MATHEMATICS OF COMPUTATION
VOLUME 40, NUMBER 161
JANUARY 1983, PAGES 405-411

Odd Perfect Numbers Not Divisible By 3. II

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Abstract. We prove that odd perfect numbers not divisible by 3 have at least eleven distinct prime factors.

1. N is called a perfect number if $\sigma(N) = 2N$, where $\sigma(N)$ is the sum of positive divisors of N. Twenty-seven even perfect numbers are known; however, no odd perfect (OP) numbers have been found.

Suppose N is OP and $\omega(N)$ is the number of distinct prime factors of N. Gradstein (1925), Kühnel (1949), Weber (1951), and the author (1978, [5]) proved that $\omega(N) \geq 6$. Pomerance (1972, [7]) and Robbins (1972) proved that $\omega(N) \geq 7$. Hagis (1975, [2]) and Chein (1978, [1]) proved that $\omega(N) \geq 8$.

Hagis and McDaniel [3] proved that the largest prime factor of $N \geq 100129$, and Pomerance [8] proved that the second largest prime factor of $N \geq 139$.

If $3 \nmid N$, then Kanold (1949) proved that $\omega(N) \geq 9$, and the author (1977, [4]) proved that $\omega(N) \geq 10$.

In this paper we prove

THEOREM. *If N is OP and $3 \nmid N$, then $\omega(N) \geq 11$.*

2. In the remainder of this paper we assume that N is OP and

$$N = \prod_{i=1}^{10} p_i^{a_i},$$

where p_i's are primes, $5 \leq p_1 < \cdots < p_{10}$ and a_i's are positive integers, and we will get a contradiction. We write $p_i^{a_i} \| N$ and $a_i = V_{p_i}(N)$.

The following lemmas were proved in [4] and [7]:

LEMMA 1. *Suppose $p^a \| N$. Then*
(a) *All a's are even except for one a in which case $a \equiv p \equiv 1(4)$. We write π for p.*
(b) *If $p \equiv 1\ (3)$, $a \not\equiv 2\ (3)$.*
(c) *If $p \equiv 1\ (4)$ and $p \equiv 2\ (3)$, then $p \neq \pi$.*

LEMMA 2. *Suppose $q = 5$ or 17 and $p^a \| N$. Then*

$$V_q(\sigma(p^a)) = \begin{cases} V_q(a+1) & \text{if } p \equiv 1(q), \\ V_q(p+1) + V_q(a+1) & \text{if } p \equiv -1(q) \text{ and } p = \pi, \\ 0 & \text{otherwise}. \end{cases}$$

Received November 2, 1981; revised June 21, 1982.
1980 *Mathematics Subject Classification*. Primary 10A20.
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LEMMA 3. *Suppose $p^a \| N$, q is a prime and $q^b \| a+1$. Then N is divisible by at least c distinct primes $\equiv 1(q)$ other than p, where $c = b$ if $q^b = a+1$, and $c = 2b$ if $q^b \neq a+1$.*

The proof of the next lemma is similar to that of Lemma 6 in [4].

LEMMA 4. $p_1 = 5$, $p_2 = 7$, $p_3 = 11$, $p_4 \leqslant 17$, $p_5 \leqslant 23$, $p_6 \leqslant 37$, $p_7 \leqslant 107$, $p_9 \geqslant 139$, $p_{10} \geqslant 100129$. *If $p_7 \geqslant 103$, then $p_8 \leqslant 113$.*

LEMMA 5. *Suppose p, q are odd primes, a, b are positive integers, $p^b \mid q+1$, $p \geqslant 5$ and $2b \geqslant a$. Then $q \nmid \sigma(p^a)$.*

Proof. Since p and q are odd primes, $q \geqslant 2p^b - 1$. Suppose $\sigma(p^a) = mq$ for some integer m. Then $a \geqslant b$ and

$$\sigma(p^{b-1}) + m \equiv \sigma(p^a) + m = m(q+1) \equiv o(p^b).$$

Hence

$$m \geqslant p^b - \sigma(p^{b-1}) = (p^{b+1} - 2p^b + 1)/(p-1),$$

and

$$\begin{aligned}\sigma(p^a) = mq &\geqslant (2p^b - 1)(p^{b+1} - 2p^b + 1)/(p-1)\\ &= (2p^{2b+1} - 4p^{2b} - p^{b+1} + 4p^b - 1)/(p-1)\\ &> (p^{2b+1} - 1)/(p-1) = \sigma(p^{2b}) \geqslant \sigma(p^a),\end{aligned}$$

because $p \geqslant 5$ and $2b \geqslant a$, a contradiction. Q.E.D.

Remark. Lemma 5 also holds if $p = 3$ and $2b > a$.

The next lemma is due to Hagis.

LEMMA 6. *Suppose $p = 5$ or 17 and p^a is a component of an OP number. Then $\sigma(p^a)$ has at least one prime factor $\geqslant$ 100129 except*

(a) *if* $p = 5$, $a = 1, 2, 4, 5, 6, 8, 9, 13, 14, 17, 26, 29$.

(b) *If* $p = 17$, $a = 1, 2, 4, 5, 9$.

COROLLARY 6. *Suppose $p = 5$ or 17 and $p^a \| N$. Then $\sigma(p^a)$ has at least one prime factor $\geqslant$ 100129 except*

(a) *if* $p = 5$, $a = 2, 4, 6, 8$,

(b) *if* $p = 17$, $a = 2, 4$.

Proof. We can easily show that $5^{14} \not\| N$ and $5^{26} \not\| N$ because $\sigma(5^{14}) = 11 \cdot 13 \cdot 71 \cdot 181 \cdot 1741$ and $\sigma(5^{26}) = 19 \cdot 31 \cdot 109 \cdot 271 \cdot 829 \cdot 4159 \cdot 31051$. Then Corollary 6 follows from Lemmas 1 and 6. Q.E.D.

LEMMA 7. *If $17^a \| N$ and $a \geqslant 8$, then $p_9 \geqslant 100129$, $p_{10} \geqslant 2 \cdot 17^{a-3} - 1 > 2 \cdot 10^6$, and $17^{a-3} \mid \pi + 1$.*

Proof. If p is a prime and $p \leqslant 113$, then $p \not\equiv \pm 1$ (17) except for $p = 103$. Hence by Lemmas 1, 2 and 4 if $17 \mid \sigma(p_i^{a_i})$ for $1 \leqslant i \leqslant 7$, then $i = 7$, $p_7 = 103$ and $17 \nmid \sigma(p_8^{a_8})$.

Suppose $p_7 \neq 103$. Then $17^a \mid \sigma(p_8^{a_8} p_9^{a_9} p_{10}^{a_{10}})$. Since $a \geqslant 8$, $17^3 \mid \sigma(p_i^{a_i})$ for some $8 \leqslant i \leqslant 10$. If $p_i \equiv 1$ (17), then $17^3 \mid a_i + 1$, and by Lemma 3 N would have at least three more primes $\equiv 1$ (17), a contradiction. Hence $p_i \equiv -1$ (17) and $p_i = \pi$. Then by the same lemma $17^2 \nmid \sigma(p_j^{a_j})$ for $j \neq i$, $8 \leqslant j \leqslant 10$, $17^{a-2} \mid \sigma(p_i^{a_i})$, $17^2 \nmid a_i + 1$, and $17^{a-3} \mid p_i + 1$ by Lemma 2. By Lemma 5 $p_i \nmid \sigma(17^a)$, and by Corollary 6 $\sigma(17^a)$ has at least one prime factor $\geqslant 100129$.

The same arguments hold if $p_7 = 103$ because $17 \nmid \sigma(p_8^{a_8})$. Q.E.D.

LEMMA 8. *If $5^a \parallel N$ and $a \geqslant 14$, then $a \geqslant 16$, $p_9 \geqslant 100129$ and $p_{10} \geqslant 579281$.*

Proof. We showed that $a \neq 14$ in the proof of Corollary 6. Suppose $p^b \parallel N$, $p \equiv 1$ (5) and $p \leqslant 107$. Then $p = 11, 31, 41, 61, 71$, or 101. If $5 \mid \sigma(p^b)$, then by Lemma 2 $\sigma(p^4) \mid \sigma(p^b)$. Since $131 \cdot 21491 \mid \sigma(61^4)$, $211 \cdot 2221 \mid \sigma(71^4)$ and $31 \cdot 391 \cdot 1381 \mid \sigma(101^4)$, it is easy to show that if $5 \mid \sigma(p^b)$, $p \neq 61, 71$, or 101.

Suppose $5 \mid \sigma(41^b)$. Then $579281 \mid \sigma(41^4)$. Since the order of 5 mod 579281 is 72410 and a is even, $579281 \nmid \sigma(5^a)$, and $\sigma(5^a)$ has at least one prime factor $\geqslant 100129$ by Corollary 6. Hence we may assume that $5 \nmid \sigma(41^b)$.

Since $3001 \cdot 3221 \cdot 24151 \mid \sigma(11^{24})$ and $101 \cdot 4951 \cdot 17351 \mid \sigma(31^{24})$, $5^2 \nmid \sigma(11^b)$ and $5^2 \nmid \sigma(31^b)$.

Suppose $5 \mid \sigma(11^b)$. Then $3221 \mid \sigma(11^4)$, and if $3221^c \parallel N$, $5^2 \nmid \sigma(3221^c)$ because $151 \cdot 601 \cdot 1301 \cdot 1601 \mid \sigma(3221^{24})$. Similarly, if $5 \mid \sigma(31^b)$, then $17351 \mid \sigma(31^4)$, and if $17351^c \parallel N$, $5^2 \nmid \sigma(17351^c)$ because $101 \cdot 2351 \mid \sigma(17351^{24})$.

Suppose $p^b \parallel N$, $p \equiv -1$ (5), and $p \leqslant 107$. Then $p = 19, 29, 59, 79$, or 89, and by Lemma 1 $p \neq \pi$. Hence by Lemma 2 $5 \nmid \sigma(p^b)$.

In summary if $p^b \parallel N$, $p \leqslant 107$, and if $5 \mid \sigma(p^b)$, then $p = 11$ or 31, in which case $q^c \parallel N$ where $q = 3221$ or 17351 and $5^2 \nmid \sigma(q^c)$.

Now we will show that $5^{a-8} \mid \pi + 1$. Suppose three $p_i \equiv 1$ (5) for $1 \leqslant i \leqslant 7$. Then $p_3 = 11$, $p_6 = 31$ and $p_7 = 41$, and it is easy to show that $41 < p_8 \leqslant 61$. Hence $5 \nmid \sigma(p_8^{a_8})$. Since $p_{10} \geqslant 100129$, the above summary shows that $5^2 \nmid \sigma(\prod_{i=1}^{8} p_i^{a_i})$. Suppose $5^a \parallel \sigma(p_9^{a_9} p_{10}^{a_{10}})$. By a similar argument used in the proof of Lemma 7 we have for $i = 9$ or 10 $5^{a-4} \mid \sigma(p_i^{a_i})$, $p_i = \pi$, and $5^{a-6} \mid \pi + 1$. Suppose $5^{a-1} \parallel \sigma(p_9^{a_9} p_{10}^{a_{10}})$. Then $p_9 = 3221$ or 17351 and $5^2 \nmid \sigma(p_9^{a_9})$, $5^{a-2} \mid \sigma(p_{10}^{a_{10}})$, $p_{10} = \pi$, and $5^{a-4} \mid \pi + 1$.

Similar arguments show that if two $p_i \equiv 1$ (5) for $1 \leqslant i \leqslant 7$, then $5^{a-8} \mid \pi + 1$ and that if $p_i \equiv 1$ (5) for $1 \leqslant i \leqslant 7$, then $5^{a-5} \mid \pi + 1$.

Since $a \geqslant 16$ and $5^{a-8} \mid \pi + 1$, $\pi \nmid \sigma(5^a)$ by Lemma 5, $\pi \geqslant 2 \cdot 5^{a-8} - 1 > 7 \cdot 10^5 > 579281$, and Lemma 8 follows from Corollary 6. Q.E.D.

COROLLARY 8. *Suppose $5^a \parallel N$, $a \geqslant 14$, and $579281 \nmid N$ if $41 \mid N$. Then $a \geqslant 16$, $p_9 \geqslant 100129$, $p_{10} \geqslant 2 \cdot 5^{a-8} - 1 > 7 \cdot 10^5$, and $5^{a-8} \mid \pi + 1$.*

The next lemma is due to McDaniel [6].

LEMMA 9. *Suppose $a \geqslant 2$, $a + 1$ is a prime, and p is a prime.*

(a) *If $p^2 \mid \sigma(5^a)$, then $p > 2^{29}$.*

(b) *If $p^2 \mid \sigma(17^a)$, then $p > 2^{27}$ or $p = 48947$.*

LEMMA 10. *Suppose $p^a \parallel N$, $q \mid \sigma(p^b)$, $b + 1 \mid a + 1$, $q \leqslant 107$, and q, $b + 1$ are primes. Then*

(a) *If $p = 5$, $q = 11, 31, 59$, or 71.*

(b) *If $p = 17$, $q = 47, 59$, or 83.*

Proof. Suppose $p = 5$ or 17, and d is the order of p mod q. Then $p^d \equiv 1$ (q), and $d \mid b + 1$. Since $p \not\equiv 1$ (q), $d > 1$, and $d = b + 1$ because $b + 1$ is an odd prime. The order d is not an odd prime except for those q stated above. Q.E.D.

LEMMA 11. *Suppose* $17^a \parallel N$, $a \geqslant 8$, *and* $307 \nmid N$. *If* $p_8 < 1000$, *then* $a \geqslant 10$, $p_9 \geqslant 25646167$, *and* $p_{10} > 8 \cdot 10^8$.

Proof. Since $307 \mid \sigma(17^8)$, $a \geqslant 10$, and by Lemma 7, $17^{a-3} \mid \pi + 1$ and $p_{10} \geqslant 2 \cdot 17^{a-3} - 1 > 8 \cdot 10^8$. Suppose $p_8 < 1000$ and $100129 \leqslant p_9 < 25646167 < 2^{27}$. Choose b such that $b+1$ is a prime and $b + 1 \mid a + 1$. Then $\sigma(17^b) \mid \sigma(17^a)$, and $b \neq 2$, 4, or 6 because $307 \mid \sigma(17^2)$, $88741 \mid \sigma(17^4)$ and $25646167 \mid \sigma(17^6)$. Hence $b \geqslant 10$. If $1 \leqslant i \leqslant 7$ and $p_i \mid \sigma(17^b)$, then by Lemmas 4, 10 $i = 7$ and $p_7 = 47$, 59, or 83. Since $\pi = p_{10} \nmid \sigma(17^a)$ by Lemma 5, we have $\sigma(17^b) \mid p_7 p_8 p_9$ by Lemma 9. Then $\sigma(17^{10}) \leqslant 83 \cdot 1000 \cdot p_9$, or $p_9 > 25646167$, a contradiction. Hence $p_9 \geqslant 25646167$. Q.E.D.

COROLLARY 11. *If* $p_7 \leqslant 29$ *and* $p_8 < 6203$, *then* $a \geqslant 10$, $p_9 \geqslant 25646167$, *and* $p_{10} > 8 \cdot 10^8$.

Proof. As in Lemma 11 $\sigma(17^b) \mid p_8 p_9$. Then $\sigma(17^{10}) \leqslant 6203 \cdot p_9$, or $p_9 > 25646167$, a contradiction. Q.E.D.

LEMMA 12. *Suppose* $5^a \parallel N$, $a \geqslant 14$, $579281 \nmid N$ *if* $41 \mid N$, *and* $p \nmid N$ *if* $p = 31$, 71, 191, 409, *or* 19531. *If* $p_8 \leqslant 41$, *then* $a \geqslant 22$, $p_9 \geqslant 12207031$, *and* $p_{10} \geqslant 2 \cdot 5^{a-8} - 1 > 10^{10}$.

Proof. $a \geqslant 22$ because $a \neq 14$ as before, $409 \mid \sigma(5^{16})$, $191 \mid \sigma(5^{18})$, and $19531 \mid \sigma(5^{20})$. The rest of the proof is similar to that of Lemma 11 using $\sigma(5^{10}) = 12207031$ and $\sigma(5^{12}) = 307175781$. Q.E.D.

COROLLARY 12. *If* $p_7 \leqslant 29$ *and* $p_8 < 6203$, *then* $a \geqslant 22$, $p_9 \geqslant 12207031$, *and* $p_{10} \geqslant 2 \cdot 5^{a-8} - 1 > 10^{10}$.

LEMMA 13. *Suppose* $5^a \parallel N$, $a = 10$ *or* 12, $p_8 \leqslant 151$, $p_9 > 3011$, *at most two* $p_i \equiv 1$ (5) *for* $1 \leqslant i \leqslant 8$, $p_i = 11$, 31, 41, *or* 151 *if* $p_i \equiv 1$ (5) *and* $1 \leqslant i \leqslant 8$, $p_i = 19$, 29, 59, 79, 89, 109, *or* 149 *if* $p_i \equiv -1$ (5) *and* $1 \leqslant i \leqslant 8$, $p \nmid N$ *if* $p = 131$, 3221, *or* 17351, *and* $579281 \nmid N$ *if* $41 \mid N$. *Then* $p_9 \geqslant 3 \cdot 10^6$, *and* $p_{10} \geqslant 12207031$.

Proof. Suppose $3011 < p_9 < 3 \cdot 10^6$. Then $p_{10} = \sigma(5^a) = 12207031$ or 305175781, and $5 \nmid \sigma(p_{10}^{a_{10}})$ because $131 \mid \sigma(12207031^4)$ and $3011 \mid \sigma(305175781^4)$. Suppose $5 \mid \sigma(p_i^{a_i})$, $1 \leqslant i \leqslant 8$. Since $3221 \mid \sigma(11^4)$, $17351 \mid \sigma(31^4)$, $579281 \mid \sigma(41^4)$ and $104670301 \mid \sigma(151^4)$, $p_i \not\equiv 1$ (5). Since $p_i \neq \pi$ if $p_i = 19$, 29, 59, 79, 89, and 149 by Lemma 1, we have $p_8 = 109 = \pi$ by Lemmas 2, 4. If $5^4 \mid \sigma(p_8^{a_8})$, then $5^3 \mid a_8 + 1$ by Lemma 2, and N would have at least six prime factors $\equiv 1$ (5), a contradiction. Hence $5^4 \nmid \sigma(p_8^{a_8})$, and so $5^{a-3} \mid \sigma(p_9^{a_9})$, $p_9 \neq \pi$, $p_9 \equiv 1$ (5) and N would have at least 7 more prime factors $\equiv 1$ (5), a contradiction. Hence $5 \nmid \sigma(p_i^{a_i})$ for $1 \leqslant i \leqslant 8$.

Then $5^a \mid \sigma(p_9^{a_9})$, $p_9 = \pi$, $5^2 \nmid a_9 + 1$, $5^{a-1} \mid p_9 + 1$, and $p_9 \geqslant 2 \cdot 5^{a-1} - 1 > 3 \cdot 10^6$, a contradiction. Q.E.D.

Definition. Suppose $M = \prod_{i=1}^r p_i^{a_i}$. Then

$$S(M) = \sigma(M)/M,$$

$$a(p_i) = \min\Big\{a_i \mid p_i^{a^i+1} > 10^{10} \text{ where } p_i^a \text{ satisfies the restrictions implied by Lemma 1}\Big\},$$

$$b_i = \begin{cases} a_i & \text{if } a_i < a(p_i), \\ a(p_i) & \text{if } a_i \geqslant a(p_i). \end{cases}$$

LEMMA 14. *Suppose* $M = \prod_{i=1}^{10} p_i^{b_i}$. *Then* $0 \leqslant \log 2 - \log S(M) < 10 \cdot 10^{-10}$.

Proof. Suppose $p^a \| N$ and $a \geqslant a(p)$. Then

$$\begin{aligned} 0 &\leqslant \log S(p^a) - \log S(p^{a(p)}) < \log \frac{p}{p-1} - \log \frac{p^{a(p)+1}-1}{p^{a(p)}(p-1)} \\ &= \log \frac{p^{a(p)+1}}{p^{a(p)+1}-1} = \log\left(1 + \frac{1}{p^{a(p)+1}-1}\right) \\ &< \frac{1}{p^{a(p)+1}-1} \leqslant 10^{-10}. \end{aligned}$$

Hence

$$\begin{aligned} 0 &\leqslant \log S(N) - \log S(M) \\ &\leqslant \sum_{i=1}^{10} |\log S(p_i^{a_i}) - \log S(p_i^{b_i})| < 10 \cdot 10^{-10}. \end{aligned}$$

Q.E.D.

The proof of the next two lemmas is easy.

LEMMA 15. *If q is a prime, $q \mid \sigma(p_i^{a_i})$ for some $1 \leqslant i \leqslant 7$ with $a_i < a(p_i)$, and if $q \leqslant p_7$, then $q = 2$ or $q = p_i$ for some $1 \leqslant i \leqslant 7$.*

LEMMA 16. *Suppose $M = \prod_{i=1}^{7} p_i^{b_i}$ and $L = M \cdot \prod_{j=1}^{r} q_j^{c_j}$ where q_j is a prime, $q_j > p_7$, $q_j \mid \sigma(p_i^{b_i})$ for some $1 \leqslant i \leqslant 7$ with $b_i < a(p_i)$, $q_1 < \cdots < q_r$, and c_j is the minimum allowable power of q_j determined by Lemma 1. If there is no such q_j, then $r = 0$ and the product is defined to be 1. Then*

(a) $r \leqslant 3$ *and* $\log S(L) \leqslant \log 2$.

(b) *If* $r = 3$, *then* $p_8 = q_1, p_9 = q_2, p_{10} = q_3$ *and*

$$\log 2 < \log S(M) + 7 \cdot 10^{-10} + \sum_{j=1}^{3} \log q_j/(q_j - 1).$$

(c) *If* $r = 2$ *and* $q_2 < 100129$, *then* $p_8 = q_1, p_9 = q_2$ *and*

$$\log 2 < \log S(M) + 7 \cdot 10^{-10} + \sum_{j=1}^{2} \log q_j/(q_j - 1) + \log 100129/100128.$$

LEMMA 17. $p_8 < 3011$.

Proof. Suppose $p_8 \geqslant 3011$. We used a computer (PDP 11/70 at the University of Toledo) to find $M = \prod_{i=1}^{7} p_i^{b_i}$ satisfying Lemmas 1, 4, 15, 16, $\log S(M) < \log 2$, and

$$\begin{aligned} \log 2 < \log S(M) &+ 7 \cdot 10^{-10} + \log 3011/3010 \\ &+ \log 3019/3018 + \log 100129/100128. \end{aligned}$$

The results were:

$$5^{14}7^{12}11^{10}13^{9}17^{8}23^{a_6}29^{6},$$
$$5^{14}7^{12}11^{10}13^{9}17^{6}23^{a_6}29^{6},$$
$$5^{14}7^{12}11^{10}13^{6}17^{8}23^{a_6}29^{6},$$
$$5^{12}7^{12}11^{10}13^{9}17^{8}23^{a_6}29^{6},$$
$$5^{10}7^{12}11^{10}13^{9}17^{8}23^{a_6}29^{6},$$

where $a_6 = 6$ or 8. Since

$$\frac{5}{4}\,\frac{7}{6}\,\frac{11}{10}\,\frac{13}{12}\,\frac{17}{16}\,\frac{23}{22}\,\frac{29}{28}\,\frac{6203}{6202}\,\frac{6211}{6210}\,\frac{100129}{100128} < 2,$$

$3011 \leqslant p_8 < 6203$. By Corollaries 11 and 12 $p_9 \geqslant \min\{25646167, 12207031\}$ and $p_{10} \geqslant \min\{8 \cdot 10^8, 10^{10}\}$. Then N is not OP because

$$\frac{5}{4}\,\frac{7}{6}\,\frac{11}{10}\,\frac{13}{12}\,\frac{17}{16}\,\frac{23}{22}\,\frac{29}{28}\,\frac{3011}{3010}\,\frac{12207031}{12207030}\,\frac{800000000}{799999999} < 2. \quad \text{Q.E.D.}$$

The proof of the next lemma is also easy.

LEMMA 18. *Suppose* $M = \prod_{i=1}^{9} p_i^{b_i}$, $q = \max\{p \mid p$ *is a prime and* $\log S(M) + \log S(p^a) \geqslant \log 2$ *where a is the minimum allowable power of p*$\}$ *and* $r = \min\{p \mid p$ *is a prime and* $\log S(M) + 9 \cdot 10^{-9} + \log p/(p-1) < \log 2\}$. *Then* $q < p_{10} < r$; *in particular, if there are no primes between q and r, N is not OP.*

LEMMA 19. $p_9 < 3011$.

Proof. Suppose $p_8 < 3011 \leqslant p_9$. We used a computer to find $M = \prod_{i=1}^{8} p_i^{b_i}$ satisfying Lemmas 1, 4, 7, 8, 15, 16, $\log S(M) < \log 2$, and

$$\log 2 < \log S(M) + 8 \cdot 10^{-10} + \log 3011/3010 + \log 100129/100128.$$

There were seventy-two such M's. However, none of them satisfied Lemmas 11, 12 and 13 except

$$5^{2}7^{12}11^{10}13^{10}19^{10}23^{8}31^{6}59^{6}.$$

It is easy to show that $7753 \leqslant p_9 \leqslant 8389$, $a_2 \geqslant 22$ ($\sigma(7^{12})$ is a prime), $a_3 \geqslant 16$, $a_4 \geqslant 16$, $a_5 = 10$ or $\geqslant 16$ (if $\sigma(19^{10})$ is a prime, $a_5 \geqslant 16$), $a_6 \geqslant 12$, $a_7 \geqslant 12$, and $a_8 \geqslant 12$. Then for each p_9 with $7753 \leqslant p_9 \leqslant 8389$ Lemma 18 is not satisfied. Hence N is not OP. Q.E.D.

Proof of Theorem. By Lemma 19 $p_9 < 3011$. We used a computer to find $M = \prod_{i=1}^{9} p_i^{b_i}$ satisfying Lemmas 1, 4, 7, 8, 15, 16, $\log S(M) < \log 2$, and

$$\log 2 < \log S(M) + 9 \cdot 10^{-10} + \log 100129/100129.$$

There were thirty-nine such M's; however, none of them satisfied Lemma 18. Hence N is not OP. Q.E.D.

Acknowledgement. The author would like to thank Professor P. Hagis, Jr. for supplying the table in Lemma 6.

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