to be easy to pull out parts, substitute etc. The condensed mathematical style would probably make this book unsuitable for a first self-study book of the subject.

It is remarkable what an ideal student, with previous knowledge of computer programming, could do after reading this book and working its many exercises. Faced with an approximation problem (not involving differential equations or multi-dimensional functions on general domains), he or she could:

(i) Make a rational choice of method.

(ii) Program it, or use canned programs.

(iii) Furnish meaningful error estimates and insight in the properties and expected behavior of the method.

The second point is not stressed, but there is always given just enough algorithmic detail where it matters.

With the time constraints of an (US) undergraduate curriculum, where typically not more than two courses are devoted to Numerical Analysis, it is not likely that a whole course will be given in Approximation Theory. The demand to include Numerical Linear Algebra, numerical quadrature (to a larger extent than in this book) and numerical solution of integral equations, ordinary and partial differential equations, will preclude this. Therefore, this book can be expected to find its (US) audience among graduate students.

In conclusion, I call this a perfect no-nonsense introduction to Approximation Theory for a mathematically mature audience.

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**23[2.25].**—JET WIMP, *Sequence Transformations and Their Applications*, Mathematics in Science and Engineering, Vol. 154, Academic Press, New York, 1981, xix + 257 pp., $23\frac{1}{2}$ cm. Price $38.50.

It was remarked by Benjamin Disraeli that whenever he wished to learn something of a subject, he wrote a book about it. This principle is widely practiced by writers upon mathematics and computer science; their works are offered less as statements of existing knowledge than as exercises soliciting the appraisal of the informed reader. A summary judgement upon the book under review is that its contents derive more from uncritical reading of recent papers than from profound study.

The scope of the book is very briefly indicated by the following chapter synopses. 1 gives definitions concerning comparison of rates of convergence, and miscellaneous results dealing with some special sequences. 2–4 deal with Toeplitz methods, Richardson extrapolation, special methods using known properties of orthogonal polynomials and other linear methods. 5–8 consider nonlinear algorithms—Aitken's $\delta^2$-process, the $\varepsilon$- and $\rho$-algorithms—and their connections with the algebraic theory of continued fractions. 9 deals briefly with the acceleration of sequences of vectors and with other nonlinear algorithms. 10, 11 deal with a general theory having roughly the following import: most convergence acceleration algorithms produce numbers which may be represented as components in the solutions of sets of linear algebraic equations; the point of each algorithm is that these numbers may alternatively be obtained by use of a simple algebraic recursion; the theory places emphasis upon the replacement of the simple recursion by the solution of a set of linear equations at each stage. 12 concerns acceleration methods based upon statistical considerations. 13 considers analytic transformations of two-dimensional infinite sums. The level of treatment is indicated by the author's prefatory remarks that it was not feasible to include very detailed and computational proofs, and that where he thinks that abstraction confuses rather than elucidates he has left well alone.

The author's choice of level of treatment has the immediately visible consequence that the proofs of many of the given theorems are stated as "*Obvious*." or "*Left to the reader*." or "*Trivial*.". A further consequence, which becomes apparent upon reading, is that the superficial is often preferred to the valuable. For example, one of the earlier theorems stated (attributed to Brezinski) is that if $\{v_j\}$ is a totally monotone sequence with $v_0 < \rho$, $c_i \geqslant 0$ $(i = 0, 1, \ldots)$ and the series $\sum c_i \rho^i$ converges, then the sequence $\sum c_i v_j^i$ is totally monotone. As the author remarks, this result is an obvious consequence of the slightly more general result that if $\{v_{i,j}\}$ $(i = 0, 1, \ldots)$ are all totally monotone, and with all $c_i \geqslant 0$, $\sum c_i v_{i,0}$ converges, then $\sum c_i v_{i,j}$ is totally monotone; the latter result itself being obvious. If results concerning power series and total monotonicity are to be given, perhaps the above might have been replaced by some auxiliary results given in a classic paper by Fejér [1], namely that if $\{c_j\}$ is totally monotone, then for $|\rho| < 1$ and $v_j = \sum_{i=j}^{\infty} c_i \rho^i$, the sequences $\{\mathrm{Re}\, v_j\}$, $\{\mathrm{Im}\, v_j\}$ and $|v_j|^2$ are also totally monotone; these results, not particularly difficult to derive, would interest the good student far more. Reading through the book it is often possible to suggest alternative results more interesting and useful than those stated.

The absence of treatment in depth results in some serious omissions, particularly with regard to acceleration algorithms connected with the analytic theory of continued fractions. This theory is equipped with *a posteriori* error estimates, deriving from the first stage in a convergence proof, and with *a priori* error bounds, deriving from comparison, made in the second stage, of these estimates with terms of a sequence tending to zero. Furthermore, the continued fractions obtained from power series whose coefficients are moments of a bounded nondecreasing function are associated with functions having a positive imaginary part in the upper half-plane, i.e., with solutions, having one sign, of Laplace's equation over a half-plane which can be used as a convenient reference domain to derive results concerning other domains. Naturally such functions have wide application in applied mathematics; in particular, much recent work in theoretical physics is the formulation of simple corollaries to the above convergence theory. As the reviewer has shown [2], classical series of numerical analysis—Newton's interpolation series, Newton's series for the derivative, the Euler-Maclaurin integration series—derived from extensive classes of functions, also generate continued fractions of the above type and may be accelerated with the security of rigorous error bounds. In the book under review, the analytic theory of continued fractions, together with its important applications, are entirely neglected.

Even within the limited frame of reference adopted, there are some significant gaps in the presentation. One of the most effective devices for the transformation of power series whose coefficients are, with alternating sign, moments of a bounded nondecreasing function, is a variant of the $\varepsilon$-algorithm in which a staircase sequence of numbers in the $\varepsilon$-array is taken as the initial sequence for the construction of a further array. For example, the sum of the first six terms of the series $\Sigma\binom{-1/2}{i}(i+1)^{-1} = 2(2^{1/2}-1) = 0.82842\ 71247\ 43\ldots$ is $0.81\ldots$; simple application of the $\varepsilon$-algorithm to these terms yields the estimate $0.82840\ldots$ and repeated application $0.82842\ 71247\ 49\ldots$. This mode of repetition of the $\varepsilon$-algorithm is not mentioned. (The author bases a number of his comparisons of numerical performance upon a recent survey by Ford and Smith in which similar omission occurs.) There are other omissions of the same kind.

Treatments of convergence behavior, where given, are largely concerned with comparisons of rates of convergence of initial and derived sequences. Thus with $\Sigma\, a_i, \Sigma\, b_i$ two convergent series, the second is said to converge faster than the first if $\Sigma_{i=n}\, b_i = o\{\Sigma_{i=n}\, a_i\}$ and so on. The given results might be of some help to the student of mathematics as elementary exercises in the use of the symbols $O$, $o$ before he is ready for more substantial analysis, but they are of little use to the working numerical analyst who wishes to know, for example, the law concealed by the symbol $o$, and the numerical values of the dominant constants in this law.

In essence, the contents of the book reduce to a collection of algorithms, each presented with a motivation, some without error analysis, and some even without convergence proof. The numerical performance of the algorithms considered is for the most part illustrated with respect to standard series, $\Sigma\,(-1)^i(i+1)^{-1}$, $\Sigma\,(i+1)^{-2}$, $\Sigma\,(-1)^i i!$ and so on. Where error analyses and convergence proofs are not given, the reader is thus encouraged to judge the significance of a transformed sequence produced by a method when applied to an example for which the correct limit is unknown, from the appearance of the sequence. This is highly dubious practice. After some experience, every working numerical analyst encounters convergent sequences which initially appear to converge to the wrong limit. Indeed Gautschi [3] gives a nice example of such spurious convergence, and takes the trouble to explain how it arises. Such cases may, of course, be dismissed as examples of bad luck; but if one gambles often enough, one is sure to encounter misfortune.

The subject of the book is of prime importance. The fact that information otherwise to be obtained from billions of iterations of a recursive process can, subject to suitable preliminary theoretical investigation, be extracted with complete security from a half dozen or so iterations, has evident implications in all branches of applied mathematics and in numerical analysis in particular. But it is precisely its wide range of application that makes of convergence acceleration a difficult matter upon which to write. It is required of an author who writes with authority that he should be firmly grounded in the function theoretic bases of the algorithms considered, that he should be conversant with the branches of science in which they are applied, and that he should have sufficient practical experience to distinguish that which is useful from that which is not. In default of such an author, it is to be expected that Disraeli's principle will frequently be invoked; we may comfortably look forward to a number of books upon sequence transformations and their applications.

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1. L. Fejér, "Potenzreihen mit mehrfach monotoner Koeffizientenfolge und ihre Legendre-Polynome," *Proc. Cambridge Philos. Soc.*, v. 31, 1935, pp. 307–316.
2. P. Wynn, "Accélération de la convergence de séries d'opérateurs en analyse numérique," *C. R. Acad. Sci. Paris Ser. A-B*, v. 276A, 1973, pp. 803–806.
3. W. Gautschi, "Anomalous convergence of a continued fraction for ratios of Kummer functions," *Math. Comp.*, v. 31, 1977, pp. 994–999.

**24[4.05.2, 4.10.3, 4.15.3].**—E. P. Doolan, J. J. H. Miller & W. H. A. Schilders, *Uniform Numerical Methods for Problems with Initial and Boundary Layers*, Boole Press, Dublin, 1980, xvi + 324 pp., 24 cm. Price $60.00.

This monograph systematically addresses a relatively new class of numerical methods for singularly perturbed initial and boundary value problems, typical examples of which are

$$\text{(IVP)} \qquad \varepsilon u_x(x) + a(x)u(x) = f(x) \quad \text{for } x > 0,\ u(0) = A,$$

and

$$\text{(BVP)} \qquad \varepsilon u_{xx}(x) + a(x)u_x(x) - b(x)u(x) = f(x) \quad \text{for } 0 < x < 1,$$
$$u(0) = A \text{ and } u(1) = B.$$

In these problems $\varepsilon$ is a positive constant in $(0, 1]$ which may be very small, $a(x) > 0$, $b(x) \geqslant 0$, and $A$ and $B$ are given constants. When $\varepsilon$ is small, near $x = 0$ the solution $u(x)$ of (IVP) and (BVP) displays a boundary layer, i.e., a large gradient.

The presentation is expository while centering around the authors' research on finite difference methods for problems of the type (IVP) and (BVP) whose convergence is *uniform* for $\varepsilon$ in $(0, 1]$ in the sense described below. Many of the results are new and have appeared previously in at most an abbreviated form.

Denoting the approximate solution obtained using a given finite difference scheme on an equally spaced mesh of size $h$ by $u^h$ (having value $u_i^h$ at the $i$th mesh point), the scheme is said to be *uniformly convergent with order p* if the difference between $u^h$ and the exact solution $u$ at all the grid points is bounded by $Ch^p$ where $C$ and $p$ are independent of $h$ *and* $\varepsilon$. Uniformly convergent methods can be expected to be reliable for all values of $\varepsilon$ even on coarse meshes. Such methods may thus also provide a sound starting point for various mesh refinement algorithms.

When $\varepsilon$ is small relative to the mesh size, use of classical "centered" difference methods is quickly seen to lead to instability; e.g., defining $\rho = h/\varepsilon$ and approximating the solution of (IVP) when $a \equiv 1$ and $f \equiv 0$ with

$$\text{(C1)} \qquad \varepsilon(u_{i+1} - u_i)/h + (u_{i+1} + u_i)/2 = 0, \quad u_0 = A,$$

leads to

$$\text{(C2)} \qquad u_{i+1} = (1 - \rho/2)u_i/(1 + \rho/2)$$

which oscillates when $\rho > 2$. This type of instability can be suppressed by the use of "upwinding", e.g.,

$$\text{(W1)} \qquad \varepsilon(u_{i+1} - u_i)/h + u_{i+1} = 0, \quad u_0 = A,$$