385	0.03273080	0.02976489
1.75	0.05670914	0.04973115	0.04428457	0.03991448	0.03633017
2.00	0.06296778	0.05528364	0.04527404	0.04444463	0.04047846
2.25	0.06248000	0.05820379	0.05190532	0.04683506	0.04267515
2.50	0.06770664	0.0595652	0.05308225	0.04791236	0.04366158
2.75	0.06826144	0.06000355	0.05353241	0.04832372	0.04404030
3.00	0.06844312	0.06016674	0.05368057	0.048455938	0.04416542
3.25	0.06849450	0.06021306	0.05372272	0.04849803	0.04420112
3.50	0.06850713	0.06022449	0.05373313	0.04850755	0.04420591
3.75	0.06850570	0.06022684	0.05373529	0.04850559	0.04421182
4.00	0.06851017	0.06022731	0.05373571	0.04850998	0.04421218
4.25	0.06851023	0.06022737	0.05373575	0.04851004	0.04421221
4.50	0.06851017	0.06022735	0.05373575	0.04851002	0.04421221
4.75	0.06851017	0.06022735	0.05373576	0.04851002	0.04421221
5.00	0.06851029	0.06022741	0.05373575	0.04851006	0.04421223

N = 4	F	P		X	
		0.2	0.4	0.6	0.8
0.10	0.00000703	0.00000411	0.00000291	0.00000225	0.00000183
0.25	0.00046195	0.00030844	0.00023170	0.00018558	0.00015479
0.50	0.00868721	0.00649892	0.00519806	0.00433321	0.00371592
0.75	0.04053261	0.03207676	0.02665756	0.02281941	0.01995380
1.00	0.122455	0.08518910	0.07260239	0.06330442	0.05614055
1.25	0.18759524	0.15843356	0.13735682	0.12133491	0.10871339
1.50	0.27248722	0.23327116	0.20507071	0.18276685	0.16495168
1.75	0.34035426	0.29486030	0.26062232	0.23376495	0.21206450
2.00	0.38479167	0.33217173	0.29784405	0.26819026	0.24408484
2.25	0.40908861	0.35712890	0.31855106	0.28747410	0.26212865
2.50	0.42032385	0.36814851	0.32826579	0.29657322	0.27068532
2.75	0.42475700	0.37228590	0.33214506	0.30022466	0.27413923
3.00	0.42622916	0.37359621	0.33347251	0.30149323	0.27532399
3.25	0.4266827	0.37410933	0.33386367	0.30185062	0.27567714
3.50	0.42681056	0.37421602	0.33396359	0.30194598	0.27576774
3.75	0.42683566	0.37423992	0.33398771	0.30196708	0.27578813
4.00	0.42683786	0.37424529	0.33399266	0.30197269	0.27579314
4.25	0.42683852	0.37424296	0.33399105	0.30197147	0.27579254
4.50	0.42683840	0.37424296	0.33399096	0.30197155	0.27579236
4.75	0.42683852	0.37424284	0.33399096	0.30197155	0.27579308
5.00	0.42683893	0.37424326	0.33399141	0.30197157	0.27579254

P	X		4.0	5.0
	1.5	2.0		
0.10	0.00000125	0.00000095	0.00000064	0.00000049
0.25	0.00010943	0.00008463	0.00005824	0.00003587
0.50	0.00274106	0.00217181	0.00153473	0.00096739
0.75	0.01519468	0.01227292	0.00866660	0.00694131
1.00	0.04379417	0.03591667	0.02642665	0.02090822
1.25	0.08632224	0.07160078	0.05352283	0.04272683
1.50	0.13280928	0.11124164	0.08404332	0.06756288
1.75	0.17236555	0.14533126	0.11075163	0.08951795
2.00	0.19962472	0.16906291	0.12960452	0.10515606
2.25	0.215217164	0.18271810	0.14058781	0.11434203
2.50	0.222611769	0.18931001	0.14594954	0.11886060
2.75	0.22564590	0.19200927	0.14816695	0.12074208
3.00	0.22664905	0.19255388	0.14895004	0.12141097
3.25	0.22701389	0.19323784	0.14918751	0.12161499
3.50	0.22709519	0.19331175	0.14924961	0.12166870
3.75	0.22711343	0.19332826	0.14926362	0.12168080
4.00	0.22711712	0.19333160	0.14926642	0.12168318
4.25	0.22711867	0.19333255	0.14926690	0.12168372
4.50	0.22711736	0.19333291	0.14926744	0.12168407
4.75	0.22711754	0.19333190	0.14926666	0.12168372
5.00	0.22711778	0.19333220	0.14926708	0.12168413

F	X		9.0	10.0
	6.0	7.0		
0.10	0.00000033	0.00000028	0.00000025	0.00000020
0.25	0.0000309	0.00002592	0.00002276	0.00002026
0.50	0.00081650	0.00070633	0.00062236	0.00055623
0.75	0.00484011	0.00420394	0.00371560	0.00332892
1.00	0.01475141	0.01285875	0.01139667	0.01023322
1.25	0.03044982	0.02662434	0.02365640	0.02128278
1.50	0.04854954	0.04256514	0.03785501	0.03414897
1.75	0.06474203	0.05687869	0.05072064	0.04576690
2.00	0.07640368	0.06722492	0.06001819	0.05420876
2.25	0.08332449	0.07338615	0.06557012	0.05926104
2.50	0.08976165	0.07645607	0.06834358	0.06179039
2.75	0.08820617	0.07775021	0.06951555	0.06286144
3.00	0.08872384	0.07821536	0.06993800	0.06324810
3.25	0.08888298	0.07835871	0.07006842	0.06336778
3.50	0.08892518	0.07839680	0.07010317	0.06339967
3.75	0.08893478	0.07840556	0.07011122	0.06340712
4.00	0.08893675	0.07840729	0.07011128	0.06340855
4.25	0.08893710	0.07840765	0.07011300	0.06340885
4.50	0.08893728	0.07840776	0.07011318	0.06340891
4.75	0.08893698	0.07840747	0.07011306	0.06340891
5.00	0.08893716	0.07840765	0.07011306	0.06340879

P	0.2	0.4	X	0.8	1.0
N = 5	0.10	0.00000058	0.00000034	0.00000024	0.00000019
0.25	0.00009441	0.00006342	0.00004778	0.00003833	0.00003201
0.50	0.00349796	0.00263584	0.00211657	0.00176884	0.00151949
0.75	0.02392180	0.01916183	0.01599800	0.01373701	0.01203872
1.00	0.07960975	0.06519304	0.05670729	0.04962519	0.04412822
1.25	0.17469484	0.14884126	0.12980056	0.11514676	0.10350096
1.50	0.29120827	0.25221467	0.22370083	0.19950294	0.18075289
1.75	0.40102309	0.35114998	0.31272016	0.28208160	0.25702858
2.00	0.48386610	0.42661675	0.38211763	0.34627211	0.31674039
2.25	0.53514844	0.47387451	0.42583543	0.38698673	0.35483706
2.50	0.56163257	0.49843538	0.44873804	0.40843892	0.37501174
2.75	0.57317674	0.50921327	0.45884097	0.41794842	0.38399357
3.00	0.57745954	0.51323277	0.46262753	0.42152756	0.38734716
3.25	0.57881953	0.51451540	0.46384072	0.42267859	0.38848168
3.50	0.57919151	0.51486766	0.46417511	0.42259670	0.38873518
3.75	0.57927853	0.51495141	0.46425480	0.42307270	0.38885796
4.00	0.57929790	0.51496840	0.46426952	0.42308724	0.38887310
4.25	0.57930088	0.51497132	0.46427395	0.42309129	0.38887584
4.50	0.57930166	0.51497197	0.46427453	0.42309177	0.38887626
4.75	0.57930076	0.51497138	0.46427375	0.42309093	0.38887560
5.00	0.57930118	0.51497197	0.46427423	0.42309147	0.38887602

P	1.5	2.0	X	4.0	5.0
0.10	0.00000010	0.00000008	0.00000005	0.00000004	0.00000003
0.25	0.00022666	0.00001754	0.00001208	0.00000922	0.00000745
0.50	0.00112383	0.00089178	0.00063122	0.00048852	0.00039485
0.75	0.00520220	0.00744667	0.00539270	0.00422729	0.00347622
1.00	0.03457750	0.02843542	0.02098882	0.01663584	0.01377922
1.25	0.08267105	0.06885308	0.05162828	0.04130144	0.03442901
1.50	0.14649159	0.12322223	0.09357538	0.07545412	0.06322169
1.75	0.21054488	0.17843097	0.13683826	0.11102176	0.09341854
2.00	0.26138717	0.22269857	0.17201000	0.14019566	0.11834794
2.25	0.29421043	0.25153130	0.19520414	0.15959930	0.13502038
2.50	0.31177074	0.26707733	0.20784944	0.17025614	0.14422941
2.75	0.31965899	0.27410907	0.21362680	0.17515898	0.14848512
3.00	0.32266225	0.27680296	0.21586043	0.17706674	0.15015256
3.25	0.32363784	0.27767829	0.21659642	0.17766981	0.15070665
3.50	0.32391030	0.27719290	0.21680427	0.17787850	0.15086478
3.75	0.32397374	0.27719892	0.21685469	0.17792249	0.15090352
4.00	0.32398579	0.27800220	0.21686566	0.17793160	0.15091187
4.25	0.32399195	0.27800304	0.21686721	0.17793345	0.15091324
4.50	0.32399237	0.27800471	0.21686765	0.17793369	0.15091342
4.75	0.32399178	0.27800399	0.21686745	0.17793363	0.15091336
5.00	0.32399166	0.27800405	0.21686727	0.17793328	0.15091318

P	6.0	7.0	X	9.0	10.0
0.10	0.00000003	0.00000002	0.00000002	0.00000002	0.00000002
0.25	0.00000625	0.00000538	0.00000473	0.00000421	0.00000380
0.50	0.00033643	0.00029111	0.00025911	0.00022933	0.00020733
0.75	0.00295182	0.00256493	0.00226777	0.00203223	0.00184105
1.00	0.01176023	0.01025746	0.00904532	0.00816977	0.00741521
1.25	0.02951577	0.02583037	0.02296350	0.02066963	0.01879254
1.50	0.05440603	0.04774984	0.04254574	0.03836508	0.03493284
1.75	0.08064133	0.07094258	0.06332844	0.05719146	0.05213951
2.00	0.10240233	0.09024972	0.08067906	0.07294554	0.06656629
2.25	0.11702013	0.10326368	0.09240626	0.08361763	0.07635736
2.50	0.12512749	0.11050498	0.09894848	0.08958405	0.08184105
2.75	0.12888885	0.11387491	0.10200055	0.09237349	0.08440930
3.00	0.13036776	0.11520356	0.10320705	0.09347785	0.08542764
3.25	0.13086098	0.11564803	0.10361165	0.09384894	0.08577043
3.50	0.13100213	0.11577541	0.10372776	0.09395576	0.08586525
3.75	0.13103706	0.11580694	0.10375643	0.09398216	0.08589381
4.00	0.13104439	0.11581367	0.10376265	0.09398782	0.08589500
4.25	0.13104576	0.11581492	0.10376376	0.09398878	0.08589989
4.50	0.13104588	0.11581510	0.10376400	0.09398896	0.08590007
4.75	0.13104588	0.11581498	0.10376394	0.09398896	0.08590001
5.00	0.13104564	0.11581492	0.10376382	0.09398878	0.08589989

N = 6	P	0.2		0.4		X		0.8	1.0
		0.00000005	0.00000003	0.00000002	0.00000002	0.00000001	0.00000001		
0.25	0.00001995	0.00001346	0.00001016	0.00000816	0.00000682				
0.50	0.00146190	0.00110716	0.00089152	0.00074642	0.00064201				
0.75	0.01478853	0.01190816	0.0097410	0.00858329	0.00753416				
1.00	0.06437927	0.05382412	0.04627702	0.04060124	0.03617315				
1.25	0.17205834	0.14746082	0.12911689	0.11487979	0.10349602				
1.50	0.33241856	0.28977442	0.25703961	0.23105770	0.20990610				
1.75	0.51046044	0.45020503	0.40303284	0.36499965	0.33363593				
2.00	0.66512263	0.59122515	0.53262430	0.48487129	0.44515526				
2.25	0.77348346	0.69096303	0.62501192	0.57092375	0.52567565				
2.50	0.83597219	0.74892330	0.67905575	0.62154734	0.57328647				
2.75	0.86605680	0.77700663	0.70538729	0.64633310	0.59669769				
3.00	0.87827456	0.78847808	0.71611896	0.65654439	0.60637915				
3.25	0.88249296	0.79245043	0.719595223	0.66011399	0.60977459				
3.50	0.88373792	0.79362905	0.72107095	0.66117665	0.61079001				
3.75	0.88405347	0.79392898	0.72135663	0.66145122	0.61105114				
4.00	0.88412237	0.79399514	0.72149981	0.66151184	0.61110502				
4.25	0.88413602	0.79400736	0.72143179	0.66152316	0.61112064				
4.50	0.88413811	0.79400927	0.72143370	0.66152513	0.61112189				
4.75	0.88413805	0.79400724	0.72143406	0.66152495	0.61112189				
5.00	0.88413638	0.79400820	0.72143245	0.66152400	0.61112076				

P	1.5		2.0		X		4.0	5.0
	1.0	0.00000001	1.5	0.00000001	2.0	0.00000000		
0.25	0.000004493	0.00000374	0.00000258	0.00000197	0.00000009			
0.50	0.00047575	0.00037794	0.00026784	0.00020742	0.00016925			
0.75	0.00577258	0.00467953	0.00339475	0.00266370	0.00219179			
1.00	0.02843509	0.02343050	0.01733488	0.01375794	0.01140525			
1.25	0.08299029	0.06929106	0.05211242	0.04176748	0.03485227			
1.50	0.17092258	0.14421575	0.10993356	0.08834370	0.07455152			
1.75	0.27484733	0.23380280	0.18014944	0.14657730	0.12357134			
2.00	0.36981487	0.31649846	0.24586385	0.20109701	0.17015583			
2.25	0.43919706	0.37745023	0.29490000	0.24211562	0.20541012			
2.50	0.48064190	0.41414350	0.3274565	0.26727319	0.22715062			
2.75	0.50120014	0.43247151	0.33981019	0.28005493	0.23825246			
3.00	0.50976145	0.44016033	0.34618461	0.28549629	0.2430152			
3.25	0.51279896	0.44288975	0.34846675	0.28745985	0.24472219			
3.50	0.51371002	0.44371575	0.34916258	0.28806096	0.24525130			
3.75	0.51394528	0.44392697	0.34934443	0.28821874	0.24539065			
4.00	0.51399779	0.44397801	0.34938550	0.28825444	0.24542242			
4.25	0.51400810	0.44398743	0.3493913	0.28826141	0.24542868			
4.50	0.51400948	0.44398880	0.34939474	0.28826267	0.24542964			
4.75	0.51400971	0.44398874	0.34939468	0.28826249	0.24542981			
5.00	0.51400870	0.44398880	0.34939486	0.28826290	0.24542559			

P	6.0		7.0		X		9.0	10.0
	6.0	7.0	8.0	9.0	10.0			
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
0.25	0.00000134	0.00000115	0.00000101	0.00000090	0.000000891	0.000000891		
0.50	0.00014294	0.00012371	0.00010905	0.00009749	0.00009749	0.00009749		
0.75	0.00181959	0.00161841	0.00143122	0.00128284	0.001164917	0.001164917		
1.00	0.00973594	0.00849909	0.00753873	0.00677341	0.00614917	0.00614917		
1.25	0.02990271	0.02618469	0.02328977	0.02097060	0.01907191	0.01907191		
1.50	0.06422400	0.05641136	0.05029425	0.04537451	0.04133179	0.04133179		
1.75	0.10681570	0.09406549	0.08403677	0.07594150	0.06926972	0.06926972		
2.00	0.14748162	0.13014728	0.11646307	0.10538524	0.09623313	0.09623313		
2.25	0.17839211	0.15766692	0.14126283	0.12795419	0.11693937	0.11693937		
2.50	0.19753259	0.17467314	0.15670925	0.14204121	0.12988704	0.12988704		
2.75	0.20734507	0.18355459	0.16467196	0.14931786	0.13688661	0.13688661		
3.00	0.21155816	0.18734127	0.16810971	0.15246546	0.13548941	0.13548941		
3.25	0.21308929	0.18871965	0.16936374	0.15361619	0.14055216	0.14055216		
3.50	0.21356195	0.18914670	0.16975325	0.15397388	0.14088309	0.14088309		
3.75	0.21368688	0.18925971	0.16985643	0.15406907	0.14097130	0.14097130		
4.00	0.21371531	0.18928564	0.16988009	0.15409088	0.1409157	0.1409157		
4.25	0.21372086	0.18929052	0.16988474	0.15409499	0.14095550	0.14095550		
4.50	0.21372175	0.18929130	0.16988552	0.15409577	0.14099604	0.14099604		
4.75	0.21372199	0.18929154	0.16988566	0.15409607	0.14099634	0.14099634		
5.00	0.21372187	0.18929148	0.16988581	0.15409625	0.14099652	0.14099652		

P	0.2	0.4	X	0.8	1.0
N = 7	0.10	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000432	0.00000292	0.00000221	0.00000178	0.00000149
0.50	0.00062739	0.00047690	0.00038484	0.00032263	0.00027776
0.75	0.00941758	0.00761203	0.00639083	0.00550866	0.00484112
1.00	0.05385637	0.04520261	0.03856606	0.03425127	0.03055509
1.25	0.17628360	0.15171111	0.13322526	0.11879152	0.10719925
1.50	0.39759314	0.34916706	0.30985278	0.27923030	0.25417006
1.75	0.68681711	0.60826826	0.54708970	0.49690968	0.45527303
2.00	0.97598702	0.87252140	0.78943676	0.72110987	0.66385591
2.25	1.20521641	1.08352852	0.98490703	0.90317512	0.83423799
2.50	1.35279274	1.2041798	1.1123503	1.02274990	0.94670117
2.75	1.43125057	1.29365826	1.18122959	1.08740711	1.00776291
3.00	1.46612835	1.32639408	1.21206856	1.11654758	1.03540325
3.25	1.47920390	1.33872795	1.2237376	1.12761784	1.0493372
3.50	1.48338032	1.34267616	1.22748566	1.13118458	1.04933643
3.75	1.48450375	1.34375477	1.22851181	1.13216496	1.05027390
4.00	1.48478127	1.34400749	1.22875404	1.13239670	1.05049610
4.25	1.48483276	1.34405804	1.22880268	1.13244343	1.05054092
4.50	1.48484135	1.34406567	1.22880936	1.13245010	1.05054760
4.75	1.48484325	1.34406662	1.22881031	1.13245010	1.05054855
5.00	1.48484802	1.34407043	1.22881603	1.13245583	1.05055332

P	1.5	2.0	X	4.0	5.0
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000165	0.00000082	0.00000056	0.00000043	0.00000035
0.50	0.00020614	0.00016389	0.00011626	0.00009008	0.00007352
0.75	0.00371641	0.00301621	0.00219103	0.00172048	0.00141634
1.00	0.02407959	0.01987123	0.01472757	0.01170046	0.00970572
1.25	0.08620965	0.07211322	0.05435798	0.04362511	0.03643404
1.50	0.20769250	0.17564517	0.13427585	0.10870200	0.09131514
1.75	0.37663805	0.32130295	0.24845546	0.20259953	0.17105156
2.00	0.55428863	0.47601378	0.37141937	0.3046217	0.25823063
2.25	0.70111769	0.60501271	0.47521240	0.39145052	0.33286172
2.50	0.79902381	0.69169490	0.54513661	0.45069275	0.38423210
2.75	0.85265619	0.73951381	0.58502805	0.48424041	0.41319788
3.00	0.87712336	0.76146203	0.60322827	0.49978572	0.42676473
3.25	0.83650978	0.76992828	0.61030728	0.50586849	0.43209678
3.50	0.88956165	0.77269495	0.61263317	0.50788242	0.43386530
3.75	0.89040720	0.77346486	0.61329144	0.50849494	0.43437070
4.00	0.89060879	0.77364916	0.61344892	0.50858670	0.43449223
4.25	0.89064968	0.77368689	0.61348110	0.50861508	0.43451750
4.50	0.89065689	0.77369368	0.61348712	0.50862032	0.43452209
4.75	0.89065778	0.77369469	0.61348784	0.50862080	0.43452293
5.00	0.89065820	0.77369453	0.61348832	0.50862110	0.43452275

P	6.0	7.0	X	9.0	10.0
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000025	0.00000025	0.00000020	0.00000020	0.00000018
0.50	0.00006211	0.00005376	0.00004735	0.00004237	0.00003832
0.75	0.00120360	0.00104642	0.00092556	0.00082973	0.00075188
1.00	0.00829257	0.00723857	0.00642233	0.00577154	0.00524051
1.25	0.03127913	0.02740255	0.02438114	0.02195999	0.01997633
1.50	0.07873291	0.06919742	0.06172304	0.05570652	0.05075790
1.75	0.14801395	0.13044983	0.11661446	0.10543394	0.09621048
2.00	0.24212074	0.19797981	0.17730510	0.16052314	0.14667851
2.25	0.28955919	0.25624233	0.22980510	0.20832574	0.19051838
2.50	0.32478677	0.29664010	0.26631027	0.24161440	0.22111487
2.75	0.36038971	0.31957906	0.28708720	0.26060212	0.23859680
3.00	0.37242496	0.33039349	0.29690516	0.26959223	0.24688774
3.25	0.37171098	0.33467001	0.30079681	0.27316219	0.25018519
3.50	0.37875408	0.33609974	0.30210072	0.27436060	0.25129384
3.75	0.37920320	0.33650458	0.30247225	0.27470243	0.25161105
4.00	0.37931257	0.33660567	0.30256313	0.27478629	0.25168836
4.25	0.37933534	0.33662641	0.30258220	0.27480394	0.25170475
4.50	0.37933946	0.33663034	0.30258594	0.27480704	0.25170773
4.75	0.37934017	0.33663088	0.30258632	0.27480757	0.25170827
5.00	0.37934017	0.33663076	0.30258644	0.27480763	0.25170827

N =	P	X				
		0.2	0.4	0.6	0.8	1.0
8	0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
	0.25	0.00000095	0.00000065	0.00000049	0.00000039	0.00000033
	0.50	0.00027463	0.00020935	0.00016921	0.00014200	0.00012235
	0.75	0.00613066	0.00496937	0.00417968	0.00360725	0.00317305
	1.00	0.04619315	0.03888338	0.03358483	0.02956345	0.02640533
	1.25	0.18593347	0.16050583	0.14125508	0.12615705	0.11399096
	1.50	0.49218148	0.43243325	0.38578338	0.34831572	0.31752556
	1.75	0.96293914	0.85677195	0.77204829	0.70277202	0.64502817
	2.00	1.50383595	1.3507640	1.22592258	1.12269592	1.03572750
	2.25	1.98986626	1.79749966	1.63996601	1.50836945	1.39667034
	2.50	2.33868885	2.12108040	1.94711425	1.79104710	1.66254711
	2.75	2.54345466	2.31225109	2.12097740	1.95979577	1.82195091
	3.00	2.64309120	2.40575790	2.20907307	2.04307365	1.90090847
	3.25	2.6836149	2.440403076	2.24528333	2.07743454	1.93359375
	3.50	2.69763537	2.45726013	2.24578392	2.08938313	1.94459302
	3.75	2.70173645	2.461113682	2.26153374	2.09290791	1.94836521
	4.00	2.70275497	2.461210957	2.26245975	2.09379387	1.94921303
	4.25	2.70297718	2.46231842	2.26266003	2.09398651	1.94939804
	4.50	2.70301723	2.46236038	2.26269913	2.09402466	1.94943523
	4.75	2.70302391	2.46236324	2.26270350	2.09402752	1.94944000
	5.00	2.70302677	2.46236801	2.26270580	2.09403038	1.94944191

P	X				
	1.5	2.0	3.0	4.0	5.0
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000023	0.00000018	0.00000012	0.00000010	0.00000008
0.50	0.00009099	0.00007232	0.00005134	0.00003679	0.00003249
0.75	0.00243956	0.00198174	0.00144108	0.00113226	0.00093245
1.00	0.02084504	0.01722197	0.01278165	0.01016260	0.00843459
1.25	0.09187567	0.07656360	0.05811632	0.04668584	0.03902020
1.50	0.26016110	0.22040945	0.16887194	0.13689172	0.11510348
1.75	0.53534466	0.45769477	0.35492015	0.28990346	0.24504340
2.00	0.80815269	0.74755841	0.58532935	0.48105961	0.40843362
2.25	1.17923260	1.02088547	0.80527204	0.66510701	0.56660044
2.50	1.41071415	1.22585201	0.97204173	0.80568057	0.68809104
2.75	1.55072975	1.35068798	1.07463074	0.89275944	0.76373023
3.00	1.62063122	1.41339207	1.12663174	0.93717265	0.80249196
3.25	1.64976531	1.43967056	1.14860435	0.95605260	0.81904268
3.50	1.65598745	1.44893932	1.15641403	0.96280009	0.82498181
3.75	1.66303062	1.45171070	1.15876484	0.96484202	0.82678646
4.00	1.66380624	1.45241737	1.15936852	0.96536869	0.82725376
4.25	1.66396809	1.45256901	1.15949917	0.96548223	0.82735491
4.50	1.66399956	1.45259953	1.15952396	0.96550453	0.82737440
4.75	1.66400337	1.45260429	1.15952969	0.96550977	0.82737941
5.00	1.66400719	1.45260334	1.15952872	0.96551073	0.82738024

P	X				
	6.0	7.0	8.0	9.0	10.0
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000006	0.00000006	0.00000005	0.00000004	0.00000004
0.50	0.00002745	0.00002376	0.00002095	0.00001873	0.00001694
0.75	0.000079260	0.000068923	0.000060971	0.000054664	0.000049540
1.00	0.00720895	0.00629437	0.00558575	0.00502055	0.00455523
1.25	0.03351561	0.02937248	0.02614115	0.02355050	0.02142708
1.50	0.09930241	0.08731771	0.07791519	0.07034129	0.06410968
1.75	0.21221703	0.18715191	0.16738498	0.15139627	0.13819695
2.00	0.35486269	0.31372744	0.28114474	0.25469714	0.23279529
2.25	0.49355352	0.43721300	0.39243007	0.35597616	0.32572359
2.50	0.60052311	0.53276247	0.47876275	0.43471372	0.39809442
2.75	0.66737938	0.59266347	0.53301972	0.48429811	0.44374728
3.00	0.70176572	0.62356216	0.56107217	0.50998586	0.46743774
3.25	0.71649873	0.63683730	0.57315207	0.52106750	0.47767353
3.50	0.72180313	0.64162892	0.57752198	0.52508324	0.48138893
3.75	0.72341955	0.64309275	0.57885939	0.52631521	0.48253018
4.00	0.72383916	0.64347410	0.57920843	0.52663672	0.48282784
4.25	0.72393209	0.64355856	0.57928592	0.52670819	0.48289496
4.50	0.72394812	0.64357316	0.57929545	0.52672076	0.48290652
4.75	0.72395271	0.64357704	0.57930315	0.52672434	0.48290992
5.00	0.72395331	0.64357769	0.57930368	0.52672458	0.48290998

P	0.2	0.4	X	0.8	1.0
N = 9					
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000021	0.00000014	0.00000011	0.00000009	0.00000007
0.50	0.00012207	0.00009326	0.00007547	0.00006339	0.00005465
0.75	0.00405902	0.00329740	0.00277735	0.00239935	0.00211210
1.00	0.020438458	0.01306985	0.00947232	0.02597247	0.0211789
1.25	0.020045524	0.017347980	0.015292901	0.13675648	0.12369102
1.50	0.06251103	0.05087423	0.049237460	0.44519502	0.40631759
1.75	1.39313221	1.24301147	1.12249088	1.02350044	0.94069105
2.00	2.40846062	2.16910648	1.97378254	1.81117821	1.67360592
2.25	3.437179564	3.11676979	2.85180092	2.62908554	2.43910885
2.50	4.26307392	3.88238049	3.56578541	3.29797459	3.06826496
2.75	4.79794121	4.38171864	4.0340387	3.73878193	3.48466494
3.00	5.08266926	4.64898491	4.28584385	3.97682858	3.71037674
3.25	5.20826951	4.76782608	4.39826584	4.08349228	3.81184864
3.50	5.25551987	4.81214523	4.44034672	4.12354565	3.85005665
3.75	5.27021503	4.82610893	4.45364475	4.13623810	3.86219692
4.00	5.27414894	4.82985401	4.45722103	4.13965607	3.86547479
4.25	5.27503872	4.83070660	4.45803738	4.14044189	3.86622506
4.50	5.27521324	4.83087158	4.45815569	4.14059639	3.86637688
4.75	5.27522659	4.83088493	4.45820904	4.14060879	3.86639118
5.00	5.27524471	4.83090401	4.45822811	4.14062500	3.86640739

P	1.5	2.0	X	4.0	5.0
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000005	0.00000004	0.00000003	0.00000002	0.00000002
0.50	0.00004064	0.00003235	0.00002298	0.00001782	0.00001455
0.75	0.00162581	0.00132168	0.00096151	0.00075613	0.00062289
1.00	0.01835563	0.01517928	0.01127811	0.00897282	0.00745022
1.25	0.05986836	0.08375460	0.06333286	0.05092285	0.04258121
1.50	0.33360553	0.28302830	0.23032339	0.17628026	0.14832878
1.75	0.78270274	0.57032957	0.52055856	0.42610532	0.36050183
2.00	1.40710545	1.21421814	0.95335060	0.78494287	0.66717374
2.25	2.06592886	1.79401016	1.41996002	1.17534828	1.00277328
2.50	2.61474037	2.27909470	1.81467533	1.50808239	1.29035091
2.75	2.98052406	2.60524654	2.08272839	1.73560619	1.48799133
3.00	3.18034267	2.78450298	2.23139477	1.86255937	1.59883022
3.25	3.27079773	2.86610222	2.29962826	1.92123127	1.65023041
3.50	3.30506992	2.89717102	2.32580757	1.94384956	1.67013931
3.75	3.31601715	2.90714264	2.33427143	1.95119953	1.67663193
4.00	3.31899257	2.90986252	2.33656886	1.95322323	1.67842960
4.25	3.31967545	2.91049250	2.33712959	1.95365911	1.67885399
4.50	3.31980991	2.91061592	2.33723736	1.95379257	1.67893696
4.75	3.31983757	2.91063690	2.33725548	1.95380688	1.67895126
5.00	3.31983948	2.91062832	2.33724689	1.95380116	1.67894459

P	6.0	7.0	X	9.0	10.0
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000001	0.00000001	0.00000001	0.00000001	0.00000001
0.50	0.00001230	0.00001065	0.00000535	0.00000839	0.00000759
0.75	0.00052957	0.00046058	0.00040749	0.00036537	0.00033115
1.00	0.00636948	0.00556260	0.00493718	0.00443820	0.00403082
1.25	0.03658846	0.03207479	0.028525264	0.02572760	0.02341133
1.50	0.12803197	0.11262316	0.1002568	0.09077573	0.08275002
1.75	0.31241673	0.27565557	0.24663872	0.22315073	0.20374870
2.00	0.58016610	0.51325047	0.46018374	0.41706765	0.38134193
2.25	0.87445426	0.77528358	0.69633400	0.63198936	0.57853663
2.50	1.12763743	1.00147057	0.90070754	0.81838596	0.74986398
2.75	1.30236816	1.15799904	1.04249001	0.94796127	0.86916757
3.00	1.40669580	1.24625601	1.12271023	1.02141762	0.93691307
3.25	1.44645119	1.28758240	1.16022587	1.05583477	0.96870404
3.50	1.46423054	1.30364799	1.17487526	1.06929874	0.98115921
3.75	1.47004890	1.30891514	1.17968845	1.07373047	0.98526543
4.00	1.47166252	1.31037598	1.18103027	1.07496738	0.98641258
4.25	1.47204399	1.31072712	1.18134785	1.07526112	0.98668522
4.50	1.47212029	1.31079674	1.18141270	1.07532024	0.98674113
4.75	1.47213268	1.31080914	1.18142319	1.07533073	0.98675025
5.00	1.47213459	1.31081009	1.18142509	1.07533169	0.98675185

N = 10	P	X				
		0.2	0.4	0.6	0.8	1.0
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000005	0.00000003	0.00000002	0.00000002	0.00000002	0.00000002
0.50	0.00005493	0.00004204	0.00003406	0.00002862	0.00002469	0.00002469
0.75	0.00272363	0.00221648	0.00186903	0.00161595	0.00142334	0.00142334
1.00	0.03584117	0.03029019	0.02623474	0.02314016	0.02070025	0.02070025
1.25	0.21997249	0.19067973	0.16831416	0.15066642	0.13638043	0.13638043
1.50	0.81108451	0.71582013	0.64074504	0.58001363	0.52985144	0.52985144
1.75	2.06551933	1.84694099	1.67055334	1.52534580	1.40345478	1.40345478
2.00	3.97225761	3.58630848	3.26968002	3.00500870	2.78034306	2.78034306
2.25	6.15727425	5.59811974	5.13375376	4.74156761	4.40747452	4.40747452
2.50	8.11164556	7.41120720	6.82468891	6.32577801	5.85589214	5.85589214
2.75	9.50955105	8.71644688	8.04871084	7.47810841	6.98445988	6.98445988
3.00	10.32384668	9.48078346	8.76886654	8.15891361	7.62999630	7.62999630
3.25	10.7153769	9.84998512	9.11865262	8.49032021	7.94526577	7.94526577
3.50	10.8723373	9.9985189	9.25923347	8.62460613	8.07338142	8.07338142
3.75	10.9252405	10.04848491	9.30710506	8.67029858	8.11709118	8.11709118
4.00	10.9403439	10.0632334	9.32084274	8.68344307	8.12968622	8.12968622
4.25	10.9440088	10.0667448	9.32420731	8.68667412	8.13278770	8.13278770
4.50	10.9447689	10.0674582	9.32488823	8.68733406	8.13517857	8.13517857
4.75	10.9449053	10.0675898	9.32502365	8.68745136	8.13354301	8.13354301
5.00	10.9449253	10.0676107	9.32503891	8.68747139	8.13356209	8.13356209

P	X				
	1.5	2.0	3.0	4.0	5.0
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000001	0.00000001	0.00000001	0.00000000	0.00000000
0.50	0.00001837	0.00001463	0.00001040	0.00000807	0.00000659
0.75	0.00109671	0.00089209	0.00064971	0.00051092	0.00042099
1.00	0.11638466	0.01355956	0.01008376	0.00802679	0.00666700
1.25	0.11026883	0.09256208	0.07007289	0.05638000	0.04716545
1.50	0.43576062	0.37011248	0.28447515	0.23104733	0.19452399
1.75	1.17009163	1.00348663	0.78126519	0.63972402	0.54163739
2.00	2.34329033	2.02551270	1.59289687	1.31417847	1.11808205
2.25	3.74443817	3.25681114	2.58494759	2.14344025	1.83096481
2.50	5.04204464	4.40594006	3.52008915	2.93178940	2.51246153
2.75	5.99827862	5.25857639	4.22089005	3.52665806	3.02916241
3.00	6.56989861	5.77140421	4.64622402	3.89000702	3.34625816
3.25	6.80579098	6.02478409	4.85811424	4.07207966	3.50587749
3.50	6.96583557	6.12909508	4.94601059	4.14796829	3.57273293
3.75	7.00525856	6.16499805	4.97647381	4.17448711	3.59611797
4.00	7.01667786	6.17543983	4.98539543	4.18227196	3.60301971
4.25	7.01950073	6.17802620	4.98762226	4.18422508	3.60475349
4.50	7.02008724	6.17857552	4.9880861	4.18463612	3.60512257
4.75	7.02019310	6.17867279	4.98817444	4.18470860	3.60518837
5.00	7.02020931	6.17868614	4.98818588	4.18472290	3.60520077

P	X				
	6.0	7.0	8.0	9.0	10.0
0.10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.25	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.50	0.00005577	0.0000482	0.0000425	0.0000380	0.0000344
0.75	0.0035799	0.0031138	0.0027552	0.0024706	0.0022394
1.00	0.00570123	0.00497989	0.00442060	0.00397426	0.00360579
1.25	0.04054044	0.03554776	0.03165005	0.02852279	0.02595799
1.50	0.16797590	0.14780605	0.13196182	0.11518616	0.10866618
1.75	0.46964645	0.41455531	0.37103641	0.33578902	0.30665886
2.00	0.97295594	0.86119559	0.77248216	0.70034426	0.64053357
2.25	1.598111306	1.41785336	1.27416611	1.15693760	1.05947495
2.50	2.19810390	1.95381832	1.75849337	1.59863567	1.46546745
2.75	2.65492630	2.36313629	2.12921810	1.93748474	1.77746105
3.00	2.93624496	2.615593056	2.35873699	2.14765549	1.97129250
3.25	3.07833672	2.74396515	2.47524643	2.25459862	2.07007790
3.50	3.13804436	2.79784298	2.52437552	2.29969311	2.11179256
3.75	3.15899088	2.81688023	2.54177380	2.31571293	2.12663556
4.00	3.16519260	2.82251356	2.54653222	2.32046586	2.13104916
4.25	3.16675854	2.82392883	2.54822922	2.32166862	2.13216305
4.50	3.16708851	2.82423306	2.54851151	2.32192707	2.13240623
4.75	3.16714764	2.82428932	2.54856014	2.32197666	2.13244820
5.00	3.16716003	2.82429495	2.54857159	2.32198238	2.13245487