

Besides further discussion of topics presented in the earlier edition such as optimization algorithms, the appendices also contain new applications of the ideas of variational inequalities. These include the solution of nonlinear Dirichlet problems, a brief discussion of quasi-variational inequalities, and the numerical simulation of the transonic potential flow of ideal compressible fluids.

With the additional material now included in the present volume, this book is certainly an essential reference for anyone interested in the numerical solution of problems that can be formulated as variational inequalities.

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26[2.05.3].—HERBERT E. SALZER, NORMAN LEVINE & SAUL SERBEN, *Tables for Lagrangian Interpolation Using Chebyshev Points*, manuscript of 54 pages typewritten text + 267 pages of tables, xeroxed and slightly reduced from computer print-out sheets, deposited in the UMT file.

For n -point Lagrangian interpolation for $f(x)$ given at the Chebyshev points $x_{n,i} = -\cos[(2i - 1)\pi/2n]$, $i = 1(1)n$, there are two tables. The first, which is an auxiliary table of $x_{n,i}$ for every n , and $s_{n,i} = \sin[(2i - 1)\pi/2n]$ for the odd values of n , for $n = 2(1)25(5)50(10)100$, to 25 significant figures, is intended primarily for storage in a computer program for calculating the interpolation coefficients in barycentric form. The second, which is the main table, giving the interpolation coefficients themselves, just for $n = 20$, but for $x = -1(0.001)1$, to 20 significant figures, is convenient also for desk calculation with small computers.

The following topics are included in the introductory text: Relation of tables, use of tables for interpolation and quadrature, possible application to equally spaced arguments, advantages in Chebyshev-point interpolation (minimal remainder term, with convergence and stability of coefficients for increasing n), use of tables for Chebyshev economization as an alternative to the methods of C. Lanczos and C. W. Clenshaw, further development of computational methods using interpolation at Chebyshev nodes (especially in numerical integration), description of computation and checking of the tables, and 44 references.

These are some of the more important points in the text which have not been sufficiently noted or emphasized elsewhere in the literature: For practical applications, the advantage in the much smaller upper bound for the classical remainder term is not nearly so important as the *convergence of the interpolation polynomial* as $n \rightarrow \infty$ for the wide class of continuous functions satisfying the Dini-Lipschitz condition in the real interval $[-1, 1]$ (this includes functions with a bounded first derivative which in turn includes analytic functions) *in conjunction with the much smaller interpolation coefficients* (e.g., for $n = 100$ the largest barely exceeds 1, whereas for equal spacing some coefficients exceed 10^{25} ; furthermore, since the sum of the absolute values of the coefficients $\leq 1 + (2/\pi)\ln n$, the factor for total round-off error is < 4). On the basis of the preceding remarks, instead of the global methods of Lanczos which employ the properties of the Chebyshev polynomials to

produce coefficients of an economized polynomial, or of Clenshaw which operate with the coefficients in Chebyshev series expansions, here functional values $f(x_{n,i})$ replace the polynomial or Chebyshev coefficients, for use over the entire range in x (normalized to $[-1, 1]$). After all operations and calculations pertaining to any of a wide class of problems have been completed, we end up with a skeletal set of final answers $f(x_{n,i})$, $i = 1(1)n$, from which $f(x)$ is found immediately by using these tables which are capable of *global interpolation*.

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27[9.00].—M. I. KNOPP, (Editor), *Analytic Number Theory* (Proc. Conf. held at Temple University, Philadelphia, May 12–15, 1980), Lecture Notes in Math., vol. 899, Springer-Verlag, Berlin and New York, 1981, x + 478 pp., 22 cm. Price \$24.50.

The conference mentioned in the title was held on the occasion of the proforma retirement of Emil Grosswald.

The volume contains detailed versions of most of the lectures given at the conference and covers a wide range of subjects in analytic number theory. Of particular interest from the standpoint of computation are the following six articles, for which we include capsule reviews:

(1) Ronald Alter, “Computations and generalizations of a remark of Ramanujan.” This paper presents extensive tables of $r(m, n, s)$, the smallest positive integer that can be expressed as a sum of m positive n th powers in s different ways.

(2) Robert J. Anderson and Harold Stark, “Oscillation theorems.” The authors give an illuminating discussion of oscillation theorems for the sum-functions of some familiar arithmetic functions; specifically, they discuss various methods for obtaining numerical estimates for the \limsup and \liminf of $x^{-1/2}M(x)$, where $M(x) = \sum_{n \leq x} \mu(n)$ and μ denotes the Möbius function.

(3) Harold G. Diamond and Kevin S. McCurley, “Constructive elementary estimates for $M(x)$.” The paper shows how arguments akin to those of Chebyshev can be combined with a finite amount of computation to produce elementary numerical upper estimates of very small size for $\limsup x^{-1} |M(x)|$; needless to say, the prime number theorem implies that this \limsup is actually zero.

(4) Steven M. Gonek, “The zeros of Hurwitz’s zeta function on $\sigma = \frac{1}{2}$.” The author shows that for certain rational values of x the proportion of zeros of $\zeta(s, \alpha)$, which have real part $\frac{1}{2}$, is definitely less than one.

(5) Peter Hagis, Jr., “On the second largest prime divisor of an odd perfect number.” On the basis of extensive computer calculations and searches the paper proves that the prime mentioned in the title must be greater than 1000, under the assumption that odd perfect numbers exist.

(6) Julia Mueller, “Gaps between consecutive zeta zeros.” Assuming the Riemann Hypothesis, the author proves that, if the zeros of the Riemann zeta function in the