REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

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21[2.10].—H. ENGELS, *Numerical Quadrature and Cubature*, Academic Press, London, 1980, xiv + 441 pp. Price $74.00.

When I was asked to review this book, I was not completely unfamiliar with its contents. I had read it cursorily and did not have too favorable an opinion of it. The book appeared to be loaded with descriptions of the author's researches and did not give a realistic picture of the state-of-the-art in numerical integration, even up to June 1978, the date the book was submitted for publication as indicated in the Preface. I had also recalled the following sentence which may give some indication of the author's viewpoint: 'Somewhat exotic methods for the approximate calculation of integrals are based on statistical methods using random numbers' (p. 78). Finally, I had had the impression that the book was priced too highly. Thus I had a problem in deciding whether to accept the editor's request to review this book. I accepted despite my initial bias against it since I believe I would be able to give it a fair review. I leave it to the reader to judge whether I have succeeded.

After a thorough reading of this work, I was left with a feeling of sloppiness, tedium, irrelevance and narrowness punctuated by a few bright spots which I shall mention later. Since it is much easier to spot errors in the work of others than in one's own work, I have found not only an abundance of typographical errors but also some errors of substance. One example is the statement on p. 344 essentially repeated on p. 353: 'If $f \in C[-1,1]$, then $f(x)$ possesses a convergent series expansion of the type $f(x) = \sum_{k=0}^{\prime\infty} a_k \omega_k(x)$. The polynomials $\omega_k(x)$ are assumed to be orthogonal with respect to a certain weight function...'. Another example is the statement on p. 371 which states essentially that if the error functional does not have an asymptotic expansion in integer powers of h, then polynomial extrapolation should not be used. There are also many errors in English, both vocabularly and usage, which would make it difficult for a reader not already familiar with the subject matter to follow the text. Thus Engels uses 'edges' in place of 'vertices', 'normed' instead of 'normalized', 'however' in place of 'moreover', 'so that' instead of 'such that', etc. In addition, there are several very problematic statements, of which we give two examples: 'The orthogonal polynomials in two or more dimensions are of comparable importance for cubature formulae to the one-dimensional orthogonal polynomials for quadrature formulae' (p. 239), and 'The requirement of analyticity is not such a serious disadvantage as it might appear, because we use only discrete values of a function and there always exist analytic functions passing

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through these discrete values' (p. 124 in the introduction to Section 3.5: Error bounds without derivatives for quadratures on analytic function spaces). One final example of sloppiness occurs in Section 2.7 on the Monte Carlo method. After a rather unsatisfactory description of this method, the author gives a FORTRAN program implementing this method. Unfortunately, the program does not implement the method given in the text but a different method which, in fact, is better than that given there.

So much for sloppiness. As for tedium, time after time I made notations that certain derivations and treatments were tedious and that certain results and tables were uninteresting. However, it would be too tedious to list examples of these. The following list of topics either unmentioned or barely touched upon is the basis for my criticism of narrowness. Numerical integration of data and of functions with singularities including Cauchy principal value integrals, nonlinear transformation of the independent variable, the epsilon algorithm, the Golub-Welsch method for generating Gaussian abscissas and weights, the fast Fourier transform for use with the Clenshaw-Curtis method, the Patterson extension of the Kronrod scheme and other topics of interest are not mentioned. Adaptive integration, methods for oscillatory integrands, integrals over an infinite range, and sampling methods are only briefly mentioned. (The adaptive scheme proposed by the author is of little merit.) On the other hand, a lot of space is devoted to subjects of little practical importance such as the Davis construction of positive cubature rules, Wilf's optimal formulas, Möller's work on cubature formulas with a minimal number of nodes, equally-weighted quadrature formulas, etc. This does not mean that these subjects should not be treated; only that this should not be done at the expense of those subjects which were omitted.

I shall now briefly list those sections of the book which were of interest inasmuch as they contained material not available in other books on the subject. The section on general Lagrange and Hermite interpolation is quite good as is part of the section on composite cubature rules. Section 3.5 on error bounds on analytic function spaces contains some worthwhile material as does Section 5.3 on implicitly-defined orthogonal polynomials. Also good are the above-mentioned sections on the work of Davis and Möller. Chapter 7 'Refined Interpolatory Quadrature' has quite a few interesting sections and is the most rewarding chapter in the book although it also suffers from the same shortcomings as the other chapters. The titles of these chapters are: 1. Introduction, 2. Construction Principles for Quadrature and Cubature Formulae, 3. Error Analysis for Quadrature and Cubature Formulae, 4. Convergence of Quadrature and Cubature Procedures, 5. Orthogonal Polynomials, 6. Interpolatory Quadrature and Cubature Formulae-Preassigned Nodes or Weights, 8. Non-Interpolatory Quadratures, 9. Auxiliary Material.

The contents of Chapter 9 deserves some further comment. In Table 9.2.1, there is a list of values of the integrals of a set of test functions for testing one-dimensional integration programs. While this list is useful, a more useful list would be of multiple integrals since not many numerical values of multiple integrals, which are not products of one-dimensional integrals, are readily available. Another useful addition would have been a set of families of integrals parametrized by one or two parameters, which have proved useful in comparison studies. There is also a list of published programs which, with some omissions, is up-to-date until 1975, except for one program by the author from 1977. In some cases, I have the impression that the author has not personally inspected the papers he lists, but classifies them, incorrectly, according to their titles. Thus, the programs by Gautschi and by Golub and Welsch should be listed under 'Computation of nodes and weights' rather than under 'Gaussian quadrature programs'. Boland's programs entitled 'Product-type formulae' are not cubature programs but quadrature programs while the two references to Welsch which appear in the list of tables, 'Abscissas and weights for Gregory/Romberg quadrature' belong in the list of programs.

I could report on other amusing and not-so-amusing flaws in the book, but I shall conclude with the following evaluation. For specialists and researchers in the field of numerical integration, this book contains some items of interest. However, the nonspecialist who is interested more in the practical aspects of numerical integration is advised to refer to the standard texts, *Methods of Numerical Integration* by Davis and Rabinowitz and *Approximate Calculation of Multiple Integrals* by Stroud.

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22[2.05].—M. J. D. POWELL, *Approximation Theory and Methods*, Cambridge Univ. Press, New York, 1981, ix + 339 pp., $23\frac{1}{2}$ cm. Price \$57.50 hardcover, \$19.95 paperback.

This book grew out of the material of an undergraduate course in Approximation Theory given by Professor Powell at the University of Cambridge. There are 24 chapters, from 9 to 15 pages in length, and, quoting from the preface, ..."it is possible to speak coherently on each chapter for about an hour...".

A wide range of topics, from classical to current, are covered. The selection of topics agrees with this reviewer; some are treated in more detail like minimax approximation and various topics in spline theory. Here is a list of the chapters: 1. The approximation problem and existence of best approximations. 2. The uniqueness of best approximations. 3. Approximation operators and some approximating functions. 4. Polynomial interpolation. 5. Divided differences. 6. The uniform convergence of polynomial approximations. 7. The theory of minimax approximation. 8. The exchange algorithm. 9. The convergence of the exchange algorithm. 10. Rational approximation by the exchange algorithm. 11. Least squares approximation. 12. Properties of orthogonal polynomials. 13. Approximation to periodic functions. 14. The theory of best L_1 approximation. 15. An example of L_1 approximation and the discrete case. 16. The order of convergence of polynomial approximations. 17. The uniform boundedness theorem. 18. Interpolation by piecewise polynomials. 19. B-splines. 20. Convergence properties of spline approximations. 21. Knot positions and the calculation of spline approximations. 22. The Peano kernel theorem. 23. Natural and perfect splines. 24. Optimal interpolation.

In spite of the topical nature, notation and style are unified throughout the book. A recurrent theme is that of Lebesgue constants. It must not be inferred from the shortness of the chapters that the treatment is fast and loose: It is not. What topics are taken up are exposed in good detail and with rigor. For a lecture course, it ought